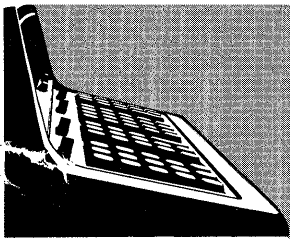
HEWLETT-PACKARD CALCULATOR

Model 9100B

Program Library



INTRODUCTION TO 9100B CALCULATOR

Congratulations, you have just become the proud owner of a new Hewlett-Packard 9100B Calculator. This calculator is a big brother to the successful Hewlett-Packard 9100A. The 9100B however, has several features which more than triple its computing power, these being:

1. A TWO PAGE memory allowing up to 392 program steps or 32 data storage registers.

2. A true SUBROUTINE capability permitting instant access to subroutines from any point in a program.

3. A convenient X ◄— ( ) operator allowing rapid data recall to the X register.

4. A STEP PROGRAM "dual display" greatly simplifying program editing and modification.

A detailed explanation of the use of these features is found in the 9100B Operating and Programming Manual (Hewlett-Packard Part No. 09100-90021) supplied with your calculator.

We hope you find the 9100B Calculator and Library a useful tool and guide for your computational work. To better serve you and other 9100 Calculator users, we welcome you to send us program solutions you have written. Please write them up in a format similar to the programs furnished in the Program Library and send them to us. Also send in any comments you might have on the 9100B Calculator and Program Library to our Applications Group. We at Hewlett-Packard look forward to serving you.

Sincerely,

HEWLETT-PACKARD COMPANY

Dave Cole
Applications

P.O. Box 301
Loveland, Colorado 80537

INTRODUCTION TO LIBRARY

This Program Library is intended to illustrate practical problem solutions available with the 9100B Calculator. Approximately two thirds of the programs in this library are compatible with the 9100A Calculator. Those programs which are applicable only to the "B" calculator will have a 9100B ONLY above the program number.

Many programs are short enough so that they may be incorporated as subroutines in other programs. Each program consists of a description, equations (with references), examples and a list of the program steps. The programs have been placed in a three-ring binder with classification dividers so that changes, additions and reorganizations can be easily made. Each classification has a corresponding part number and each program within a category is numbered according to the classification part number.

In each Program Library Classification is found a listing of the programs in that section. The first grouping contains 9100A programs (with "A" program numbers). The second grouping contains "B" only programs for that classification. An inspection of the first group program numbers will disclose that certain "A" programs have been removed from the "B" Library. These deleted programs have been replaced by more extensive "B" programs.

PROGRAM LIBRARY CLASSIFICATIONS

HP Part No.	
09100-70000	Mathematics
09100-70800	Statistics
09100-71000	Electronics
09100-72000	Mechanics
09100-73000	Business
09100-73200	Physics
09100-73700	Thermodynamics
09100-74000	Surveying
09100-74200	Structures
09100-75000	Fluid Mechanics
09100-75200	Life Sciences
09100-75500	Chemical
09100-75800	Secondary Education
09100-76000	Miscellaneous

You are invited to submit programs for inclusion in the Hewlett-Packard KEYBOARD, a periodic publication which contains useful information about the Hewlett-Packard 9100 Calculator systems. Submitted programs having a broad appeal or illustrating ingenious programming techniques will be included in the KEYBOARD with credit.

Please return the Program Library card (in the pocket on the rear cover) indicating the Program Library disciplines in which you are interested.

The following accessories are available:

HP Part No.	
09100-90020	Five Programming Pads
09100-90021	Operating and Programming Manual
09100-90022	9100B Program Library
09100-90024	Diagnostic Card and Envelope
5060-5919	Box of Ten Magnetic Program Cards
4040-0350	Dust Cover
9320-1183	Pull-Out Card (English)
9320-1184	Pull-Out Card (French)
9320-1185	Pull-Out Card (German)
9320-1186	Pull-Out Card (Italian)
9320-1187	Pull-Out Card (Spanish

Program Library Usage

To facilitate the use of the 9100B Program Library, brief discussions of:

Manual Program Entry,
Program Loading from Magnetic Card,
and 9100B Peripherals

are included here. The Operating and Programming Manual (HP Part No. 09100-90021) covers these topics in greater detail and should be consulted for further information.

Manual Program Entry

Initially, the program steps must be manually entered into the calculator step-by-step. Then, you may record the program on a magnetic card furnished with your calculator. The use of the magnetic card eliminates repeated manual re-entry. Programs recorded on the magnetic card may be entered instantly, using the ENTER button on the calculator. After entering the program steps into the calculator, it is suggested that you use the sample data to verify test answers. If you do not get the correct answers, it is suggested that you carefully check each program step to see if the program is correctly entered into the appropriate location and/or re-read the user instructions. This may be done by comparing the step location and the code number to the printed program. You can do this very conveniently using the STEP PROGRAM key in the program mode.

Program Loading

In general, 9100B programs will occupy both the (+) and (-) calculator pages. The most efficient manner to load a 2-page program is to load Side A into the (+) page and Side B into the (-) program page. This can easily be accomplished by placing no END statements on the (+) program page. The END statement may be included on the (-) program page. Load the program as follows:

PRESS: END

ENTER PROGRAM: Side A

(Since there is no END on Side A, the calculator will advance to (-)(0)(0) automatically).

ENTER PROGRAM: Side B

(With or without an END, the calculator will advance to (+)(0)(0) ready for execution).

In the Program Library the above two instructions will be replaced by the single instruction:

ENTER PROGRAM: Side A followed by Side B

To insure that an extraneous END statement does not get recorded on Side A, all unused registers on the (+) page should be filled with CONTINUE's prior to recording.

9100B Peripherals

Several peripheral devices are available to complement the 9100B Calculator, these being:

9125A X - Y Plotter
9120A Printer
9160A Optical Card Reader
9150A Display Monitor

The "B" library has been written to simplify the attachment of the 9125A and 9120A to the 9100B. At appropriate input and output points in a program CONTINUE's have been placed for use in calling the printer or plotter. In most cases the plotting programs incorporate plotting Subroutines allowing the user to specify units in terms of inches, or centimeters.

9100B/ 9100A Program Library Cross-Reference

The 9100B Program Library contains several B Only programs which previously were "A" programs. These programs were either multiple pass or storage limited on the 9100A Calculator. After conversion, these programs offer the advantages of being single pass, having ample storage, and in some cases yielding plotted output. The table given below lists these converted programs.

"A" Library Part Number	Program Title	"B" Library Part Number
70015	NUMERICAL INTEGRATION USING SIMPSON'S RULE WHEN f(x) IS KNOWN	70409
70018	CONVOLUTION	70408
70019	4th OR 5th DEGREE POLYNOMIAL ROOTS	70405
70020	INTEGRAL OF THE FORM $F(X) = \int_A^X f(u)du$	70410
70810	WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA	70902
70814	MULTIPLE LINEAR REGRESSION	70901
70815	NON-LINEAR REGRESSION-LEAST SQUARES PARABOLA	70903
71002	TCHEBYSHEFF FILTER DESIGN--FINITE TERMINATIONS	71501
71011	S PARAMETER TO Y PARAMETER CONVERSION	71502
74008	COORDINATE GEOMETRY AND ENCLOSED AREA	74102
74011	TRAVERSE WITH COMPASS RULE ADJUSTMENT OPTION	74101

MATHEMATICS 09100-70000

70001 - n!
Calculates n! for positive integer n. ($n < 70$)

70002 - POLYNOMIAL EVALUATION
Evaluates polynomials of the form:

$$f(z) = C_n z^n + C_{n-1} z^{n-1} + \ldots + C_1 z + C_0$$

for complex C_i, i=1, . . . , n and complex z.

70003 - NUMERICAL INTEGRATION USING SIMPSON'S ONE-THIRD RULE
Uses Simpson's rule to obtain the area under a curve. The equation used is:

$$A = \frac{h}{3} (Y_0 + 4Y_1 + Y_2)$$

70006 - 1st ORDER DIFFERENTIAL EQUATIONS
Solves differential equations of the form:

$$y' = f(x, y)$$

70007 - RAISING A NUMBER TO A POWER
Solves the equation:

$$Z_3 = Z_1^{Z_2} \text{ where } Z_i = X_i + jY_i \quad i = 1, 2, 3$$

70008 - 2nd ORDER DIFFERENTIAL EQUATIONS
Solves differential equations of the form:

$$y'' = f(x, y, y')$$

70009 - QUADRATIC EQUATION

Solves $ax^2 + bx + c = 0$ for the roots.

70010 - FINITE DIFFERENCE INTERPOLATION USING GAUSS'S BACKWARD FORMULA
Uses Gauss's backward formula for interpolation in tabular data with equal abscissa spacing. The program fits a cubic equation through the tabular data.

70011 - CUBIC EQUATION

Solves $x^3 + px^2 + qx + r = 0$ for the real and complex roots.

70013 - FACTORS OF n
Gives all factors of an integer n.

70014 - REAL ROOTS OF f(x)
Calculates real roots of f(x) by starting from x_0 and incrementing until f(x) changes sign, then converges on the root. f(x) is programmed in by the user.

70016 - SIMULTANEOUS SOLUTION OF TWO EQUATIONS IN TWO UNKNOWNS
The program solves two independent equations of the form:

$$ax + by = e$$
$$cx + dy = f$$

x and y are the unknowns to be found.

70017 - n! $(n < 10^{12})$
Calculates n! for positive integer n.

70021 - POLYNOMIAL EVALUATION $(1 \leq n \leq 10)$
Repeatedly evaluates for a given x, a real polynomial of the form:

$$f(x) = A_n x^n + A_{n-1} x^{n-1} \ldots A_1 x + A_0 \quad \text{for } 1 \leq n \leq 10$$

70022 - 3 X 3 MATRIX INVERSION OR SIMULTANEOUS SOLUTION OF THREE EQUATIONS IN THREE UNKNOWNS
Solves three linear independent equations in three unknowns simultaneously or inverts a 3 x 3 matrix.

70023 - FOURIER SERIES
Calculates the Fourier Series coefficients that represent a periodic time function f(t) with period T. The specific f(t) is programmed into the calculator by the user.

70024 - GAMMA FUNCTION
Evaluates the gamma function $\Gamma(\nu)$ for $0 \leq \nu \leq 10^9$ where $\Gamma(\nu) = \int_0^\infty e^{-t} t^{\nu-1} dt$

70025 - BESSEL FUNCTION
Calculates the value of the Bessel function $J_n(x)$ of the first kind of integer order n where

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(-\frac{x^2}{4}\right)^k}{k!\,(n+k)!}$$

70401 - HYPERGEOMETRIC SERIES EXPANSION
Given a, b, and c, this program determines the coefficients of the hypergeometric series F(a, b, c; X). This program is useful in solving Gauss's differential equation.

70402 - (3 X 3) MATRIX MULTIPLICATION
Given two (3 x 3) matrices A and B, this program determines the product matrix $C = A \cdot B$.

70403 - ROOTS OF A 4th DEGREE POLYNOMIAL
This program determines the roots (real and complex) of a 4th degree polynomial of the form

$$X^4 + a_1 X^3 + a_2 X^2 + a_3 X + a_4$$

where a_i is real.

70404 - NUMERICAL SOLUTION OF TWO FIRST ORDER DIFFERENTIAL EQUATIONS
This program may be used to solve a wide variety of pairs of first order differential equations of the form

$$\frac{dy}{dx} = f(X, Y, Z),$$

$$\frac{dz}{dx} = g(X, Y, Z).$$

70405 - ROOTS OF A 6th DEGREE POLYNOMIAL
The program determines the roots (real and complex) of a 6th degree polynomial of the form

$$X^6 + a_1 X^5 + a_2 X^4 + a_3 X^3 + a_4 X^2 + a_5 X + a_6$$

where a_i is real.

70406 - CHARACTERISTIC EQUATION OF A (3 X 3) MATRIX AND EIGENVALUE DETERMINATION
Given a (3 x 3) matrix A, this program computes the characteristic equation

$$\lambda^3 + p\,\lambda^2 + q\,\lambda + r = 0$$

and then determines the eigenvalues by using Program 09100-70011 as a Sub-Program.

70407 - SIMULTANEOUS SOLUTION OF FOUR LINEAR EQUATIONS IN FOUR UNKNOWNS
Given a system of four linear equations in four unknowns defined by the matrix equation

$$[A_{ij}]\ [X_i] = [P_i] ,$$

this program uses Cholewski's method to determine the X_i's.

70408 - CONVOLUTION INTEGRAL WITH PLOT
This program evaluates and plots y(t), the convolution of e(t) and h(t). Mathematically

$$y(t) = \int_0^t e(\tau)\, h(t-\tau)\, d\tau .$$

70409 - NUMERICAL INTEGRATION USING SIMPSON'S RULE WHEN f(x) IS KNOWN
The specific f(x) is programmed into the calculator by the user and is then used by the general solution to evaluate the integral. Execution time is dependent on the number of panels. Note f(x) should not have any singularities in the integration interval.

70410 - INTEGRAL OF THE FORM: $F(x) = \int_A^x f(u)\, du$ WITH PLOT
This program calculates the integral of a known function f(u) between any lower limit A and a successively incremented upper limit X. Simpson's rule is used to perform the integration. A special application of this program is when f(u) is a probability density function. F(x) then represents the cumulative distribution function.

70411 - MAX - MIN OF Z = Z(X, Y)
This program determines the approximate range of a function Z of two independent variables X and Y given a range for X and Y. This program can be used in conjunction with Program 09100-70412, PLOT OF Z = Z(X, Y).

70412 - PLOT OF Z = Z(X, Y)
Given a function Z of two independent variables X and Y, this program creates a three dimensional plot over a prescribed range of X and Y.

STATISTICS 09100-70800

70801 - MEAN AND STANDARD DEVIATION
Calculates the mean and standard deviation of n data points.

70802 - STANDARD DEVIATION AND MEAN OF GROUPED DATA
Calculates the mean and standard deviation of data points of certain frequencies.

70803 - LINEAR REGRESSION
Calculates the best fit of a set of data points to the line $y = ax + b$, i.e., the program computes the estimates $\hat{a}$ and $\hat{b}$. It also gives the correlation coefficient r.

70804 - NORMAL PROBABILITY INTEGRAL
Evaluates the integral under the normal density function.

70805 - χ^2 - CHI SQUARE DISTRIBUTION
Calculates the integral of the Chi Square distribution from 0 up to a value of χ^2 for a given number of degrees of freedom.

70806 - χ^2 - CHI SQUARE EVALUATION EXPECTED VALUES EQUAL ($E_i = E$)
Chi square calculation where the expected value of each observation is equal.

70808 - χ^2 - CHI SQUARE EVALUATION EXPECTED VALUES UNEQUAL ($E_i \neq E_j$)
Chi square calculation where the expected values of the observations are not necessarily equal.

70811 - LEAST SQUARES FIT-POWER CURVE
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form: $y = ax^b$

70812 - LEAST SQUARES FIT-EXPONENTIAL
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form: $y = ae^{bx}$

70813 - POISSON DENSITY
Calculates the various summations associated with the Poisson density to give a probability based on an input parameter and summation endpoints.

70816 - RANDOM NUMBER GENERATOR
Random numbers (RN) in the range $0 \leq RN \leq 1$ are calculated; more than 10,000 random numbers may be generated before any previous value is repeated.

70901 - MULTIPLE LINEAR REGRESSIONS
Given a set of data points (X_i, Y_i, Z_i), this program determines the coefficients of the linear equation

$$Z = a_0 + a_1X + a_2Y$$

70902 - WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA
Calculates the parameters for the Weibull distribution and thus estimates of times to failure percentages may be made.

70903 - NON-LINEAR REGRESSION - LEAST SQUARES PARABOLA
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form:

$$y = a_0 + a_1x + a_2x^2$$

70904 - NORMAL (GAUSSIAN) CURVE PLOT
Given mean (M) and variance σ^2, this program generates a normal curve given by

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(X - M)^2}{2\sigma^2}}$$

This program can be used with Program 70905, Histogram Generation.

70905 - HISTOGRAM GENERATION WITH PLOT
This program generates and plots a histogram of ten windows given a set of positive numbers. The mean M and variance σ^2 of the data set are computed and stored for use by Program 70904, NORMAL CURVE PLOT.

70906 - HISTOGRAM GENERATION
This program generates a histogram table of ten windows given a data set of positive numbers. In addition it determines the mean M and the variance σ^2 of the data set.

70907 - ONE WAY ANALYSIS OF VARIANCE
This program separates the total variance in a table of data into that due to chance and that due to differences between the population means underlying each column of sample data.

70908 - F DISTRIBUTION
This program evaluates the F distribution density function for given values of F, V_1, and V_2.

70909 - TWO WAY ANALYSIS OF VARIANCE (m X 4)
This program analyses the total statistical variance in a table of data by separating the total variance into two parts, the variance among rows, and the variance between columns. These variances are then compared to the variance due to random influence.

70910 - TWO WAY ANALYSIS OF VARIANCE WITH REPLICATES
This program analyses the total statistical variance of a table of data by separating the total variance into three parts; the variance among rows, the variance between columns, and the variance due to interaction.

ELECTRONICS 09100-71000

71001 - TCHEBYSHEFF FILTER DESIGN
Calculates component values for Tchebysheff low pass filters with equal terminations.

71003 - BUTTERWORTH FILTER DESIGN
Calculates component values for Butterworth low pass filters between equal terminations.

71004 - MINIMUM LOSS PADS
Calculates resistive minimum loss pad and gives resistor values and loss in dB.

71005 - TCHEBYSHEFF EVALUATION
Used to determine filter order or the frequency response of a particular Tchebysheff filter.

71006 - ATTENUATOR PADS T OR Π
Calculates resistor values for either T or Π pads.

71007 - BAND PASS FILTER DESIGN
Calculates ideal component values and evaluates the frequency response by the image parameter method for a band pass filter.

71008 - STUB MATCHED TRANSMISSION LINE
Calculates the distance from a load to a point where a shorted stub is to be placed and the length of the stub to match a transmission line.

71009 - TRANSMISSION LINE
Calculates the impedance at any point on a transmission line either toward the generator or toward the load, the voltage reflection (magnitude and phase) and the VSWR on the line.

71010 - WYE → DELTA AND/OR DELTA → WYE CONVERSION
Transforms impedances wired in delta configuration to the equivalent wye configuration and vice-versa. Loop and nodal analyses are used to perform the transformations.

71501 - TCHEBYSHEFF FILTER DESIGN - FINITE TERMINATIONS
Calculates component values for Tchebysheff low pass filters with finite terminations (equal or unequal).

71502 - S PARAMETER TO Y PARAMETER CONVERSION
Converts S parameters for linear (active or passive) circuits to Y parameters.

71503 - FREQUENCY RESPONSE FROM POLES AND ZEROES WITH PLOT
Given the zeroes and poles of a complex function f(s), the magnitude and phase response is computed over a specified frequency range. The program can consider any combination of six poles and zeroes of the form $r_i = \quad + jw$.

MECHANICS 09100-72000

72002 - TRANSCENDENTAL EQUATION (ARC INVOLUTE IN GEAR DESIGN)
Solves for the angle ϕ in radians in the following expression

$$\text{INV}(\phi) = \text{TAN}\,\phi - \phi$$

where the INV (ϕ) is given. $1\text{x}10^{-17} < \phi < 1\text{x}10^{6}$ (ϕ is in radians).

72003 - SPRING DESIGN - COMPRESSION AND EXTENSION SPRINGS
This program calculates one of three variables (d - diameter of wire, N - number of turns of wire, and D - mean coil diameter), the remaining two being set, and calculates the maximum allowable stress. Other inputs are set to predetermined values.

72004 - STRESS AND STRAIN FROM A RECTANGULAR ROSETTE
Calculates the principal strains and stresses given rectangular rosette and strain gauge inputs.

72501 - PROPERTIES OF AREAS
This program determines the properties of any area which can be approximated by a set of rectangles. The properties determined are:

1. Area
2. Moments of inertia
3. Distances from axes to the centroid
4. Products of inertia about the centroid

BUSINESS 09100-73000

73001 - ANNUAL INTEREST
Calculates i that satisfies the equation:

$$R = \frac{P\,i(1+i)^n}{(1+i)^n - 1}$$

73008 - AMORTIZED LOAN
Calculates the monthly payment on the principal of a loan for a specified term, the amount of payments toward principal and interest, and a running total of the amount of payments toward principal and interest to date.

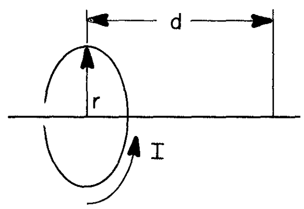
73101 - RETURN ON INVESTMENT FOR UP TO 16 VARYING CASH FLOWS
This program solves for the value of interest per period for up to sixteen varying cash flows. The program solves the equation:

$$\text{PV (Present Value)} = \sum_{k=1}^{16} \frac{C_k}{(1+i)^k}$$

where C_k represents a cash flow per period.

PHYSICS 09100-73200

73202 - FLUX DENSITY
Calculates the flux density along an axis of a circular loop for a specified radius r, current I, and distance d.



73203 - VENTURI METER
Calculates flow Q by using Bernoulli's equation.

73204 - ORBITAL MECHANICS
Calculates the velocity of a satellite at apogee and perigee and the orbital time in seconds of a satellite orbiting a body in space.

THERMODYNAMICS 09100-73700

73851 - HEAT TRANSFER THROUGH A MULTILAYERED CYLINDER
This program calculates the rate of heat flow through a multilayered cylinder. Both convective and conductive layers are considered.

73852 - TRANSIENT CONDUCTION IN A SLAB
Given the initial temperature of a slab and a fluid, this program determines the temperature (as a function of time and distance) of the slab as it is immersed into the fluid.

SURVEYING 09100-74000

74003 - INVERSE TRAVERSE FROM COORDINATES
Calculates bearing, distance, and quadrant code from end-point coordinates.

SURVEYING (CON'T)

74004 - THREE POINT PROBLEM
Calculates the coordinates of an observer's position given coordinates of three other points and two reference angles to the observer's position.

74101 - TRAVERSE WITH COMPASS RULE ADJUSTMENT OPTION
Traverses by bearing and distance, calculates coordinates, closure error, total traverse distance and precision ratio. The program may then be used to distribute the closure error by the Compass Rule.

74102 - COORDINATE GEOMETRY AND ENCLOSED AREA
Traverses by bearing and distance and calculates coordinates, enclosed area, and the following curve data:

1. Arc length
2. Chord length
3. Central angle in decimal degrees
4. Coordinates of tangent points

STRUCTURES 09100-74200

74203 - CANTILEVER BEAM - INTERMEDIATE LOAD
Calculates shear, moment, and deflection.

74204 - CANTILEVER BEAM - TRIANGULAR LOAD
Calculates shear, moment, and deflection.

74205 - COORDINATES OF EQUALLY SPACED POINTS ON A CIRCLE
Computes the rectangular coordinates of equally spaced points on a circle given the center point coordinates, radius and offset angle of the circle, and the number of coordinate points desired on the circle.

FLUID MECHANICS 09100-75000

75001 - CHEZY-MANNING EQUATION
Finds channel flow when channel is circular pipe and flowing full.

75003 - RECTANGULAR WEIR
Calculates the rate of fluid flow over a rectangular weir.

75004 - WATER FLOW IN PIPE
Calculates the loss factor (f) and the head loss for a pipe of specified dimensions with a known flow rate.

LIFE SCIENCES 09100-75200

75201 - RADIOACTIVE DECAY
Calculates the mass loss between any two times given the half-life, initial mass and present mass; displays the decay curve; or calculates the age of the mass based upon the present mass.

CHEMICAL 09100-75500

75502 - ELEMENTAL PERCENTAGE AND MOLECULAR WEIGHT - 6 ELEMENT
Calculates percentages and molecular weight of compounds containing 6 elements or less.

75503 - CHN ANALYSIS [K VALUES]
Calculates K values given C, H and N, blank values, and percentages for a known standard.

75504 - CHN PERCENTAGES
Given C, H, and N values and using previously calculated K values and known blank values, calculates C, H and N percentages.

75505 - MOLECULAR WEIGHT BY VPO
Calculates molecular weight for an unknown based on a series of vapor pressure osmometer (VPO) readings at various dilutions by extrapolating a least squares curve fit to infinite dilution.

75506 - MEMBRANE OSMOMETER
Determines the number-average molecular weight by extrapolating a least squares curve fit to infinite dilution.

SECONDARY EDUCATION 09100-75800

75802 - PRIME NUMBERS
Calculates all prime numbers between any two numbers.

75901 - CONIC SECTION DETERMINATION WITH PLOT
Given the generating angle of a cone, β , and the intersection angle with a plane, α , this program determines and plots the conic section.

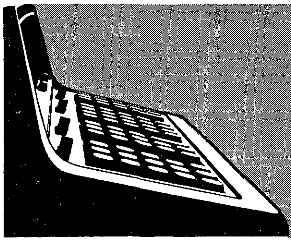
MISCELLANEOUS 09100-76000

76003 - NAVIGATIONAL COURSE CALCULATION
Calculates course settings for one or more adjoining legs of a proposed journey, the length of each leg, and the total distance covered on completion of the journey.

76004 - CIRCLE DETERMINED BY THREE POINTS
Calculates the radius and center point (in rectangular coordinates) of the circle defined by three given points.

76005 - AREA OF A RECTILINEAR SURFACE POLYGON
Calculates the area of any rectilinear polygon given the rectangular coordinates of the vertices.

76501 - DIAGNOSTIC (EXERCISER)
The program exercises each calculator operation and memory location with the exception of the FMT, PRINT, and error conditions.



9100B MATHEMATICS PROGRAM LISTING

70001 - n!
Calculates n! for positive integer n. ($n < 70$)

70002 - POLYNOMIAL EVALUATION
Evaluates polynomials of the form:

$$f(z) = C_n z^n + C_{n-1} z^{n-1} + \ldots + C_1 z + C_0$$

for complex C_i, i=1, . . . , n and complex z.

70003 - NUMERICAL INTEGRATION USING SIMPSON'S ONE-THIRD RULE
Uses Simpson's rule to obtain the area under a curve. The equation used is:

$$A = \frac{h}{3}(Y_0 + 4Y_1 + Y_2)$$

70006 - 1st ORDER DIFFERENTIAL EQUATIONS
Solves differential equations of the form:

$$y' = f(x, y)$$

70007 - RAISING A NUMBER TO A POWER
Solves the equation:

$$Z_3 = Z_1^{Z_2} \text{ where } Z_i = X_i + jY_i \quad i = 1, 2, 3$$

70008 - 2nd ORDER DIFFERENTIAL EQUATIONS
Solves differential equations of the form:

$$y'' = f(x, y, y')$$

70009 - QUADRATIC EQUATION

Solves $ax^2 + bx + c = 0$ for the roots.

70010 - FINITE DIFFERENCE INTERPOLATION USING GAUSS'S BACKWARD FORMULA
Uses Gauss's backward formula for interpolation in tabular data with equal abscissa spacing. The program fits a cubic equation through the tabular data.

70011 - CUBIC EQUATION

Solves $x^3 + px^2 + qx + r = 0$ for the real and complex roots.

70013 - FACTORS OF n
Gives all factors of an integer n.

70014 - REAL ROOTS OF f(x)
Calculates real roots of f(x) by starting from x_0 and incrementing until f(x) changes sign, then converges on the root. f(x) is programmed in by the user.

70016 - SIMULTANEOUS SOLUTION OF TWO EQUATIONS IN TWO UNKNOWNS
The program solves two independent equations of the form:

$$ax + by = e$$
$$cx + dy = f$$

x and y are the unknowns to be found.

70017 - n! ($n < 10^{12}$)
Calculates n! for positive integer n.

70021 - POLYNOMIAL EVALUATION ($1 \leq n \leq 10$)
Repeatedly evaluates for a given x, a real polynomial of the form:

$$f(x) = A_n x^n + A_{n-1} x^{n-1} \ldots A_1 x + A_0 \quad \text{for } 1 \leq n \leq 10$$

70022 - 3 X 3 MATRIX INVERSION OR SIMULTANEOUS SOLUTION OF THREE EQUATIONS IN THREE UNKNOWNS
Solves three linear independent equations in three unknowns simultaneously or inverts a 3 x 3 matrix.

70023 - FOURIER SERIES
Calculates the Fourier Series coefficients that represent a periodic time function f(t) with period T. The specific f(t) is programmed into the calculator by the user.

70024 - GAMMA FUNCTION
Evaluates the gamma function $\Gamma(\nu)$ for $0 \leq \nu \leq 10^9$ where $\Gamma(\nu) = \int_0^\infty e^{-t} t^{\nu-1} dt$

70025 - BESSEL FUNCTION
Calculates the value of the Bessel function $J_n(x)$ of the first kind of integer order n where

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(-\frac{x^2}{4}\right)^k}{k!\,(n+k)!}$$

9100B ONLY

70401 - HYPERGEOMETRIC SERIES EXPANSION
Given a, b, and c, this program determines the coefficients of the hypergeometric series F(a, b, c; X). This program is useful in solving Gauss's differential equation.

70402 - (3 X 3) MATRIX MULTIPLICATION
Given two (3 x 3) matrices A and B, this program determines the product matrix $C = A \cdot B$.

70403 - ROOTS OF A 4th DEGREE POLYNOMIAL
This program determines the roots (real and complex) of a 4th degree polynomial of the form

$$X^4 + a_1 X^3 + a_2 X^2 + a_3 X + a_4$$

where a_i is real.

70404 - NUMERICAL SOLUTION OF TWO FIRST ORDER DIFFERENTIAL EQUATIONS
This program may be used to solve a wide variety of pairs of first order differential equations of the form

$$\frac{dy}{dx} = f(X,Y,Z),$$

$$\frac{dz}{dx} = g(X,Y,Z).$$

70405 - ROOTS OF A 6th DEGREE POLYNOMIAL
The program determines the roots (real and complex) of a 6th degree polynomial of the form

$$X^6 + a_1 X^5 + a_2 X^4 + a_3 X^3 + a_4 X^2 + a_5 X + a_6$$

where a_i is real.

70406 - CHARACTERISTIC EQUATION OF A (3 X 3) MATRIX AND EIGENVALUE DETERMINATION
Given a (3 x 3) matrix A, this program computes the characteristic equation

$$\lambda^3 + p\ \lambda^2 + q\ \ \lambda + r\ =\ 0$$

and then determines the eigenvalues by using Program 09100-70011 as a Sub Program.

70407 - SIMULTANEOUS SOLUTION OF FOUR LINEAR EQUATIONS IN FOUR UNKNOWNS
Given a system of four linear equations in four unknowns defined by the matrix equation

$$[A_{ij}]\ [X_i]\ =\ [P_i],$$

this program uses Cholewski's method to determine the X_i's.

70408 - CONVOLUTION INTEGRAL WITH PLOT
This program evaluates and plots y(t), the convolution of e(t) and h(t). Mathematically

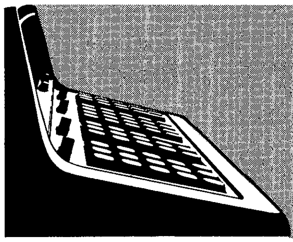
$$y(t) = \int_0^t e(\tau)\, h(t - \tau)\, d\tau\ .$$

70409 - NUMERICAL INTEGRATION USING SIMPSON'S RULE WHEN f(x) IS KNOWN
The specific f(x) is programmed into the calculator by the user and is then used by the general solution to evaluate the integral. Execution time is dependent on the number of panels. Note f(x) should not have any singularities in the integration interval.

70410 - INTEGRAL OF THE FORM: $F(x) = \int_A^x f(u)\, du$ WITH PLOT
This program calculates the integral of a known function f(u) between any lower limit A and a successively incremented upper limit X. Simpson's rule is used to perform the integration. A special application of this program is when f(u) is a probability density function. F(x) then represents the cumulative distribution function.

70411 - MAX - MIN OF Z = Z(X, Y)
This program determines the approximate range of a function Z of two independent variables X and Y given a range for X and Y. This program can be used in conjunction with Program 09100-70412, PLOT OF Z = Z(X, Y).

70412 - PLOT OF Z = Z(X, Y)
Given a function Z of two independent variables X and Y, this program creates a three dimensional plot over a prescribed range of X and Y.



PART NO.
09100-70001

n!

This program calculates n! for integer n where $0 \leq n \leq 69$.

$$n! = n(n-1) \cdots 3 \cdot 2 \cdot 1$$

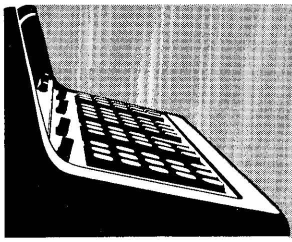
EXAMPLES	USER INSTRUCTIONS
0! = 1 6! = 720	ENTER PROGRAM (Starting Address is 0-0) PRESS: GO TO (0) (0) [or END] → PRESS: CONTINUE ENTER DATA: n ⟶ X PRESS: CONTINUE DISPLAY 0 —— Z n! —— Y n —— X

JULY, 1969
PART NO. 09100-70001

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	N	0	0	ENTER N					
	2	$x \to ( )$	23									
	3	f	15									
	4	IF $x = y$	50	IF N IS ZERO ENTER 1								
	▲ 5	arc ▾	72									
	6	1	01									
	7	↑	27									
	8	↑	27									
	9	1	01	DECREMENT N								
	a	−	34									
	b	IF $x > y$	53	BRANCH WHEN N REACHES ZERO								
	c	1	01									
	d	6	06									
1	0	ROLL ↓	31									
	1	×	36									
	2	ROLL ↑	22									
	3	GO TO ( )( )	44									
	4	0	00									
	5	a	13									
	6	0	00									
	7	ROLL ↓	31									
	8	f	15									
	9	END	46	N	N!	0	DISPLAY					
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

▲ Denotes Revision

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70002

POLYNOMIAL EVALUATION

This program evaluates polynomials of the form

$$f(Z) = C_n Z^n + C_{n-1} Z^{n-1} + \cdots + C_1 Z + C_0$$

for complex C_i, $i = 0, 1, \cdots, n$ and complex Z.

EXAMPLES

$P(Z) = (3 + 4i)Z^2 + (-2 + i)Z^1 + (1 - i)$

for $Z = 2 + i$ ReZ = 2 ImZ = 1

$P(2 + i) = -11 + 23i$

$P(Z) = 49.6Z^4 + 18Z^3 + 52.4Z^2 + 8Z + 12.8$

for $Z = i$ ReZ = 0 ImZ = 1

$P(i) = 10 - 10i$

GENERAL FORM

$$f(Z) = C_nZ^n + C_{n-1}Z^{n-1} + \ldots + C_1Z + C_o$$

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: n → Z, Im Z → Y,
Re Z → X

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
n_i	——	Z

n_i is an indicator or coefficient entry number. When $n_i = 0$ enter C_o.

ENTER DATA: $ImCn_i$ → Y,
$ReCn_i$ → X

PRESS: CONTINUE

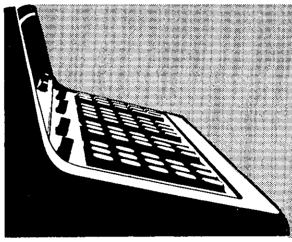
DISPLAY

0	——	Z
ImP(Z)	——	Y
ReP(Z)	——	X

HEWLETT · PACKARD

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST DATA ENTRY								
	2	STOP	41	ReZ	ImZ	n	ENTER Z AND n [THE DEGREE OF P(Z)]					
	3	TO POLAR	62									
	4	x→( )	23	STORE DATA (steps 0-4 to 0-a)								
	5	d	17									
	6	y→( )	40									
	7	c	16									
	8	ROLL ↑	22									
	9	x→( )	23									
	a	b	14									
	b	CLEAR	20									
	c	b	14	RECALL n IN THE X REGISTER TO INDICATE COEFFICIENT TO BE ENTERED								
	d	STOP	41	ReCn	ImCn	0	ENTER Cn					
1	0	TO POLAR	62									
	1	x→( )	23									
	2	a	13									
	3	y→( )	40									
	4	9	11									
	5	b	14	DECREMENT n_i (steps 1-5 to 1-8; FROM 3d → 1-5)								
	6	x⇄y	30									
	7	1	01									
	8	−	34									
	9	↓	25									
	a	IF x<y	52	BRANCH WHEN $n_i = -1$ (steps 1-a to 1-c)								
	b	4	04									
	c	0	00									
	d	x→( )	23									
2	0	b	14	RECALL n_i IN THE X REGISTER TO INDICATE COEFFICIENT TO BE ENTERED								
	1	STOP	41	$ReCn_i$	$ImCn_i$	0	ENTER Cn_i					
	2	ACC +	60									
	3	c	16									
	4	↑	27									
	5	d	17									
	6	↑	27									
	7	a	13									
	8	×	36									
	9	x⇄y	30									
	a	y⇄( )	24									
	b	9	11									
	c	ROLL ↓	31									
	d	+	33									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	↓	25									
	1	x⇄y	30									
	2	TO RECT	66	CALCULATE P(Z) BY USE OF THE EQUATION								
	3	ACC +	60	$P(Z) = \cdots \{[(C_nZ + C_{n-1})Z + C_{n-2}]Z + C_{n-3}\} + \cdots + C_0$								
	4	RECALL	61									
	5	TO POLAR	62									
	6	x→( )	23									
	7	a	13									
	8	y→( )	40									
	9	9	11									
	a	CLEAR	20									
	b	GO TO ( )( )	44									
	c	1	01									
	d	5	05									
4 (FROM 1c)	0	y⇄( )	24	RECALL P(Z)								
	1	9	11									
	2	a	13									
	3	TO RECT	66									
	4	END	46	ReP(Z)	ImP(Z)	0	DISPLAY					
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



PART NO.
09100-70003

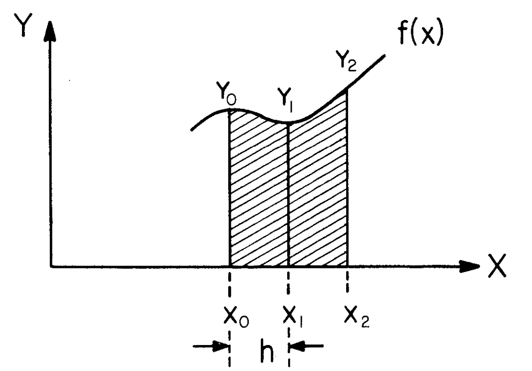
NUMERICAL INTEGRATION USING SIMPSON'S ONE-THIRD RULE

This program evaluates the area under a curve represented by discrete points. The equation used is Simpson's One-Third Rule ie.,

$$\int_{X_0}^{X_2} f(x)dx = \frac{h}{3}(Y_0 + 4Y_1 + Y_2) - \frac{1}{90} h^5 f^{(4)}(\xi)$$

where $X_0 < \xi < X_2$

Graphically the integration is performed over two panels (each of width h) as shown below



The application of Simpson's Rule over 2n panels between $X_0 = a$, and $X_{2n} = b$ gives

$$\int_a^b f(x)dx = \frac{h}{3}(Y_0 + 4Y_1 + 2Y_2 + 4Y_3 + 2Y_4 + \cdots + 4Y_{n-1} + Y_n)$$

$$- \frac{1}{180}(b-a)h^4 f^{(4)}(\xi)$$

where $a < \xi < b$ and $f^{(4)}(\xi)$ is the fourth derivative of $f(x)$ evaluated at ξ .

Thus to use the program divide the abscissa into 2n panels. (The method requires an even number of panels of width h.) Since the error term is neglected, choose h (the distance between points) such that the error term is small. Usually $h << 1$ when $f^{(4)}(\xi)$ is unknown.

Reference: Numerical Analysis
by Kaiser S. Kunz

McGraw - Hill Book Co., Inc. 1957

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: h ⟶ X

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: X_0 ⟶ X

PRESS: CONTINUE

⟶ DISPLAY

A_{i-1}	——	Z
X_i	——	Y
0	——	X

Note: Subsequent X_i's are calculated from X_0 and h and are not entered.

A - the area shows every other time starting after third y entry

ENTER DATA: Y_i ⟶ X (Corresponds to calculated X_i displayed in Y)

PRESS: CONTINUE (To restart a new problem PRESS: END, PRESS: CONTINUE)

To Change Increment (new h)

This can only be done if area or Z register is blank

ENTER DATA: Y_i ⟶ X

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: new h ⟶ X

PRESS: CONTINUE (Program branches to area display and ordinate entry. Proceed as before.)

EXAMPLES

INCREMENT CONSTANT

X	Y
0	2
.25	2.8
.50	3.8
.75	5.2
1.00	7.0
1.25	9.2
1.50	12.1
1.75	15.6
2.00	20

h = .25

$$\int_0^2 f(x)dx = 16.58$$

INCREMENT CHANGE

X	Y	
0	2	
.25	2.8	
.50	3.8	h = .25
.75	5.2	
1.00	7.0	
1.50	12.1	h = .5
2.00	20	

$$\int_0^2 f(x)dx = 16.62$$

Step	Key	Code	Display x	Display y	Display z	Storage f / e / d / c / b / a
0 0	CLEAR	20				
1 (FROM 4-4)	1	01	DISPLAY 1 TO INDICATE FIRST DATA ENTRY			
2	STOP	41	h	0	0	ENTER INCREMENT
3	$x\to()$	23				
4	d	17				
5	IF FLAG	43	BRANCH WHEN CHANGE h			
6	1	01				
7	6	06				
8	CLEAR	20				
9	2	02	DISPLAY 2 TO INDICATE SECOND DATA ENTRY			
a	STOP	41	X_0	0	0	ENTER X_0
b	$x\to()$	23				
c	f	15				
d	↑	27				
1 0	0	00				
1	↑	27				
2	ROLL ↓	31				DISPLAY X_0
3	STOP	41	Y_0	X_0	0	ENTER Y_0 CORRESPONDING TO X_0
4	$x\to()$	23				
5	c	16				
6	CLEAR x	37				
7	↑	27				
8 (FROM 4-7)	d	17	CALCULATE $X_i = X_{i-1} + h$			
9	ACC +	60				
a	RECALL	61				
b	↑	27				
c	CLEAR x	37				DISPLAY $AREA_{i-1}, X_i$
d	STOP	41	Y_i	X_i	$AREA_{i-1}$	ENTER Y_i CORRESPONDING TO X_i
2 0	$x\to()$	23				
1	b	14				
2	CLEAR x	37				
3	↑	27				
4	d	17	CALCULATE $X_{i+1} = X_i + h$			
5	ACC +	60				
6	f	15				
7	↑	27				
8	CLEAR x	37				DISPLAY X_{i+1}
9	STOP	41	Y_{i+1}	X_{i+1}	0	ENTER Y_{i+1} CORRESPONDING TO X_{i+1}
a	↑	27				
b	$y\rightleftarrows()$	24				
c	c	16				
d	+	33				

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	b	14									
	1	↑	27									
	2	4	04									
	3	×	36	CALCULATE $\left(\frac{Y_{i-1}+4Y_i+Y_{i+1}}{3}\right)h=A$								
	4	0	00									
	5	ROLL ↓	31									
	6	+	33									
	7	3	03									
	8	÷	35									
	9	d	17									
	a	×	36									
	b	0	00									
	c	ACC +	60	SUM A TO PREVIOUS AREA								
	d	ROLL ↓	31									
4	0	IF FLAG	43									
	1	SET FLAG	54									
	2	GO TO ( )( )	44	CHECK FOR FLAG, IF SET BRANCH TO h INPUT								
	3	0	00									
	4	1	01									
	5	GO TO ( )( )	44									
	6	1	01	BRANCH TO ENTER NEW ORDINATE AT SAME INCREMENT								
	7	8	10									
	8	END	46									
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

1st ORDER DIFFERENTIAL EQUATIONS

This program may be used to solve a wide variety of first order (homogenous or non-homogenous, linear or non-linear) differential equations of the form

$$Y' = f(X,Y)$$

The solution is a numerical solution which calculates Y_i for X_i. The X values are closely spaced with increment h over the desired range. Specifically the solution used in this program is a Runge-Kutta Method (third-order) which uses the equations,

$$Y_{i+1} = Y_i + \frac{1}{6}(p+2q+2r+s)$$

where

$$p = hf(X_i, Y_i)$$

$$q = hf\left(X_i + \frac{h}{2},\ Y_i + \frac{p}{2}\right)$$

$$r = hf\left(X_i + \frac{h}{2},\ Y_i + \frac{q}{2}\right)$$

$$s = hf(X_i + h,\ Y_i + r)$$

$$h = X_{i+1} - X_i$$

Reference: Numerical Analysis
by Kaiser S. Kunz

McGraw-Hill Book Co. Inc. 1957

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: (GO TO) (4) (a)

Place mode switch to PROGRAM

Starting at 4-a, enter the program steps which take the independent variable from the X register, the dependent variable from the Y register and calculate f(X, Y). Place f(X, Y) in the Y register and exit to location 8-b. Note there is a maximum of 57 steps (4-a through 8-a) available for storing and positioning f(X, Y).

Place mode switch to RUN

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA: (Initial conditions and increment)

h → Z, Y_0 → Y, X_0 → X

PRESS: CONTINUE

The Calculator will display answers at every increment of the independent variable in the form

h —— Z
Y_i —— Y
X_i —— X

To stop the solution at the next increment depress PAUSE until display. To restart press CONTINUE.

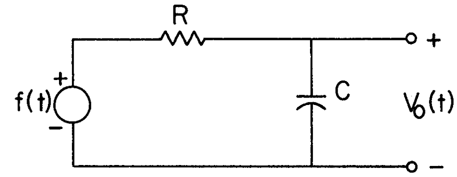
To start from a new set of initial conditions

PRESS: STOP

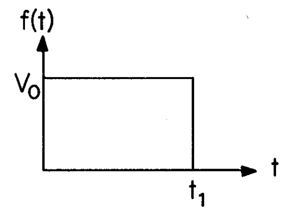
PRESS: GO TO (0) (0) [or END]

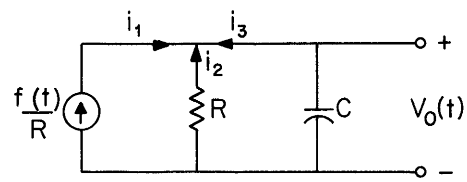
PRESS: CONTINUE

Enter initial conditions and increment as before.



$V_0(0) = 0$
$t_0 = 0$
$h = .01$





$$i_1 + i_2 + i_3 = 0$$

$$\frac{f(t)}{R} - \frac{V_0(t)}{R} - C\frac{dV_0(t)}{dt} = 0$$

Initial Conditions $\left\{ \begin{array}{l} t_0 = 0 \\ V_0(0) = 0 \end{array} \right.$

$$\frac{dV_0(t)}{dt} = -\frac{V_0(t)}{CR} + \frac{f(t)}{CR}$$

Increment h = .01

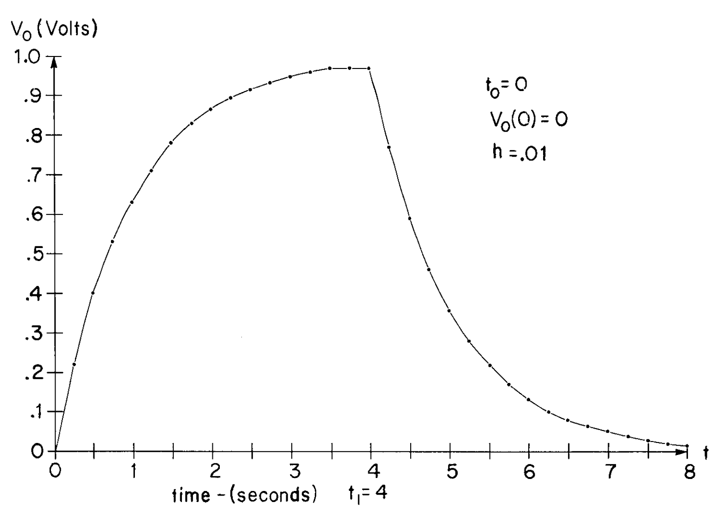
Let $V_0 = 1$ V, C = 1 F, R = 1 Ω and $t_1 = 4$ sec.

The equation becomes

$$\frac{dV_0(t)}{dt} = -V_0(t) + 1; \quad t \le 4$$

$$\frac{dV_0(t)}{dt} = -V_0(t); \quad t > 4$$

The program steps that form $V_0' = f(t, V_0)$ appear on Page 1. See User Instructions.



Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
4	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9			t	V_0							
	a	↑	27		t	V_0						
	b	4	04	4								
	c	IF $x<y$	52									
	d	5	05									
5	0	9	11									
	1	ROLL ↑	22	V_0	4	t						
	2	CHG SIGN	32	$-V_0$								
	3	↑	27		$-V_0$	4						
	4	1	01	1								
	5	+	33		$1-V_0$							
	6	GO TO ( )( )	44									
	7	8	10									
	8	b	14									
	9	ROLL ↑	22	V_0	4	t						
	a	CHG SIGN	32	$-V_0$								
	b	↑	27		$-V_0$	4						
	c	GO TO ( )( )	44									
	d	8	10									
6	0	b	14									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Steps 4b–50: COMPARE t TO 4 AND BRANCH TO APPROPRIATE DIFFERENTIAL EQUATION

Steps 51–55: $t \leq 4$

$$\frac{dV_0(t)}{dt} = -V_0(t) + 1$$

Steps 56–58: EXIT

Steps 59–5b: $t > 4$

$$\frac{dV_0(t)}{dt} = -V_0(t)$$

Steps 5d–60: EXIT

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	Remarks
0 0	CLEAR	20										
1	$x\rightarrow( )$	23										
2	b	14										
3	STOP	41	X_0	Y_0	h							ENTER INITIAL CONDITIONS AND INCREMENT
4	$x\rightarrow( )$	23										
5	f	15										
6	$y\rightarrow( )$	40										
7	d	17										
8	ROLL ↑	22										
9	$x\rightarrow( )$	23										
a	c	16										
b	ROLL ↓	31										FROM 4-9
c	PAUSE	57	X_i	Y_i	h							DISPLAY AT EACH INCREMENT
d	GO TO ( )( )	44										
1 0	4	04										CALCULATE $p = hf(X_i, Y_i)$
1	a	13										
2	↓	25										FROM a-c
3	2	02										
4	÷	35										
5	d	17										
6	+	33										
7	c	16										CALCULATE $X_i + \frac{h}{2}$, $Y_i + \frac{p}{2}$
8	↑	27										
9	2	02										OR $X_i + \frac{h}{2}$, $Y_i + \frac{q}{2}$
a	÷	35										IN THE X AND Y REGISTERS RESPECTIVELY
b	f	15										
c	+	33										
d	↓	25										
2 0	IF FLAG	43										
1	CONTINUE	47										
2	SET FLAG	54										
3	GO TO ( )( )	44										
4	4	04										CALCULATE q OR r
5	a	13										FROM 9-d
6	↓	25										
7	d	17										
8	+	33										
9	c	16										
a	↑	27										CALCULATE X_i+h IN THE X REGISTER
b	f	15										AND Y_i+r IN THE Y REGISTER
c	+	33										
d	↓	25										

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	Comments
3-0	x→( )	23										
3-1	f	15										
3-2	GO TO ( )( )	44										CALCULATE S
3-3	4	04										
3-4	a	13										
3-5	e	12										FROM a-6; CALCULATE $Y_i + \frac{1}{6}(p+2q+2r+s)$
3-6	↑	27										
3-7	6	06										
3-8	÷	35										
3-9	d	17										
3-a	+	33										
3-b	f	15										
3-c	y→( )	40										REINITIALIZE POINTS FOR STARTING NEXT INCREMENT
3-d	d	17										
4-0	↑	27										
4-1	0	00										
4-2	x→( )	23										
4-3	b	14										
4-4	x→( )	23										
4-5	e	12										
4-6	c	16										
4-7	IF FLAG	43										
4-8	0	00										
4-9	b	14	X	Y								
4-a	CONTINUE	47										FROM 1-1, 2-5, 3-4
4-b												
4-c												
4-d												
5-0												
5-1												
5-2												
5-3												
5-4												
5-5												
5-6												
5-7												
5-8												
5-9												
5-a												
5-b												
5-c												
5-d												

ENTER SPECIFIC DIFFERENTIAL EQUATION HERE. NOTE LOCATION 4-9 CONTAINS X IN THE X REGISTER AND Y IN THE Y REGISTER. USE THESE VARIABLES TO PROGRAM THE SPECIFIC EQUATION Y' = f(X,Y). AFTER FORMING Y', PLACE IT IN THE Y REGISTER IN LOCATION 8-b.

FILL THE REMAINING AREA WITH CONTINUES TO ELIMINATE THE POSSIBILITY OF AN END AND THUS RECORDING AND ENTERING PROBLEMS.

SEE OPERATING MANUAL

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
7	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
8	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b (FROM 4-a thru 8-a)	c	16		Y'	NOTE: Y' MUST BE IN THE Y REGISTER AT THIS TIME.						
	c	×	36									
	d	0	00									

Step		Key	Code	Display x / y / z	Storage f	e	d	c	b	a
9	0	IF FLAG	43							
	1	ACC +	60							
	2	SET FLAG	54	CALCULATE p+2q+2r+s IN THE e REGISTER						
	3	ACC +	60							
	4	SET FLAG	54							
	5	1	01							
	6	↑	27							
	7	b	14	INCREMENT CONTENTS OF STORAGE REGISTER b						
	8	+	33							
	9	y→( )	40							
	a	b	14							
	b	3	03							
	c	IF x>y	53							
	d	a	13							
a	0	a	13							
	1	IF x=y	50							
	2	a	13							
	3	7	07							
	4	GO TO( )( )	44							
	5	3	03	BRANCH TO CALCULATE Y_i+1 AND REINITIALIZE ROUTINE						
	6	5	05							
	7	IF FLAG	43							
	8	2	02	ZEROS FLAG AND BRANCHES TO CALCULATE S						
	9	6	06							
	a	GO TO( )( )	44							
	b	1	01	BRANCH TO CALCULATE q AND r						
	c	2	02							
	d	END	46							
	0									
	1									
	2									
	3									
	4									
	5									
	6									
	7									
	8									
	9									
	a									
	b									
	c									
	d									

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

RAISING A NUMBER TO A POWER

This program solves the equation $Z_3 = Z_1^{Z_2}$

where Z_1 and Z_2 are given. Z_i may be real or complex ie. $Z_i = X_i + jY_i \qquad i = 1,2,3$

The definitions $Z_1^{Z_2} = e^{Z_2 \ln Z_1} \qquad Z_1 \neq 0$

and $\ln Z = \ln(\sqrt{X^2 + Y^2}) + j\theta \qquad (-\pi < \theta \leq \pi, \sqrt{X^2 + Y^2} > 0)$

$$\theta = \text{TAN}^{-1} \frac{Y}{X}$$

are used.

The following equations are programmed

$$Z_3 = e^{Z_2 \ln Z_1}$$

$$= e^{(X_2 + jY_2)(\ln\sqrt{X_1^2 + Y_1^2} + j\theta_1)}$$

$$= e^{X_2 \ln\sqrt{X_1^2 + Y_1^2} - Y_2\theta_1} \left\{ \text{COS}(Y_2 \ln\sqrt{X_1^2 + Y_1^2} + X_2\theta_1) + j\text{SIN}(Y_2 \ln\sqrt{X_1^2 + Y_1^2} + X_2\theta_1) \right\}$$

Reference: Complex Variable and Applications
by Churchill

McGraw-Hill 1960

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

SET: [ ] RADIANS

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
1 —— X

ENTER DATA: Y_1 → Y, X_1 → X

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
2 —— X

ENTER DATA: Y_2 → Y, X_2 → X

PRESS: CONTINUE

DISPLAY

0 —— Z
Y_3 —— Y
X_3 —— X

RESULT: $Z_3 = X_3 + jY_3$

EXAMPLES

$Z_3 = (X_1 + jY_1)^{(X_2 + jY_2)}$

1) $Z_3 = e^{(2 + j2)}$

$X_1 = e$ $Y_1 = 0$

$X_2 = 2$ $Y_2 = 2$

$Z_3 = -3.075 + j6.719$

2) $Z_3 = (2 + j3)^{.5}$

$X_1 = 2$ $Y_1 = 3$

$X_2 = .5$ $Y_2 = 0$

$Z_3 = 1.674 + j.896$

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
2	STOP	41	X_1	Y_1	0	ENTER X_1 AND Y_1					
3	$x \to ()$	23	STORE Z_1								
4	a	13									
5	$y \to ()$	40									
6	b	14									
7	↓	25									
8	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
9	STOP	41	X_2	Y_2	0	ENTER X_2 AND Y_2					
a	$x \to ()$	23	STORE Z_2								
b	c	16									
c	$y \to ()$	40									
d	d	17									
1 0	b	14	CALCULATE $\ln\sqrt{X_1^2+Y_1^2}$ AND θ								
1	↑	27									
2	a	13									
3	TO POLAR	62									
4	$\ln x$	65									
5	ROLL ↑	22									
6	c	16									
7	$y \to ()$	40	STORE $\ln\sqrt{X_1^2+Y_1^2}$								
8	e	12									
9	×	36	CALCULATE $X_2 \ln\sqrt{X_1^2+Y_1^2}$ AND $X_2\theta_1$								
a	ROLL ↑	22									
b	×	36									
c	$y \rightleftarrows ()$	24									
d	e	12									
2 0	$x \rightleftarrows y$	30	CALCULATE $Y_2 \ln\sqrt{X_1^2+Y_1^2}$								
1	$y \rightleftarrows ()$	24									
2	d	17									
3	$x \rightleftarrows y$	30									
4	×	36									
5	$y \rightleftarrows ()$	24	CALCULATE $Y_2\theta_1$								
6	d	17									
7	×	36									
8	ROLL ↓	31	CALCULATE $X_2\ln\sqrt{X_1^2+Y_1^2} - Y_2\theta$								
9	−	34									
a	$y \rightleftarrows ()$	24									
b	d	17									
c	e	12									
d	+	33									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	ROLL ↓	31									
1	↑	27	CALCULATE $\cos(Y_2 \ln\sqrt{X_1^2+Y_1^2}+X_2\theta_1)$ AND $\sin(Y_2 \ln\sqrt{X_1^2+Y_1^2}+X_2\theta_1)$								
2	cos x	73									
3	ROLL ↓	31									
4	sin x	70									
5	x⇄y	30									
6	d	17	CALCULATE $e^{X_2 \ln\sqrt{X_1^2+Y_1^2}-Y_2\theta_1}$								
7	e^x	74									
8	×	36	CALCULATE X_3 AND Y_3								
9	ROLL ↑	22									
a	×	36									
b	0	00									
c	ROLL ↓	31									
d	END	46	X_3	Y_3	0	DISPLAY					

2nd ORDER DIFFERENTIAL EQUATIONS

This program may be used to solve a wide variety of second order differential equations of the form

$$Y'' = f(X, Y, Y')$$

The solution is a numerical solution which calculates Y_i and Y_i' for a set of closely spaced values of X_i over the desired range. The method used employs a Taylor series around the point X_i. The equations used are

$$Y_{i+1} = Y_i + hY_i' + \frac{h^2}{6}(4Y_i'' - Y_{i-1}'')$$

where the first term of the error is $\frac{-h^4}{8} Y^{(4)}$
and

$$Y_{i+1}' = Y_i' + \frac{h}{12}(5Y_{i-2}'' - 16Y_{i-1}'' + 23Y_i'')$$

where the first term of the error is $\frac{-3h^4}{8} Y^{(5)}$

where $h = X_{i+1} - X_i$

In order to start the solution the equations $Y_{-1}'' = Y_0'' - h \cdot Y_0'''$ with error of $-\frac{1}{2}h^2 Y^{(4)}$

and $Y_{-2}'' = Y_0'' - 2h \cdot Y_0'''$ with error of $-2h^2 Y^{(4)}$ are used.

X_0, Y_0, and Y_0' are known from the boundary conditions. Y_0'' is calculated from $f(X_0, Y_0, Y_0')$. Y_0'' is obtained by differentiating $f(X, Y, Y')$ and substituting in values X_0, Y_0, Y_0', and Y_0''. These initial conditions are required input.

In cases where an increasing exponential predominates it may be necessary to reverse the direction of the independent variable. This may be done simply by making h negative.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: (GO TO) (5) (a)

PLACE MODE SWITCH TO PROGRAM

The general program has a blank section for storing your specific differential equation. Starting at 5-0, enter the program steps which take the independent variable from the b register, the dependent variable from the f register, and the derivative of the dependent variable in the e register, and calculate f(X, Y, Y′). Place f(X, Y, Y′) in the Y register and exit to location 9 - 3. Note there is a maximum of 59 steps (5 - 0 through 9 - 2) available for storing and positioning f(X, Y, Y′).

PLACE MODE SWITCH TO RUN

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: Y_0'' → Z, Y_0''' → Y, h → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: X_0 → Z, Y_0' → Y, Y_0 → X

PRESS: CONTINUE

The Calculator will display answers at every increment of the independent variable in the form

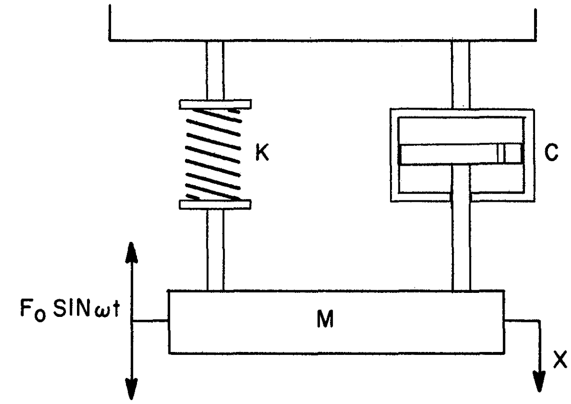
0	Z
Y_i	Y
X_i	X

To stop the solution at the next increment depress PAUSE until display. To restart depress CONTINUE.

To start from a new set of initial conditions

PRESS: STOP
PRESS: GO TO (0) (0) [or END]
PRESS: CONTINUE

EXAMPLES



$M\ddot{X} + C|\dot{X}|\dot{X} + KX = F_0 \text{ SIN } \omega t$

X(0) = 10	C = .08	ω = 4π	m = .1
$\dot{X}(0)$ = 0	F_0 = 100	K = 25	h = .001

$\ddot{X} = -.8|\dot{X}|\dot{X} - 250X + 1000 \text{ SIN } 4\pi t$

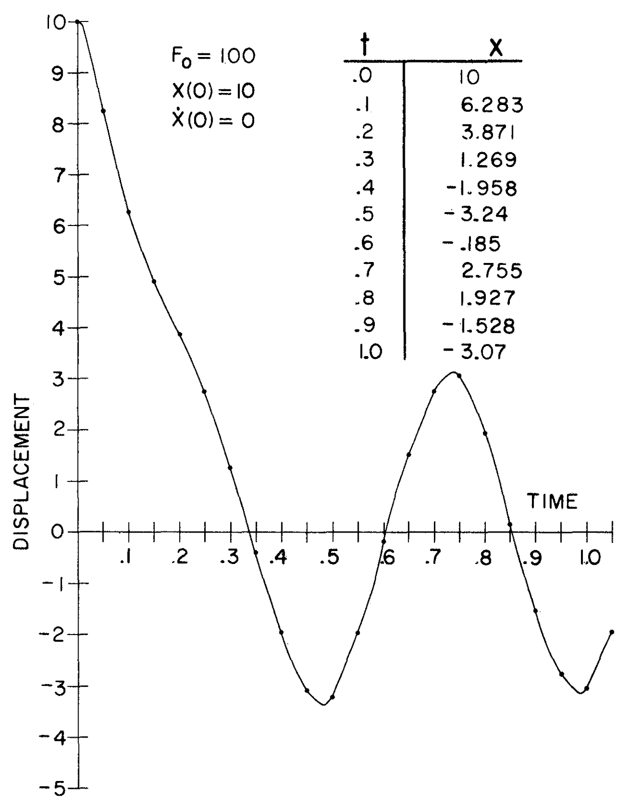
$\ddot{X}(0) = -2500$

$\dddot{X} = -1.6|\dot{X}|\ddot{X} - 250\dot{X} + 4000\pi \text{ COS } 4\pi t$

$\dddot{X}(0) = 4000\pi$

NOTE: SET [ ] [RADIANS]

The program steps that form $\ddot{X} = f(t, X, \dot{X})$ appear on Page 1. See User Instructions.



Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	Comment
5	0	e	12	$\dot{X}$			X	$\dot{X}$			t		
	1	↑	27		$\dot{X}$								
	2	|y|	55		$\|\dot{X}\|$								
	3	×	36		$\dot{X}\|\dot{X}\|$								
	4	·	21										CALCULATE $-\frac{C}{M}\|\dot{X}\|\dot{X}$
	5	8	10										
	6	CHG SIGN	32	−.8									
	7	×	36		$-.8\|\dot{X}\|\dot{X}$								
	8	f	15	X									
	9	↑	27		X	$-.8\|\dot{X}\|\dot{X}$							
	a	2	02										
	b	5	05										
	c	0	00										CALCULATE $-\frac{K}{M}X$
	d	CHG SIGN	32	−250									
6	0	×	36		−250X								
	1	↓	25	−250X	$-.8\|\dot{X}\|\dot{X}$								
	2	+	33		$-.8\|\dot{X}\|\dot{X}-250X$								
	3	4	04	4									
	4	↑	27		4	$-.8\|\dot{X}\|\dot{X}-250X$							
	5	π	56	π									
	6	×	36		4π								
	7	b	14	t									
	8	×	36		4πt								
	9	↓	25	4πt	$-.8\|\dot{X}\|\dot{X}-250X$								CALCULATE F_0 SIN ωt
	a	sin x	70	SIN 4πt									
	b	↑	27		SIN 4πt	$-.8\|\dot{X}\|\dot{X}-250X$							
	c	ENTER EXP	26										
	d	3	03	1000									
7	0	×	36		1000 SIN 4πt								
	1	↓	25	1000 SIN 4πt	$-.8\|\dot{X}\|\dot{X}-250X$								CALCULATE $\ddot{X}$
	2	+	33		$\ddot{X}$								
	3	GO TO ( )( )	44										
	4	9	11										EXIT TO 9-3
	5	3	03										
	6												
	7												
	8												
	9												
	a												
	b												
	c												
	d												

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0	0	CLEAR	20			REGISTER CONTENTS →	Y_i	Y_i'	Y_i''	Y_{i-1}''	X_i	h
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	STOP	41	h	Y_0'''	Y_0''	ENTER INCREMENT AND INITIAL CONDITIONS					
	3	$x\to()$	23									
	4	a	13									
	5	×	36									
	6	↓	25									
	7	$y\to()$	40									
	8	d	17	CALCULATE Y_{-1}'' (steps 0-5 to 0-b)								
	9	−	34									
	a	$y\to()$	40									
	b	c	16									
	c	−	34									
	d	$y\to()$	40	CALCULATE Y_{-2}'' (steps 0-c to 1-0)								
1	0	b	14									
	1	CLEAR	20									
	2	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
	3	STOP	41	Y_0	Y_0'	X_0	ENTER INITIAL CONDITIONS					
	4	ACC +	60									
	5 (FROM 9-c →)	f	15									
	6	ROLL ↓	31	RECALL AND POSITION FOR DISPLAY (steps 1-5 to 1-8)								
	7	CLEAR x	37									
	8	ROLL ↓	31									
	9	PAUSE	57	X_i	Y_i	0	DISPLAY AT EACH INCREMENT					
	a	$x \rightleftarrows y$	30									
	b	$y \rightleftarrows ()$	24									
	c	b	14									
	d	5	05									
2	0	×	36									
	1	c	16									
	2	↑	27									
	3	1	01									
	4	6	06									
	5	×	36									
	6	↓	25									
	7	−	34	CALCULATE $\frac{h}{12}(5Y_{i-2}''-16Y_{i-1}''+23Y_i'')$ (steps 1-b onward)								
	8	d	17									
	9	↑	27									
	a	2	02									
	b	3	03									
	c	×	36									
	d	↓	25									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	+	33									
1	a	13									
2	×	36									
3	1	01									
4	2	02									
5	÷	35									
6	d	17									
7	↑	27									
8	4	04									
9	×	36									
a	c	16									
b	−	34									
c	a	13									
d	×	36	CALCULATE $hY_i' + \frac{h^2}{6}(4Y_i'' - Y_{i-1}'')$								
4 0	6	06									
1	÷	35									
2	e	12									
3	+	33									
4	a	13									
5	×	36									
6	↓	25									
7	ACC +	60	CALCULATE Y_{i+1} AND Y'_{i+1}								
8	y⇄( )	24									
9	b	14									
a	a	13									
b	+	33	INCREMENT X_i								
c	y→( )	40									
d	b	14									
5 0	CONTINUE	47									
1	↓										
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d	↓										

ENTER SPECIFIC DIFFERENTIAL EQUATION HERE. REGISTER b CONTAINS X, REGISTER f CONTAINS Y, AND REGISTER e CONTAINS Y'. USE THESE VARIABLES TO PROGRAM THE SPECIFIC EQUATION Y''=f(X,Y,Y'). AFTER FORMING Y'' PLACE IT IN THE Y REGISTER AND EXIT TO LOCATION 9-3.

FILL THE REMAINING AREA WITH CONTINUES TO ELIMINATE THE POSSIBILITY OF AN END AND THUS RECORDING AND ENTERING PROBLEMS.

SEE OPERATING MANUAL

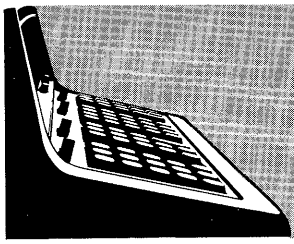
HEWLETT · PACKARD

Step		Key	Code	Display			Storage					
				x	y	z	f	e	d	c	b	a
6	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
7	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
8	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

FROM 5-0 THROUGH 9-2

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	CONTINUE	47									
	1											
	2				Y''	NOTE: Y'' MUST BE IN THE Y REGISTER AT THIS TIME						
	3	$y \rightleftarrows ()$	24									
	4	d	17									
	5	$y \rightleftarrows ()$	24									
	6	c	16									
	7	$y \rightleftarrows ()$	24	REINITIALIZE AND BRANCH FOR NEXT INCREMENT								
	8	b	14									
	9	↑	27									
	a	GO TO ( ) ( )	44									
	b	1	01									
	c	5	05									
	d	END	46									
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	*y*	*z*	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



June 1969
PART NO.
09100-70009

QUADRATIC EQUATION

This program calculates the two roots X_1 and X_2 of the equation

▲ $$ax^2 + bX + c = 0 \qquad a \neq 0$$

The equation used to calculate the roots is:

$$X_1, X_2 = -\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$$

▲ Denotes Revision

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

ENTER DATA: a → Z, b → Y, c → X

PRESS: CONTINUE

DISPLAY

real roots:

0	Z
X_2	Y
X_1	X

complex roots:

$Im(X_2)$	Z
$Im(X_1)$	Y
$Re(X_1 \text{ and } X_2)$	X

(Note Z contains zero)

EXAMPLES

GENERAL FORM

$aX^2 + bX + c = 0$

$X^2 + X + 1.25 = 0$

$X_2 = -.5 + j1$

$X_1 = -.5 - j1$

$2X^2 + 5X + 3 = 0$

$X_2 = -1.5$

$X_1 = -1.0$

$X^2 + 4 = 0$

$X_2 = j2$

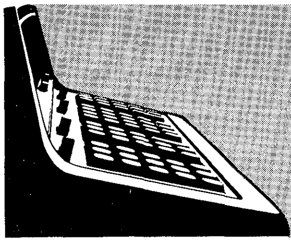
$X_1 = -j2$

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0 0	CLEAR	20									
1	STOP	41	c	b	a	ENTER		a,b,c			
2	ROLL ↑	22									
3	÷	35									
4	ROLL ↑	22									
5	$x \rightleftarrows y$	30									
6	÷	35									
7	2	02									
8	CHG SIGN	32									
9	÷	35	CALCULATE $-\frac{b}{2a}$ AND $\frac{b^2}{4a^2}-\frac{c}{a}$								
a	↓	25									
b	↑	27									
c	×	36									
d	ROLL ↓	31									
1 0	$x \rightleftarrows y$	30									
1	−	34									
2	CLEAR x	37									
3	IF $x=y$	50									
4	3	03	IF $\frac{b^2}{4a^2}-\frac{c}{a}=0$, SOLVE FOR $X_1, X_2 = -\frac{b}{2a}$								
5	4	04									
6	IF $x>y$	53									
7	2	02	IF $\frac{b^2}{4a^2}-\frac{c}{a}<0$, SOLVE FOR COMPLEX ROOTS								
8	9	11									
9	↓	25	BRANCH HERE IF $\frac{b^2}{4a^2}-\frac{c}{a}>0$								
a	$\sqrt{x}$	76									
b	↑	27									
c	CHG SIGN	32									
d	ROLL ↑	22	CALCULATE $X_1, X_2 = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2}-\frac{c}{a}}$								
2 0	+	33									
1	ROLL ↑	22									
2	+	33									
3	CLEAR x	37									
4	ROLL ↓	31	X_1	X_2	0	REAL ROOTS					
5	STOP	41	$Re(X_1 \& X_2)$	$Im(X_1)$	$Im(X_2)$	COMPLEX ROOTS		DISPLAY			
6	GO TO ( ) ( )	44									
7	0	00									
8	0	00									
9	↓	25	BRANCH HERE IF $\frac{b^2}{4a^2}-\frac{c}{a}<0$								
a	CHG SIGN	32									
b	$\sqrt{x}$	76									
c	↑	27									
d	CHG SIGN	32	CALCULATE $X = -\frac{b}{2a} \pm j\sqrt{\frac{c}{a}-\frac{b^2}{4a^2}}$								

FROM 3-3 & 3-9

FROM 1-5

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	ROLL ↑	22									
	1	GO TO ( )( )	44									
	2	2	02									
	3	5	05									
	4	↓	25	BRANCH HERE IF $\frac{b^2}{4a^2} - \frac{c}{a} = 0$								
	5	CLEAR x	37									
	6	ROLL ↓	31	CALCULATE $X_1, X_2 = -\frac{b}{2a}$								
	7	GO TO ( )( )	44									
	8	2	02									
	9	5	05									
	a	END	46									
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



FINITE DIFFERENCE INTERPOLATION USING GAUSS'S BACKWARD FORMULA

This program interpolates for data points in the region of tabulated data for uniformly spaced abscissas, with spacing h. The equation used is the backward-interpolation formula of Gauss which uses four pairs of data points and sets up the polynomial for cubic interpolation.

The equation used is:

▲ $$Y = Y_o + u\,\delta Y_{-1/2} + \frac{1}{2!}(u+1)\;\delta^2 Y_o + \frac{1}{3!}(u+1)(u)(u-1)\;\delta^3 Y_{-1/2}$$

The difference table is:

u	X	Y			
-2	X_1	Y_1			
			$Y_2 - Y_1$		
-1	X_2	Y_2		$Y_3 - 2Y_2 + Y_1$	
			$Y_3 - Y_2$		
0	X_3	Y_3 →	→	$Y_4 - 2Y_3 + Y_2$ →	$Y_4 - 3Y_3 + 3Y_2 - Y_1$
			$Y_4 - Y_3$		
1	X_4	Y_4			

where $\delta Y_{-1/2} = Y_3 - Y_2$

$\delta^2 Y_o = Y_4 - 2Y_3 + Y_2$

$\delta^3 Y_{-1/2} = Y_4 - 3Y_3 + 3Y_2 - Y_1$

and $u = \dfrac{X - X_3}{h}$

Reference: Introduction to Numerical Analysis
by F.B. Hildebrand

McGraw-Hill 1956

▲ Denotes Revision

USER INSTRUCTIONS

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: $Y_1 \rightarrow Z$, $Y_2 \rightarrow Y$, $Y_3 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: $Y_4 \rightarrow Z$, $X_3 \rightarrow Y$, $h \rightarrow X$

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
3	——	X

ENTER DATA: $X \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	——	Z
Y(X)	——	Y
X	——	X

TO RESTART PRESS: END

PRESS: CONTINUE

EXAMPLES

GENERAL FORM: Y(X)

i	X_i	Y_i
1	-1	-1
2	1	1
3	3	19
4	5	101

Y(3.1) = 21.181

Y(2.5) = 10.375

Y(-2) = -11.000

h = 2

i	X_i	Y_i
1	0	-7
2	.5	-6.75
3	1	-5
4	1.5	-.25

Y(.7) = -6.314

Y(1.2) = -3.544

h = .5

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	STOP	41	Y_3	Y_2	Y_1	ENTER Y_1, Y_2, Y_3,					
	3	$x \to ()$	23									
	4	d	17									
	5	$y \to ()$	40									
	6	c	16	STORE Y_1, Y_2, Y_3								
	7	ROLL ↑	22									
	8	$x \to ()$	23									
	9	b	14									
	a	CLEAR	20									
	b	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
	c	STOP	41	h	X_3	Y_4	ENTER Y_4, X_3, h					
	d	ACC +	60									
1	0	ROLL ↓	31									
	1	$y \rightleftarrows ()$	24									
	2	b	14									
	3	↓	25									
	4	CHG SIGN	32									
	5	↑	27									
	6	c	16									
	7	+	33									
	8	+	33									
	9	+	33	CALCULATE $\delta^3 Y_{-\frac{1}{2}}$								
	a	d	17									
	b	−	34									
	c	−	34									
	d	−	34									
2	0	b	14									
	1	+	33									
	2	$y \to ()$	40									
	3	b	14									
	4	↑	27									
	5	d	17									
	6	−	34									
	7	−	34	CALCULATE $\delta^2 Y_0$								
	8	c	16									
	9	+	33									
	a	$y \rightleftarrows ()$	24									
	b	c	16									
	c	d	17									
	d	$x \rightleftarrows y$	30									

FROM 7-3 → 3-3

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	−	34	CALCULATE $\delta Y_{-\frac{1}{2}}$								
1	y→( )	40									
2	a	13									
3	CLEAR x	37									
4	↑	27									
5	↑	27									
6	3	03	DISPLAY 3 TO INDICATE THIRD ENTRY								
7	STOP	41	X	0	0	ENTER X					
8	ROLL ↓	31	CALCULATE u								
9	↓	25									
a	e	12									
b	−	34									
c	f	15									
d	÷	35									
4 0	ROLL ↑	22	STORE X								
1	x→( )	23									
2	9	11									
3	↓	25	CALCULATE $\frac{1}{3!}(u+1)(u)(u-1)\delta^3 Y_{-\frac{1}{2}}$								
4	1	01									
5	x⇄y	30									
6	+	33									
7	×	36									
8	×	36									
9	↑	27									
a	×	36									
b	ROLL ↓	31									
c	−	34									
d	↓	25									
5 0	x⇄y	30									
1	−	34									
2	6	06									
3	÷	35									
4	b	14									
5	×	36									
6	a	13	CALCULATE $u\delta Y_{-\frac{1}{2}}$								
7	ROLL ↑	22									
8	×	36									
9	ROLL ↓	31	FORM PARTIAL SUM								
a	+	33									
b	1	01									
c	ROLL ↑	22									
d	+	33									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	×	36	CALCULATE $\frac{u(u+1)}{2!}\delta^2 Y_0$								
	1	2	02									
	2	÷	35									
	3	c	16									
	4	×	36									
	5	CLEAR x	37	FORM PARTIAL SUM								
	6	ROLL ↓	31									
	7	+	33									
	8	d	17	CALCULATE FINAL SUM Y								
	9	+	33									
	a	↓	25	POSITION DISPLAY								
	b	y⇄( )	24									
	c	9	11									
	d	x⇄y	30									
7	0	STOP	41	X	Y(X)	0	DISPLAY					
	1	GO TO ( )( )	44	BRANCH FOR X ENTRY								
	2	3	03									
	3	3	03									
	4	END	46									
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

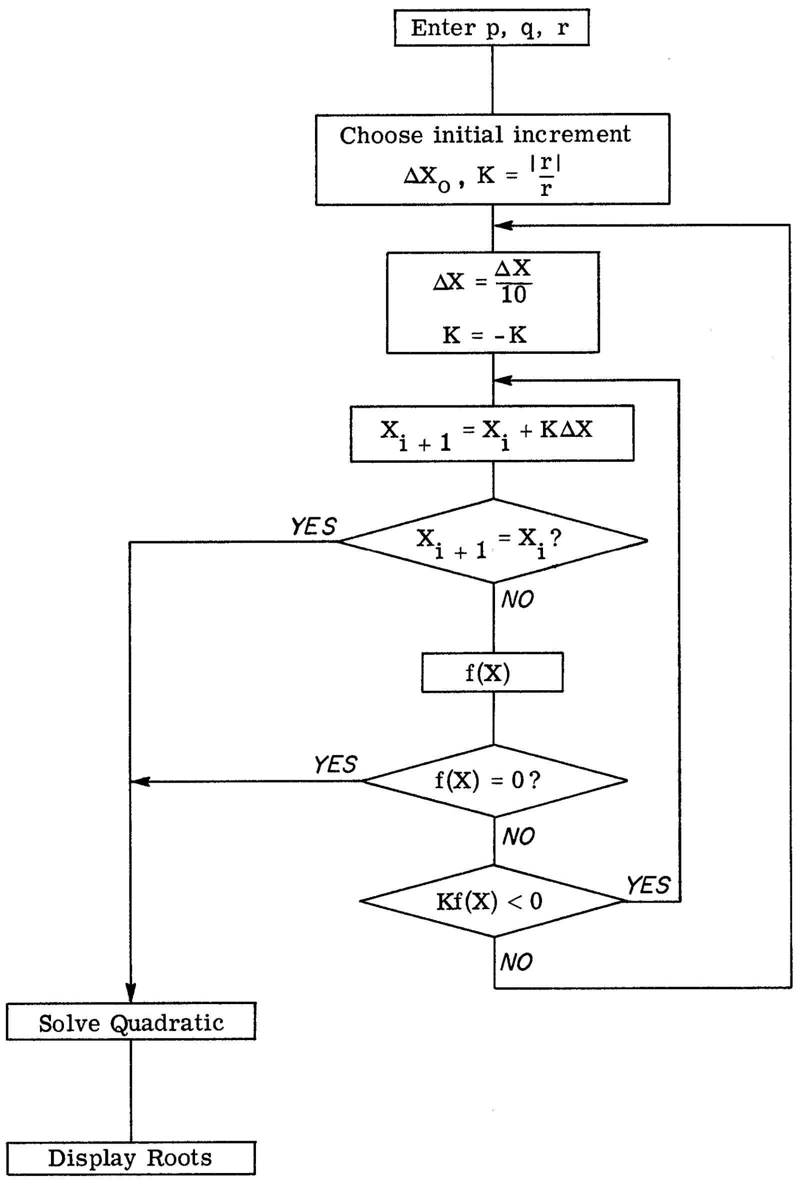
Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

CUBIC EQUATION

This program solves

$$X^3 + pX^2 + qX + r = 0$$

for real and complex roots.



USER INSTRUCTIONS

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

ENTER DATA: p → Z, q → Y, r → X

PRESS: CONTINUE

DISPLAY:

3 real roots:

0	Z
0	Y
3	X

1 real, 2 complex roots:

0	Z
0	Y
1	X

PRESS: CONTINUE

DISPLAY:

real roots: (← return to PRESS: CONTINUE)

X_3	Z
X_2	Y
X_1	X

complex roots: (← return to PRESS: CONTINUE)

(X_3)	Z
Im(X_1 and X_2)	Y
Re(X_1 and X_2)	X

EXAMPLES

General Form

$$X^3 + pX^2 + qX + r = 0$$

Example 1

$$X^3 + 3X^2 + 3X + 1$$

$$X_3 = -1$$

$$X_2 = -1$$

$$X_1 = -1$$

Example 2

$$X^3 - X^2 + X - 1$$

$$X_3 = 1$$

$$X_1, X_2 = \pm i$$

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a)	Comments
0 0	CLEAR	20					
1	STOP	41	r	q	p	ENTER p,q,r	
2	ACC +	60					
3	↓	25					
4	y→( )	40					STORE DATA
5	d	17					
6	|y|	55					
7	f	15					
8	x⇄y	30					
9	|y|	55					
a	+	33					
b	ENTER EXP	26					
c	CHG SIGN	32					
d	2	02					
1 0	↑	27					
1	1	01					
2	0	00					
3	×	36					
4	↓	25					STORE INITIAL INCREMENT ΔX_0
5	IF x<y	52					
6	1	01					
7	0	00					(branch to 1-0)
8	x→( )	23					
9	c	16					
a	CLEAR x	37					
b	↑	27					
c	f	15					
d	↑	27					CALCULATE K
2 0	|y|	55					
1	÷	35					
2	y→( )	40					
3	b	14					
4	c	16					FROM 5-8
5	x⇄y	30					
6	.	21					CALCULATE $\frac{\Delta X}{10}$
7	1	01					
8	×	36					
9	y→( )	40					
a	c	16					
b	b	14					CALCULATE −K
c	CHG SIGN	32					
d	x→( )	23					

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	b	14									
1	↓	25									
2	c	16									
3	↑	27									
4	b	14	CALCULATE $X_{i+1} = X_i + K\Delta X$								
5	×	36									
6	↓	25									
7	+	33									
8	↓	25									
9	IF $x=y$	50									
a	5	05	BRANCH IF $X_{i+1} = X_i$								
b	9	11									
c	↑	27									
d	↑	27									
4 0	d	17									
1	+	33									
2	↓	25									
3	×	36	CALCULATE f(X)								
4	e	12									
5	+	33									
6	↓	25									
7	×	36									
8	f	15									
9	+	33									
a	CLEAR x	37									
b	IF $x=y$	50	BRANCH IF f(X)=0								
c	5	05									
d	9	11									
5 0	b	14									
1	×	36									
2	CLEAR x	37	COMPARE Kf(X) TO ZERO								
3	IF $x>y$	53									
4	3	03									
5	1	01									
6	GO TO ( )( )	44									
7	2	02	BRANCH TO INCREMENT CHANGE								
8	4	04									
9	↓	25									
a	$y \to ( )$	40									
b	b	14									
c	d	17									
d	ROLL ↑	22									

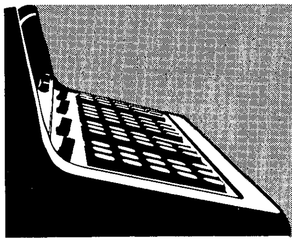
Step		Key	Code
6	0	+	33
	1	↑	27
	2	↓	25
	3	×	36
	4	e	12
	5	+	33
	6	↓	25
	7	CHG SIGN	32
	8	ROLL ↓	31
	9	2	02
	a	CHG SIGN	32
	b	÷	35
	c	↓	25
	d	↑	27
7	0	×	36
	1	ROLL ↓	31
	2	+	33
	3	ENTER EXP	26
	4	CHG SIGN	32
	5	1	01
	6	0	00
	7	ROLL ↑	22
	8	×	36
	9	|y|	55
	a	ROLL ↓	31
	b	CHG SIGN	32
	c	IF x>y	53
	d	9	11
8	0	1	01
	1	SET FLAG	54
	2	CHG SIGN	32
	3	IF x>y	53
	4	CLEAR x	37
	5	×	36
	6	↓	25
	7	√x	76
	8	−	34
	9	ROLL ↑	22
	a	+	33
	b	↓	25
	c	GO TO ( )( )	44
	d	9	11

Display (x, y, z) / Storage (f, e, d, c, b, a):

SOLVE REDUCED QUADRATIC EQUATION

FROM-8-0

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	4	04									
	1	↓	25									
	2	CHG SIGN	32									
	3	$\sqrt{x}$	76									
	4	$x\to()$	23									
	5	e	12									
	6	$y\to()$	40									
	7	f	15									
	8	CLEAR x	37									
	9	↑	27									
	a	↑	27									
	b	1	01	ENTER 1 FOR COMPLEX DISPLAY								
	c	IF FLAG	43									
	d	CLEAR x	37	FLAG IS SET FOR REAL ROOTS								
a	0	3	03									
	1	STOP	41	1 or 3	0	0	1 FOR ONE REAL AND TWO COMPLEX ROOTS, 3 FOR THREE REAL ROOTS					
	2	b	14									
	3	ROLL ↓	31	POSITION DISPLAY								
	4	RCL	61									
	5	END	46	$Re_{1,2}$	$Im_{1,2}$	X_3	DISPLAY					
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



PART NO.
09100-70013

FACTORS OF n

This program gives the factors of any positive integer n.

All repeated factors are given.

hp HEWLETT · PACKARD
hp HEWLETT · PACKARD
hp HEWLETT · PACKARD
hp HEWLETT · PACKARD

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
0	——	X

ENTER DATA: No. to be factored → X

DISPLAY

0	——	Z
0	——	Y
n	——	X

→ PRESS: CONTINUE

DISPLAY

n	——	Z
remaining factor	——	Y
Factor	——	X

FINAL DISPLAY

n	——	Z
1	——	Y
Factor	——	X

EXAMPLES

(A)

n = 50

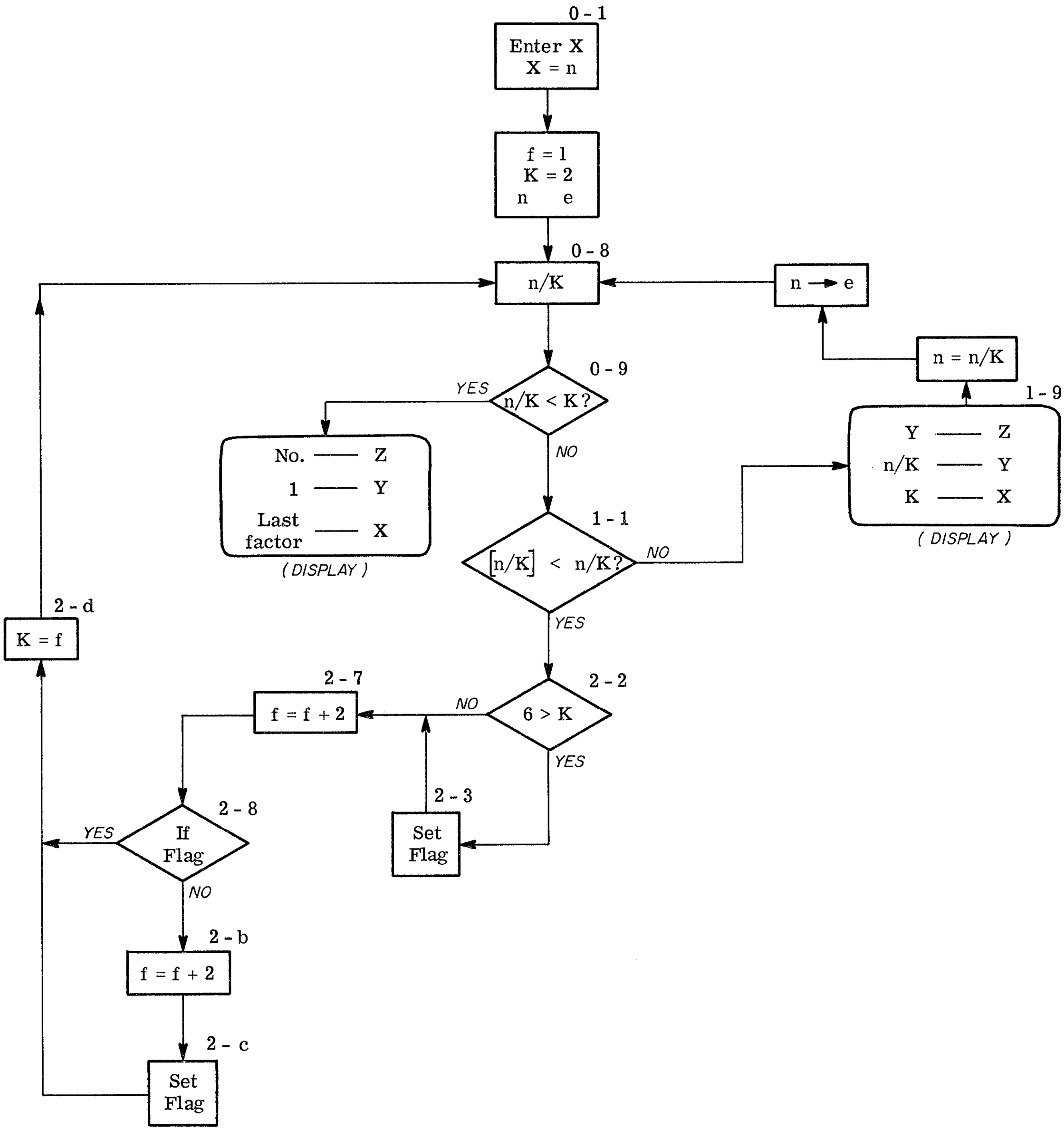
Factors are: 2, 5, 5

(B)

$n = 2.9393939 \times 10^7$

Factors are: 2.9393939×10^7

(n is a prime number)



0 - 1
Enter X
X = n
f = 1
K = 2
n e
0 - 8
n/K
n → e
n = n/K
0 - 9
YES
n/K < K ?
NO
No. —— Z
1 —— Y
Last factor —— X
(DISPLAY)
1 - 9
Y —— Z
n/K —— Y
K —— X
(DISPLAY)
1 - 1
[n/K] < n/K?
NO
YES
2 - d
K = f
2 - 7
f = f + 2
NO
6 > K
2 - 2
YES
2 - 3
Set Flag
2 - 8
YES
If Flag
NO
2 - b
f = f + 2
2 - c
Set Flag

Step	Key	Code	Display x	Display y	Display z	Remarks	Storage f	e	d	c	b	a
0 0	CLEAR	20										
1	STOP	41	X	0	0							
2	$x\rightarrow( )$	23				STORE X						
3	d	17										
4	↑	27				STORE 1 AND X						
5	1	01										
6	ACC +	60										
7	2	02				CALCULATE X/K						
8	÷	35										
9	IF $x>y$	53				BRANCH FOR FINAL DISPLAY						
a	3	03										
b	3	03										
c	$x\rightleftarrows y$	30										
d	↑	27										
1 0	int x	64				COMPARE $\left[\frac{n}{K}\right]$ TO K AND BRANCH IF K IS NOT A FACTOR						
1	IF $x<y$	52										
2	1	01										
3	d	17										
4	ROLL ↓	31				POSITION DISPLAY AND STORE UNFACTORED PORTION						
5	d	17										
6	ROLL ↓	31										
7	$y\rightarrow( )$	40										
8	e	12										
9	STOP	41	K	n/K	n							
a	GO TO ( )( )	44				BRANCH TO CONTINUE FACTORING						
b	0	00										
c	8	10										
d	CLEAR x	37										
2 0	ROLL ↓	31										
1	6	06										
2	IF $x>y$	53				SET FLAG IF $6>K$						
3	SET FLAG	54										
4	CLEAR x	37										
5	↓	25				CALCULATE $f+2$						
6	2	02										
7	ACC +	60										
8	IF FLAG	43				BRANCH WHEN FLAG IS SET						
9	2	02										
a	d	17										
b	ACC +	60				CALCULATE $f+2$ AND SET FLAG						
c	SET FLAG	54										
d	RECALL	61				SET $K=f$						

FROM 0-b

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	GO TO ( ) ( )	44									
	1	0	00	BRANCH TO CONTINUE FACTORING								
	2	8	10									
	3	d	17									
	4	↑	27									
	5	1	01	POSITION DISPLAY AND LAST FACTOR								
	6	↑	27									
	7	e	12									
	8	END	46	LAST FACTOR	1	X	DISPLAY					
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

PART NO.
09100-70014

REAL ROOTS OF f(x)

This program finds the real roots of equations of the form $f(x) = 0$. The solution evaluates $f(x)$ at a specified starting point x_0 and at successive x's by replacing x with $x + \Delta x$ (the initial Δx is also specified by the user). When $f(x)$ changes sign, Δx is replaced with $-\Delta x/10$.

The search continues until $f(x)$ is driven to zero. The value of x such that $f(x) = 0$ is a root of the equation. $f(x)$ must change sign for the technique to converge.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

SET: | PROGRAM | |

Starting at (6) (0) enter the program steps which take the independent variable from the X register and program f(x), (x is also contained in the d register). Place f(x) is the X register and exit to location 0-6. The last four steps of the f(x) subroutine should be:

GO TO
0
6
END

Note: Registers 6, 7, 8, 9, a, b, c, e, and f are available for programming and storage of f(x). Register d and the flag are unavailable.

SET: | | RUN |

PRESS: END

Store initial conditions (when applicable).

ENTER: $x_i \to X$ (x_i is initial x from which search is to begin.)

PRESS: CONTINUE

DISPLAY

$\pm f(x_{i-1}) \cdot f(x_i)$	Z
$f(x_i)$	Y
x_i	X

Note: $f(x_{i-1})$ is random the first pass and product is meaningless

Is x_i Satisfactory ?

NO (return to PRESS: END)

YES

ENTER: $\Delta x \to x$. (Δx is the searching increment. $+\Delta x$ searches right, $-\Delta x$ searches left)

USER INSTRUCTIONS (con't)

PRESS: CONTINUE

The real roots of f(x) program successively evaluates f(x).

PAUSE DISPLAY

$\pm f(x_{i-1}) \cdot f(x_i)$	Z
$f(x_i)$	Y
x_i	X

The calculation stops when x_i is a root, x_i is successively replaced by $x_i \pm \Delta x$, (See flowchart). If calculation is not converging, press: PAUSE, press: END and enter a new estimate as before.

TO ENTER A NEW PROBLEM:

PRESS: END

PRESS: GO TO (0) (0)

ENTER NEW DATA AS BEFORE

EXAMPLES

1. FIND ROOT OF x = cos x

SET: | | RADIANS |

ENTER DATA: $x_i = 0 \to X$

PRESS: END

PRESS: CONTINUE

DISPLAY

(JUNK)	Z	
-1.000	Y	(Y contains f(x))
0	X	(X contains x)

ENTER DATA: $x = 0.1 \to X$

PRESS: CONTINUE

DISPLAY

0	Z	
0	Y	
0.7390851332	X	(X contains root)

2.

$$y = \frac{\pi^2 \sqrt{x}}{1 + \cos \frac{\pi}{2} x}$$

Given the initial x condition y = 4, find x.

Rewrite equation as

$$f(x) = y(1 + \cos \frac{\pi}{2} x) - \pi^2 \sqrt{x}$$

User subroutine assumes y is in a (it will be manually entered into a as an initial condition)

SET: | | RADIANS |

ENTER DATA: y = 4 → X

PRESS: x→(a) to satisfy initial condition that y is in a.

PRESS: END

ENTER DATA: Initial x_i = 0 X

PRESS: CONTINUE

DISPLAY

(JUNK) —— Z		
8.00 —— Y		f (x, 4) in Y
0 —— X		(x in X)

Instead of entering Δ x, evaluate f(x) at a new value x_i

PRESS: END

ENTER DATA: x_i = 1 → X

PRESS: CONTINUE

DISPLAY

(JUNK) —— Z	
-5.87 —— Y	f (x, 4) in Y
1.0 —— X	(x in X)

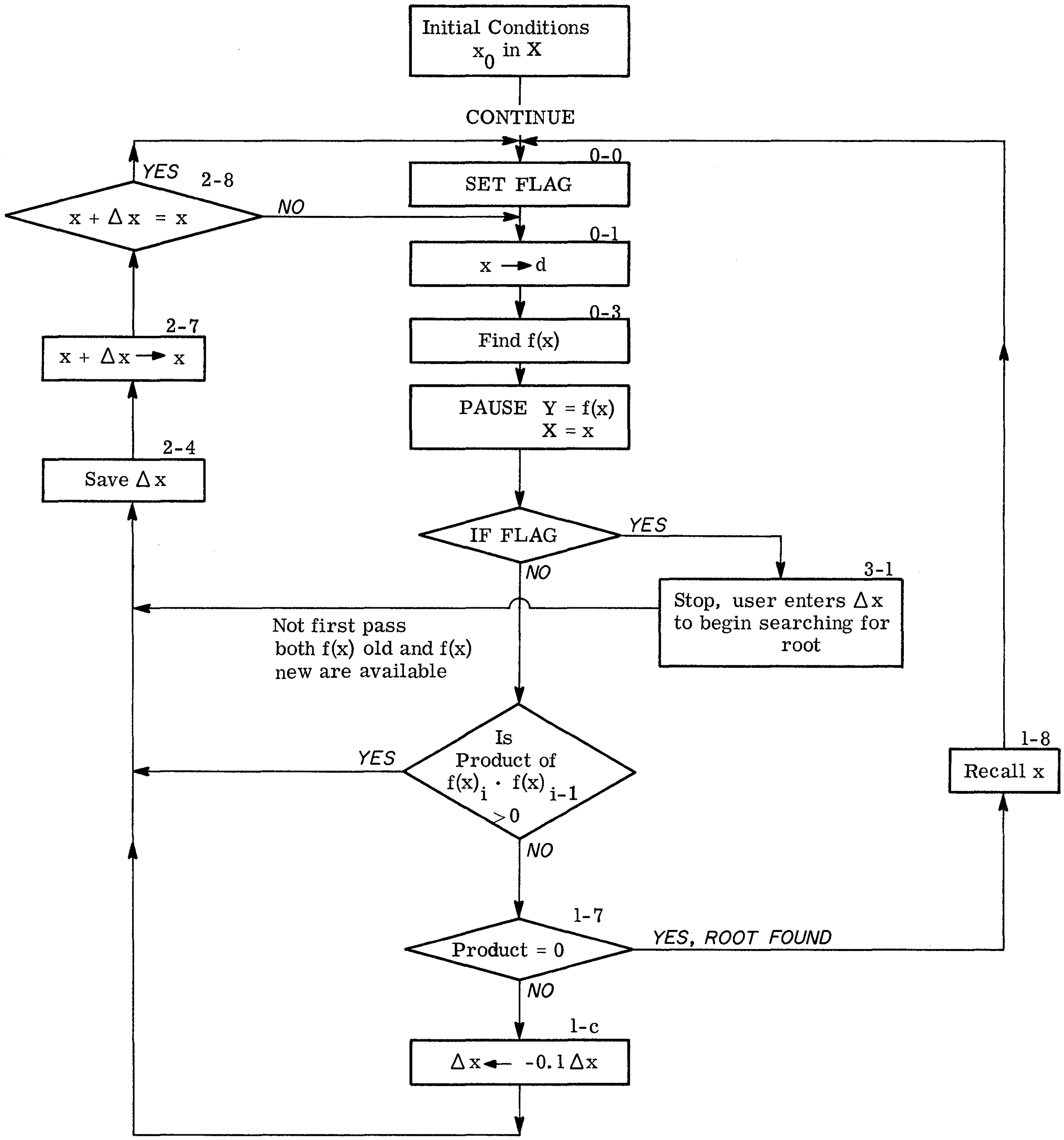
A root has been crossed between f(0) and f(1) because f(x) changed sign. Begin search to the left by entering

x = -0.1 → X

PRESS: CONTINUE

DISPLAY

4×10^{-22}		
-2	-11	f(x, 4) in Y
4.868292372	-01	X contains root



Initial Conditions
x0 in X
CONTINUE
0-0
SET FLAG
0-1
x → d
0-3
Find f(x)
PAUSE Y = f(x)
X = x
IF FLAG
YES
NO
3-1
Stop, user enters Δx to begin searching for root
Not first pass both f(x) old and f(x) new are available
Is Product of f(x)i · f(x)i-1 > 0
YES
NO
1-7
Product = 0
YES, ROOT FOUND
NO
1-c
Δx ← -0.1Δx
1-8
Recall x
2-4
Save Δx
2-7
x + Δx → x
2-8
x + Δx = x
YES
NO

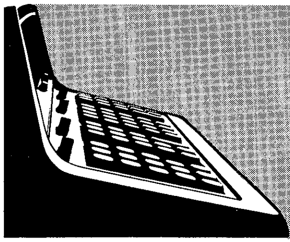
Step		Key	Code	Display x	Display y	Display z	Storage f e d c b a
6	0	↑	27	x	x		
	1	cos x	73	cos(x)			EXAMPLE 1 FINDS THE ROOT
	2	—	34		x − cos(x) = f(x)		OF x = cos x. REWRITE THE
	3	↓	25	f(x)			EQUATION AS f(x) = x − cos x.
	4	GO TO ( )( )	44				
	5	0	00				
	6	6	06				
	7	END	46				
	8						
	9						
	a						
	b						
	c						
	d	EXAMPLE 2					
6	0	↑	27	x	x		
	1	π	56	π			EXAMPLE 2 FINDS THE ROOT
	2	×	36		πx		
	3	2	02	2			OF $y = \frac{\pi^2\sqrt{x}}{1+\cos\frac{\pi}{2}x}$. GIVEN
	4	÷	35		$\pi x/2$		THE INITIAL CONDITION THAT
	5	1	01	1			y = 4, FIND x. REWRITE
	6	$x \rightleftarrows y$	30	$\pi x/2$	1		THE EQUATION AS
	7	cos x	73	$\cos(\pi x/2)$			$f(x) = y(1+\cos\frac{\pi x}{2}) - \pi^2\sqrt{x}$.
	8	+	33		$1+\cos(\pi x/2)$		
	9	a	13	y			
	a	×	36		$y[1+\cos(\pi x/2)]$		
	b	d	17	x			
	c	$\sqrt{x}$	76	$\sqrt{x}$			
	d	↑	27		$\sqrt{x}$	$y[1+\cos(\pi x/2)]$	
7	0	π	56	π			
	1	×	36		$\pi\sqrt{x}$		
	2	×	36		$\pi^2\sqrt{x}$		
	3	↓	25	$\pi^2\sqrt{x}$	$y[1+\cos(\pi x/2)]$		
	4	—	34		f(x,y)		
	5	↓	25	f(x,y)			
	6	GO TO ( )( )	44				
	7	0	00				
	8	6	06				
	9	END	46				
	a						
	b						
	c						
	d						

	Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
	0	0	SET FLAG	54									
FROM 3-0		1	x→( )	23									
		2	d	17									
		3	GO TO( )( )	44									
		4	6	06									
		5	0	00									
		6	↑	27									
		7	y⇄( )	24									
		8	4	04									
		9	×	36									
		a	↑	27	NEGATIVE PRODUCT MEANS ROOT JUST PASSED, ZERO PRODUCT MEANS ROOT FOUND								
		b	d	17									
		c	PAUSE	57	x_i	$f(x_i)$	$f(x_{i-1}) \cdot f(x_i)$						
		d	IF FLAG	43									
	1	0	3	03									
		1	1	01									
		2	ROLL ↓	31									
		3	CLEAR x	37									
		4	IF x<y	52									
		5	2	02									
		6	0	00									
		7	IF x=y	50									
		8	ROLL ↑	22									
		9	GO TO( )( )	44	RECALCULATE f(x) SET FLAG AT (0)-(0) TO STOP AT (2)-(c) ON NEXT PASS								
		a	0	00									
		b	0	00									
		c	CHG SIGN	32									
		d	•	21									
	2	0	1	01									
		1	y⇄( )	24									
		2	5	05									
		3	×	36									
FROM 3-4		4	y→( )	40									
		5	5	05									
		6	d	17									
		7	+	33									
		8	IF x=y	50	IF Xnew = Xold THEN ROOT HAS BEEN FOUND AS NEAR AS POSSIBLE								
		9	0	00									
		a	0	00									
		b	↓	25									
		c	GO TO( )( )	44									
		d	0	00									

PART NO. 09100-70014

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	1	01									
	1	STOP	41	x_i	$f(x_i)$	$f(x_i) \cdot f(x_{i-1})$						
	2	↑	27									
	3	GO TO ( ) ( )	44									
	4	2	02									
	5	4	04									
	6	CONTINUE	47									
	7											
	8											
	9											
	a											
	b											
	c											
	d											
4	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
5	0	CONTINUE	47									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	END	46	x					x			
	1			ENTER SPECIFIC f(x) STARTING AT 6-0. THE X AND d								
	2			REGISTER CONTAIN x. USE THIS VARIABLE TO PROGRAM								
	3			f(x); LEAVE f(x) IN X AND EXIT TO LOCATION 0-6								
	4			FOLLOWED BY END.								
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



SIMULTANEOUS SOLUTION OF TWO EQUATIONS IN TWO UNKNOWNS

PART NO. 09100-70016

This program solves two independent equations simultaneously for the two unknowns.

The two equations are of the following form:

$$ax + by = e$$
$$cx + dy = f$$

where x and y are the two unknowns to be found.

The solution is obtained by normalizing the coefficient of x to 1, yielding the equations:

$$x = e/a - (b/a)y \qquad (1)$$

$$x = f/c - (d/c)y \qquad (2)$$

Substracting gives the solution for y:

$$y = \frac{(e/a - f/c)}{(b/a - d/c)}$$

The value of x is then found by substituting the value of y back into the second equation.

Reference:

Numerical Analysis
by
Kaiser S. Kunz

McGraw-Hill, 1st edition
(1957)

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address is (0) (0))

PRESS: GO TO (0) (0) or [END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
0	——	X

ENTER DATA: a → Z, b → Y, e → X

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
0	——	X

ENTER DATA: c → Z, d → Y, f → X

PRESS: CONTINUE

DISPLAY

0	——	Z
y	——	Y
x	——	X

TO RESET PROBLEM:

EXAMPLE

Solve the following equations simultaneously for x and y.

(1) $2x + y = 6$

(2) $2x + 3y = 1$

Data to be entered:

2 → Z
1 → Y (1)
6 → X

2 → Z
3 → Y (2)
1 → X

Solution:

$x = 4.25$
$y = -2.50$

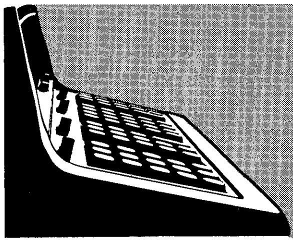
General Form:

$ax + by = e$
$cx + dy = f$

PART NO. 09100-70016

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	SET FLAG	54									
2	ACC +	60									
3	CLEAR x	37	CLEAR OUT STORAGE AND SET THE FLAG								
4	↑	27									
5	↑	27									
6	STOP	41	e	b	a	ENTER DATA: (COEFFICIENTS AND CONSTANT OF FIRST EQUATION)					
7	ROLL ↑	22									
8	÷	35									
9	ROLL ↑	22	CALCULATE e/a AND b/a ON THE FIRST PASS; f/c AND d/c ON THE SECOND PASS								
a	$x \rightleftarrows y$	30									
b	÷	35									
c	↓	25									
d	IF FLAG	43									
1 0	0	00	BRANCH TO STORE e/a AND b/a, AND ENTER COEFFICIENTS AND CONSTANT OF SECOND EQUATION								
1	2	02									
2	ACC −	63	CALCULATE (b/a − d/c) AND (e/a − f/c)								
3	$x \rightleftarrows y$	30									
4	$y \rightleftarrows ( )$	24									
5	e	12	CALCULATE y								
6	f	15									
7	÷	35									
8	e	12									
9	$x \rightleftarrows y$	30									
a	×	36	CALCULATE x								
b	ROLL ↓	31									
c	−	34									
d	CLEAR x	37									
2 0	ROLL ↓	31									
1	END	46	x	y	0						
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



n! $(n<10^{12})$

PART NO.
09100-70017

This program calculates n! for integer n where $0 \leq n < 10^{12}$

$$n! = n\ (n - 1)\ \ldots\ 3 \cdot 2 \cdot 1$$

No Reference

USER INSTRUCTIONS	EXAMPLES

ENTER PROGRAM: (Starting Address is (0) (0))

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA:

n ——→ X

PRESS: CONTINUE

DISPLAY

Exponent of 10	———	Z
Decimal Number of n!	———	Y
n	———	X

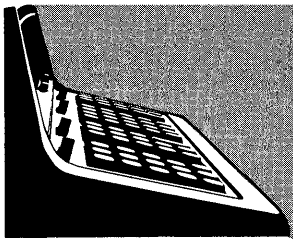
TO RESET PROBLEM:

DATA	SOLUTION	TIME
A) n = 0	n! = 1	
B) n = 7	n! = 5.040×10^3	1 sec.
C) n = 1000	n! = 4.0239×10^{2567}	16 sec.
D) n = 10^4	n! = 2.8463×10^{35659}	2 min. 53 sec.

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	STOP	41	n	0	0	ENTER n					
2	$x\to( )$	23	STORE n								
3	d	17									
4	↑	27									
5	1	01									
6	$x\rightleftarrows y$	30									
7	ROLL ↑	22									
8	ACC +	60									
9	IF $x=y$	50	BRANCH IF n=0 TO SET n!=1								
a	2	02									
b	c	16									
c	↓	25	FORM n·(n−1)·(n−2)··· 3.2.1								
d	×	36									
1 0	ENTER EXP	26									
1	9	11									
2	0	00									
3	IF $x>y$	53	BRANCH IF n! (PARTIAL) < 10^{90} TO DECREMENT (n−k) BY 1								
4	2	02									
5	3	03									
6	ENTER EXP	26	REDUCE EXPONENT OF n! (PARTIAL) BY 90's AND SUM THE VALUE OF THE EXPONENT OVERFLOW IN f								
7	CHG SIGN	32									
8	9	11									
9	0	00									
a	×	36									
b	0	00									
c	↑	27									
d	9	11									
2 0	0	00									
1	ACC +	60									
2	↓	25	DECREMENT (n−k) BY 1								
3	1	01									
4	↑	27									
5	CLEAR x	37									
6	ACC −	63									
7	RCL	61									
8	CLEAR x	37									
9	GO TO ( )( )	44	BRANCH TO MULTIPLY THE PARTIAL FACTORIAL BY (n−k−1)								
a	0	00									
b	9	11									
c	↓	25									
d	ENTER EXP	26									

FROM 3-c

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	9	11									
	1	IF $x > y$	53									
	2	3	03									
	3	d	17	REDUCE EXPONENT OF n! BY 9's AND								
	4	ENTER EXP	26	SUM THE VALUE OF EXPONENT OVERFLOW IN f								
	5	CHG SIGN	32									
	6	9	11									
	7	×	36									
	8	9	11									
	9	ACC +	60									
	a	GO TO ( )( )	44									
	b	2	02	BRANCH TO COMPARE n! TO 10^9								
	c	d	17									
	d	1	01									
4	0	0	00									
	1	IF $x > y$	53	BRANCH IF ARGUMENT $(n!) < 10^{12}$ TO RECALL								
	2	4	04	RESULTANT EXPONENT AND DISPLAY								
	3	c	16									
	4	.	21									
	5	1	01									
	6	×	36	REDUCE EXPONENT OF n! BY 1's AND SUM								
	7	1	01	THE VALUE OF EXPONENT OVERFLOW IN f								
	8	ACC +	60									
	9	GO TO ( )( )	44									
	a	3	03	BRANCH TO COMPARE n! TO 10^9								
	b	d	17									
	c	f	15									
	d	$x \rightleftarrows y$	30	RECALL n AND EXPONENT OF n!								
5	0	↑	27									
	1	d	17									
	2	END	46	n	DEC. NO. (n!)	EXP. OF 10	DISPLAY					
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



PART NO.
09100-70021

POLYNOMIAL EVALUATION ($0 \leq n \leq 10$)

This program will evaluate a polynomial of the form:

$$f(x) = A_n x^n + A_{n-1} x^{n-1} \ldots A_1 x + A_0$$

$$\text{for} \quad 1 \leq n \leq 10$$

Input data is a_n, A_{n-1} . . . A_0, and the value of x for which the polynomial is to be evaluated. Any coefficient (A_i) may be zero. If any term(s) of the polynomial is absent, then the coefficient of that term(s) must be entered as zero.

The value of f(x) for a new x may be obtained by entering the new x after the last f(x) has been calculated. To enter new coefficients for a different f(x), the program card must be re-entered and the user instructions repeated. A_i's and x's must be real numbers.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) or [END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $A_{10} \rightarrow Z$, $A_9 \rightarrow Y$, $A_8 \rightarrow X$

ENTER 0 if term does not exist.

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $A_7 \rightarrow Z$, $A_6 \rightarrow Y$, $A_5 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $A_4 \rightarrow Z$, $A_3 \rightarrow Y$, $A_2 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $A_1 \rightarrow Y$, $A_0 \rightarrow X$

PRESS: CONTINUE

0	Z
0	Y
2	X

USER INSTRUCTIONS

ENTER DATA: $x \rightarrow X$

PRESS: CONTINUE

DISPLAY

x	Z
f(x)	Y
0	X

Wish to enter a new x ?

YES → ENTER DATA: $x \rightarrow X$

NO → ENTER PROGRAM

EXAMPLES

(A) Polynomial: $x + 10$ ($A_1 = 1$, $A_0 = 10$)

x	f(x)
1	11
2	12
3	13

(B) Polynomial: $x^9 - 2x^8 + 4x^7 + x^2 + x + 100$

($A_9=1$, $A_8=-2$, $A_7=4$, $A_2=1$, $A_1=1$, $A_0=100$)

x	f(x)
1	105
2	618
.1	100.11000

General Form:

$f(x) = A_n x^n + A_{n-1} x^{n-1} \ldots A_1 x + A_0 \quad 1 \leq n \leq 10$

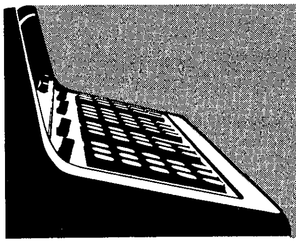
PART NO 09100-70021

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a)
0 0	CLEAR	20				
1	1	01				
2	STOP	41	A_8	A_9	A_{10}	ENTER DATA
3	x→( )	23				
4	a	13				
5	y→( )	40				
6	8	10	STORE A_{10}, A_9, AND A_8			
7	ROLL ↓	31				
8	y→( )	40				
9	9	11				
a	CLEAR	20				
b	STOP	41	A_5	A_6	A_7	ENTER DATA
c	x→( )	23				
d	b	14				
1 0	y→( )	40				
1	c	16	STORE A_7, A_6, AND A_5			
2	ROLL ↓	31				
3	y→( )	40				
4	d	17				
5	CLEAR	20				
6	STOP	41	A_2	A_3	A_4	ENTER DATA
7	x→( )	23				
8	0	00				
9	ROLL ↓	31				
a	x→( )	23	STORE A_4, A_3, A_2			
b	7	07				
c	y→( )	40				
d	1	01				
2 0	CLEAR	20				
1	STOP	41	A_0	A_1	0	ENTER DATA
2	ACC +	60	STORE A_0, AND A_1			
3	↓	25				
4	2	02	X	0	0	ENTER DATA
5 (FROM-6-C)	STOP	41	0	f(X)	X	AND DISPLAY
6	y⇄( )	24				
7	9	11				
8	y→( )	40				
9	9	11				
a	×	36				
b	ROLL ↓	31				
c	y⇄( )	24				
d	8	10				

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	y→( )	40									
1	8	10									
2	+	33									
3	↓	25									
4	×	36									
5	a	13									
6	+	33									
7	↓	25									
8	×	36									
9	d	17									
a	+	33									
b	↓	25									
c	×	36									
d	c	16									
4 0	+	33									
1	↓	25									
2	×	36									
3	b	14									
4	+	33									
5	↓	25	CALCULATE f (x)								
6	×	36									
7	↓	25									
8	y⇄( )	24									
9	1	01									
a	y→( )	40									
b	1	01									
c	+	33									
d	↓	25									
5 0	×	36									
1	↓	25									
2	y⇄( )	24									
3	7	07									
4	y→( )	40									
5	7	07									
6	+	33									
7	↓	25									
8	×	36									
9	↓	25									
a	y⇄( )	24									
b	0	00									
c	y→( )	40									
d	0	00									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6 0	+	33									
1	↓	25									
2	×	36									
3	e	12									
4	+	33									
5	↓	25									
6	×	36									
7	f	15									
8	+	33									
9	CLEAR x	37									
a	GO TO ( )( )	44	BRANCH TO DISPLAY f(x) AND ENTER A NEW x IF DESIRED								
b	2	02									
c	5	05									
d	END	46									

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



3x3 MATRIX INVERSION OR SIMULTANEOUS SOLUTION OF THREE EQUATIONS IN THREE UNKNOWNS

PART NO.
09100-70022

This program inverts a 3x3 matrix of the form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

and calculates:

$$A^{-1} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

where

$$A \cdot A^{-1} = I$$

where

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

or solves three independent equations of the form:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = K_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = K_2$$

$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = K_3$$

for x_1, x_2, and x_3.

Reference: Elementary Matrix Algebra
Franz E. Hohn
1958

This program is a destructive program

ENTER PROGRAM: (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA*:

$a_{11} \rightarrow Z$
$a_{21} \rightarrow Y$
$a_{31} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA:

$a_{12} \rightarrow Z$
$a_{22} \rightarrow Y$
$a_{32} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

*If the program for either mode should turn on the RED light in the RUN mode, interchange the second and third rows of the matrix or the second and third equations of the set and run the program again. If the RED light still comes on, the equations are dependent or the matrix is singular and a solution cannot be obtained. If the matrix or equation set is ill-conditioned, an answer will be obtained but the accuracy of the solution will suffer.

ENTER DATA:

$a_{13} \rightarrow Z$
$a_{23} \rightarrow Y$
$a_{33} \rightarrow X$

Wish to solve three equations simultaneously?

NO → [Wish to invert the input matrix]

YES (invert the input matrix):

PRESS: CONTINUE

DISPLAY

b_{11}	Z
b_{21}	Y
b_{31}	X

PRESS: CONTINUE

DISPLAY

b_{12}	Z
b_{22}	Y
b_{32}	X

PRESS: CONTINUE

DISPLAY

b_{13}	Z
b_{23}	Y
b_{33}	X

Desire check?

NO ↓

YES:

ENTER DATA:

$b_{11} \rightarrow Z$
$0 \rightarrow Y$
$0 \rightarrow X$

PRESS: CONTINUE

DISPLAY

a_{11}	Z
a_{21}	Y
a_{31}	X

YES (solve three equations simultaneously):

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

	Z
	Y
1	X

ENTER DATA:
(Do not disturb Y and Z registers when K_i's (i = 1, 2, 3) are being entered) $K_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

	Z
	Y
0	X

ENTER DATA: $K_3 \rightarrow X$

PRESS: CONTINUE

DISPLAY

	Z
	Y
0	X

ENTER DATA: $K_2 \rightarrow X$

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

PRESS: CONTINUE

DISPLAY

a_{21} — Z
a_{22} — Y
a_{32} — X

PRESS: CONTINUE

DISPLAY

a_{31} — Z
a_{32} — Y
a_{33} — X

TO RESET PROBLEM:
re-enter program and repeat USER INSTRUCTIONS.

DISPLAY

x_1 — Z
x_2 — Y
x_3 — X

TO RESET PROBLEM:
re-enter program and repeat USER INSTRUCTIONS.

EXAMPLES

(A) Find the inverse of the following matrix (A).
General form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 8 & 4 & 2 \\ 2 & 8 & 4 \\ 1 & 2 & 8 \end{bmatrix}$$

Solution:

$$A^{-1} = \begin{bmatrix} .1429 & -.0714 & 0 \\ -.0306 & .1582 & -.0714 \\ -.0102 & -.0306 & .1429 \end{bmatrix}$$

(B) Solve the following three equations simultaneously.
General form:

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = K_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = K_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = K_3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 + 2x_3 = 3 \\ 4x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 6x_3 = 4 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} x_1 = -3 \\ x_2 = 4 \\ x_3 = 1 \end{bmatrix}$$

STORAGE ALLOCATION OF THE $\{aij\}$ or $\{bij\}$ IS AS FOLLOWS:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ OR } \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

EXAMPLES (con't)

Stored in the following registers

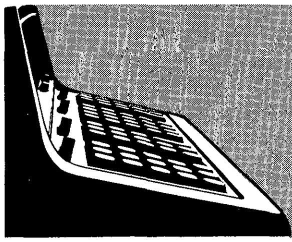
$$\begin{bmatrix} z & 0 & a \\ b & e & 1 \\ c & f & d \end{bmatrix}$$

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0 0	CLEAR	20									
1	STOP	41	a_{31}	a_{21}	a_{11}	ENTER DATA					
2	$x\to()$	23									
3	c	16									
4	$y\to()$	40									
5	b	14	STORE COEFFICIENTS								
6	↓	25									
7	$y\to()$	40									
8	a	13									
9	CLEAR	20									
a	STOP	41	a_{32}	a_{22}	a_{12}	ENTER DATA					
b	$y\rightleftarrows()$	24									
c	e	12									
d	ROLL ↑	22									
1 0	$y\rightleftarrows()$	24	STORE COEFFICIENTS								
1	f	15									
2	$x\to()$	23									
3	0	00									
4	CLEAR x	37									
5	STOP	41	a_{33}	a_{23}	a_{13}	ENTER DATA					
6	$x\to()$	23									
7	d	17									
8	CLEAR x	37									
9	ROLL ↓	31	STORE COEFFICIENTS								
a	$y\rightleftarrows()$	24									
b	a	13									
c	$x\to()$	23									
d	1	01									
2 0	CLEAR x	37									
1	↑	27									
2 (FROM 8-1 & 9-b)	$y\rightleftarrows()$	24									
3	0	00									
4	$x\rightleftarrows y$	30									
5	ROLL ↑	22									
6	÷	35									
7	$y\rightleftarrows()$	24	CALCULATE AND STORE a_{12}/a_{11} AND a_{13}/a_{11}								
8	c	16									
9	$y\rightleftarrows()$	24									
a	a	13									
b	÷	35									
c	$y\rightleftarrows()$	24									
d	f	15									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	↑	27									
1	1	01									
2	IF FLAG	43									
3	STOP	41	K_1	0	0	ENTER K_1					
4	SET FLAG	54									
5	$x \rightleftarrows y$	30	CALCULATE K_1/a_{11} (steps 4–6)								
6	÷	35									
7	$y \rightleftarrows ( )$	24									
8	d	17									
9	$y \rightleftarrows ( )$	24									
a	0	00	REPOSITION COUNTER (steps 9–c)								
b	$y \rightleftarrows ( )$	24									
c	a	13									
d	c	16									
4 0	$x \rightleftarrows y$	30									
1	×	36									
2	ROLL ↓	31	CALCULATE AND STORE $a_{33} - \frac{a_{31} a_{12}}{a_{11}}$ (steps 0–5)								
3	−	34									
4	$y \rightleftarrows ( )$	24									
5	b	14									
6	$y \rightleftarrows ( )$	24									
7	0	00									
8	f	15									
9	ROLL ↑	22									
a	×	36	CALCULATE AND STORE $a_{33} - \frac{a_{31} a_{13}}{a_{11}}$ (steps 4 6 – 5 0)								
b	ROLL ↓	31									
c	−	34									
d	$y \rightleftarrows ( )$	24									
5 0	e	12									
1	d	17									
2	ROLL ↑	22									
3	CHG SIGN	32									
4	×	36									
5	CLEAR x	37									
6	IF FLAG	43	CALCULATE AND STORE $K_3 - \frac{a_{31} K_1}{a_{11}}$ (steps 1–b)								
7	STOP	41									
8	SET FLAG	54									
9	+	33									
a	$y \rightleftarrows ( )$	24									
b	1	01									
c	$y \rightleftarrows ( )$	24									
d	0	00									

Step	Key	Code	Display x	Display y	Display z	Storage (f, e, d, c, b, a)
6 0	c	16				
1	x⇄y	30	CALCULATE $a_{22} - \frac{a_{21}a_{12}}{a_{11}}$			
2	×	36				
3	ROLL ↓	31				
4	−	34				
5	f	15				
6	ROLL ↑	22				
7	×	36				
8	x⇄y	30				
9	y⇄( )	24	CALCULATE AND STORE $a_{23} - \frac{a_{21}a_{13}}{a_{11}}$			
a	0	00				
b	−	34				
c	y⇄( )	24				
d	0	00				
7 0	d	17				
1	CHG SIGN	32				
2	×	36	CALCULATE $K_2 - \frac{a_{21}K_1}{a_{11}}$			
3	CLEAR x	37				
4	IF FLAG	43				
5	CLEAR x	37				
6	STOP	41	K_2	0	0	ENTER K_2
7	+	33				
8	y⇄( )	24				
9	a	13	INCREMENT COUNTER			
a	1	01				
b	+	33				
c	3	03				
d	IF x > y	53				
8 0	2	02	BRANCH WHEN 3 IS GREATER THAN COUNTER AND REPEAT CALCULATION			
1	2	02				
2	b	14				
3	x⇄y	30	RECALL AND POSITION SOLUTION OR b_{11}, b_{21}, b_{31}			
4	c	16				
5	STOP	41	b_{31}	b_{21}	b_{11}	DISPLAY SOLUTION OR b_{11}, b_{21}, b_{31}
6	y⇄( )	24				
7	0	00				
8	y→( )	40				
9	0	00	RECALL AND POSITION DISPLAY			
a	e	12				
b	↑	27				
c	f	15				
d	STOP	41	b_{32}	b_{22}	b_{12}	DISPLAY

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9 0	y⇄( )	24									
1	1	01									
2	y→( )	40									
3	1	01									
4	d	17	RECALL AND POSITION DISPLAY (steps 0–7)								
5	↑	27									
6	a	13									
7	ROLL ↓	31									
8	STOP	41	b_{33}	b_{23}	b_{13}	DISPLAY					
9	GO TO ( ) ( )	44									
a	2	02	BRANCH TO DISPLAY COEFFICIENTS OF ORIGINAL MATRIX IF CHECK IS DESIRED (steps 9–b)								
b	2	02									
c	END	46									
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70023

FOURIER SERIES
when f(t) is known

This program calculates the Fourier Series coefficients a_n and b_n that represent a periodic time function f(t) with period T. The Fourier Series representation of f(t) is:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{2\pi nt}{T} + b_n \sin\frac{2\pi nt}{T}$$

where

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos\frac{2\pi nt}{T}\,dt \qquad n = 0, 1, 2, \ldots.$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin\frac{2\pi nt}{T}\,dt \qquad n = 1, 2, 3, \ldots.$$

The program evaluates the coefficients by numerically integrating using Simpson's Rule which is the same procedure used in program 09100-70015.

The specific f(t) is programmed into the Calculator by the user and is then used by the general solution to evaluate the coefficients. Execution time is dependent on the number of panels.

Reference: Signals Systems and Communications
B. P. Lathi
Wiley (1965)

USER INSTRUCTIONS

SET: | RADIANS | |

ENTER PROGRAM: (Starting Address is 0 - 0)

PRESS: GO TO () ()

PRESS: 6

PRESS: 4

SET: | PROGRAM | |

ENTER: Steps to take t from the X register (t is also stored in the e register) and form f(t). Leave f(t) in the Y register and exit to location 8-4. Steps 6-4 through 8-3 are available for forming f(t). The FLAG is not available.

SET: | | RUN |

PRESS: GO TO () ()

PRESS: 9

PRESS: 9

SET: | PROGRAM | |

PRESS: sin X or cos X depending on desired sine or cosine coefficients. If series involves both sine and cosine, then initialize program to cos X.

SET: | | RUN |

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
1	———	X

(n is set to 0)

ENTER DATA:

Number of panels must be an even number → Z

$t_0 + T$ ——→ Y

t_0 ——→ X

Note: n is contained in the a register.

→ PRESS: CONTINUE

DISPLAY

$t_0 + T$	———	Z
a_n or b_n	———	Y
n	———	X

To change from cosine series to sine series--

USER INSTRUCTIONS (con't)

PRESS: GO TO () ()

PRESS: 9

PRESS: 9

SET: | PROGRAM | |

PRESS: sin X

SET: | | RUN |

PRESS: CLEAR

PRESS: X → ()

PRESS: a

(n is set to 1)

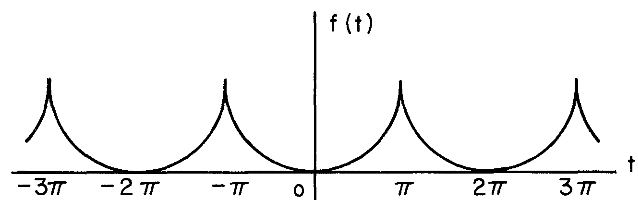
PRESS: GO TO () ()

PRESS: 5

PRESS: 7

EXAMPLES

Example #1



$f(t) = t^2 \qquad -\pi \leq t \leq \pi$

$$t^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$$

Data: Number of panels = 100
$T = 2\pi$
$t_0 = -\pi$

Solution:

$t_0 + T = 3.14159$

$a_0 = 6.57974$

$a_1 = -4.00000$

$a_2 = 1.00000$

$a_3 = -.44444$

$a_4 = .24998$

$a_5 = -.15997$

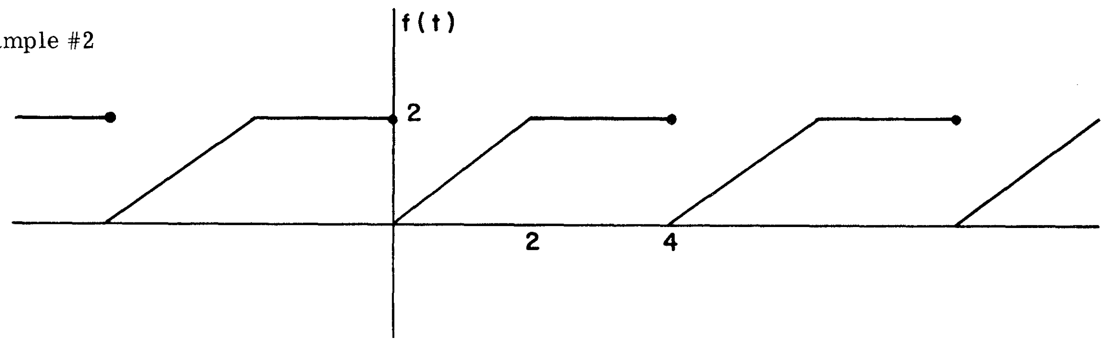
$a_6 = .11107$

$a_7 = -.08158$

$a_8 = .06243$

$b_n = 0$; $f(t) = f(-t)$

Example #2



$f(t) = t \qquad 0 < t \leq 2$

$f(t) = 2 \qquad 2 \leq t \leq 4$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T}\right)$$

Data:

$T = 4$

$t_0 = 0$

Number of panels = 50

Solution:

$t_0 + T = 4.0$

$a_0 = 3.00053$	$b_1 = -.63662$
$a_1 = -.40582$	$b_2 = -.31832$
$a_2 = .00054$	$b_3 = -.21223$
$a_3 = -.04557$	$b_4 = -.15921$
$a_4 = .00057$	$b_5 = -.12744$
$a_5 = -.01676$	$b_6 = -.10631$
$a_6 = .00062$	$b_7 = -.09128$
$a_7 = -.00883$	$b_8 = -.08009$
$a_8 = .00069$	

Note: See step page 1 for steps needed to execute examples 1 and 2.

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0				EXAMPLE 1							
	1											
	2											
	3			t				t				
	4	↑	27	t	t							
	5	×	36		$t^2 = f(t)$							
	6	GO TO ( )( )	44									
	7	8	10									
	8	4	04									
	9											
	a											
	b											
	c											
	d											
6	0				EXAMPLE 2							
	1											
	2											
	3			t				t				
	4	↑	27	t	t							
	5	2	02	2								
	6	IF $x > y$	53									
	7	8	10									
	8	4	04									
	9	$x \rightleftarrows y$	30	t	2							
	a	GO TO ( )( )	44									
	b	8	10									
	c	4	04									
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a)
0 0	CLEAR	20				
1	$x\to()$	23	INITIALIZE n TO ZERO			
2	a	13				
3 (FROM 4-3 & 4-9)	1	01				ENTER DATA: NUMBER OF PANELS (MUST BE EVEN), t_0+T, t_0
4	STOP	41	t_0	t_0+T	PANELS	
5	$y\to()$	40	STORE DATA			
6	d	17				
7	$x\to()$	23				
8	b	14				
9	—	34	CALCULATE Δt			
a	ROLL ↓	31				
b	$x\rightleftarrows y$	30				
c	÷	35				
d	$y\to()$	40				
1 0	c	16				
1	ROLL ↑	22	STORE t_0			
2 (FROM 4-9)	$x\to()$	23				
3 (FROM 6-3)	e	12				
4	GO TO ( )( )	44	BRANCH TO EVAULATE FUNCTION f(t)			
5	6	06				
6	4	04				
7 (FROM 9-d)	e	12				
8	↑	27				
9	b	14	RECALL t_0 AND BRANCH FOR INITIAL POINT			
a	IF $x=y$	50				
b	4	04				
c	4	04				
d	d	17				
2 0	—	34	COMPARE $\lvert t-(T+t_0)\rvert$ TO 10^{-9} AND STOP INTEGRATION IF $\lvert t-(T+t_0)\rvert < 10^{-9}$			
1	$\lvert y\rvert$	55				
2	ENTER EXP	26				
3	9	11				
4	CHG SIGN	32				
5	IF $x>y$	53				
6	4	04				
7	a	13				
8	e	12	COMPARE t TO $T+t_0$ AND STOP INTEGRATION IF $t>T+t_0$			
9	$x\rightleftarrows y$	30				
a	d	17				
b	IF $x<y$	52				
c	4	04				
d	a	13				

Step	Key	Code	Branch	Remarks (Display: x, y, z)
3 0	C	16		
1	ROLL ↑	22		
2	↑	27		
3	IF FLAG	43		
4	3	03		
5	b	14	→ 3-b	
6	SET FLAG	54		
7	4	04		CALCULATE $4g(t_0+\Delta t)+2g(t_0+2\Delta t)$
8	GO TO ( )( )	44		
9	3	03		
a	C	16	→ 3-c	
b	2	02		
c	×	36		
d	↓	25		
4 0	ACC +	60		
1	GO TO ( )( )	44		
2	1	01		BRANCH TO CALCULATE ANOTHER PARTIAL AREA
3	3	03		
4	C	16	FROM 1-c	RECALL Δt AND CALCULATE $t+\Delta t$.
5	ROLL ↑	22		PLACE $t+\Delta t$ AND $g(t)$ IN THE ACCUMULATORS
6	ACC +	60		
7	GO TO ( )( )	44		
8	1	01		BRANCH FOR OTHER PANEL CALCULATION
9	3	03		
a	CLEAR x	37	FROM 2-7 & 2-d	
b	ROLL ↑	22		SUM $g(t_0+T)$ TO $g(t_0)+4g(t_0+\Delta t)+2g(t_0+2\Delta t)+\cdots$
c	ACC +	60		
d	RCL	61		
5 0	↑	27		
1	C	16		
2	×	36		MULTIPLY $\Delta t / 3$ TIMES SUMMATION SERIES
3	3	03		
4	÷	35		
5	a	13		RECALL n, THE HARMONIC NUMBER
6	STOP	41		x: n; y: a_n or b_n; z: t_0+T — DISPLAY
7	CLEAR	20		
8	a	13		
9	↑	27		
a	1	01		INCREMENT n
b	+	33		
c	$y\rightarrow( )$	40		
d	a	13		

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	b	14	RECALL t_0								
	1	GO TO ( ) ( )	44	BRANCH TO CALCULATE COEFFICIENT								
	2	1	01									
	3 (FROM 1-6)	2	02									
	4	CONTINUE	47	STEPS 6-4 THROUGH 8-3 ARE AVAILABLE FOR FORMING f(t)								
	5	↓										
	6			*WHEN ENTERING THE PROGRAM FILL THIS AREA WITH CONTINUES TO AVOID THE POSSIBILITY OF AN END STATEMENT AND THUS RECORDING AND ENTERING PROBLEMS.*								
	7											
	8											
	9											
	a											
	b											
	c											
	d	↓										
7	0	CONTINUE	47									
	1	↓										
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d	↓										
8	0	CONTINUE	47									
	1	↓										
	2											
	3	↓										
	4	2	02	CALCULATE $\frac{2f(t)}{T}$								
	5	×	36									
	6	d	17									
	7	↑	27									
	8	b	14									
	9	−	34									
	a	π	56									
	b	ROLL ↓	31									
	c	÷	35									
	d	ROLL ↑	22									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	x⇄y	30									
	1	÷	35									
	2	2	02									
	3	×	36									
	4	e	12									
	5	×	36									
	6	a	13									
	7	×	36									
	8	↓	25									
	9	cos x	73									
	a	×	36									
	b	GO TO ( )( )	44									
	c	1	01									
	d	7	07									
a	0	END	46									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

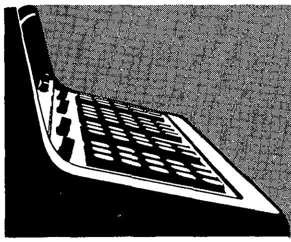
Steps 90–9a: CALCULATE $\frac{2f(t)}{T} \cos \frac{2\pi n t}{T}$ OR $\frac{2f(t)}{T} \sin \frac{2\pi n t}{T} = g(t)$

Step 99: AS ENTERED IN THIS STEP: COS CALCULATES a_n; SIN CALCULATES b_n

Steps 9b–9d: BRANCH TO INTEGRATE AND OBTAIN COEFFICIENTS

Step a0: *AFTER EXECUTION DATA IS STORED IN THIS REGISTER WHICH DESTROYS THE END STATEMENT, THEREFORE, RECORD PROGRAM ON A MAGNETIC CARD BEFORE EXECUTION.*

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



GAMMA FUNCTION

These program evaluate the Gamma Function,

where

$$\Gamma(\nu) = \int^{\infty} e^{-t} \; t^{\nu - 1} \, dt$$

Program 1 evaluates $\Gamma(\nu)$ over the range $0 < \nu \leq 69.98$

Program 2 evaluates $\Gamma(\nu)$ over the range $0 < \nu \leq 10^9$

Reference: Handbook of Mathematical Functions
National Bureau of Standards
Abramowitz & Stegan
Sixth Printing, 1967

USER INSTRUCTIONS

PROGRAM #1 $0 < \nu \leq 69.95$

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: $\nu \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	———	Z
$\Gamma(\nu)$	———	Y
ν	———	X

To enter new value for ν

PROGRAM #2 $0 < \nu \leq 10^9$

ENTER PROGRAM: (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: $\nu \rightarrow X$

PRESS: CONTINUE

DISPLAY

exponent of 10	———	Z
decimal number	—	Y
ν	———	X

To enter a new value for ν

EXAMPLE

PROGRAM #1

$\nu = 1.750$
$\Gamma(\nu) = 0.919062527$

$\nu = 51$
$\Gamma(\nu) = 3.041409320 \times 10^{64}$

PROGRAM #2

$\nu = 1.750$
$\Gamma(\nu) = 0.919062527$

$\nu = 97$
$\Gamma(\nu) = 9.916779322 \times 10^{149}$

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	ν	0	0	ENTER $\nu \rightarrow$ x					
	2	↑	27									
	3	1	01									
	4	−	34									
	5	y→( )	40									
	6	d	17									
	7	↑	27									
	8	↓	25									
	9	int x	64									
	a	−	34									
	b	CLEAR x	37									
	c	IF x > y	53									
	d	CLEAR x	37	REPLACE ν WITH $\bar{\nu}$ WHEN $5 \leq \bar{\nu} < 6$								
1	0	1	01									
	1	+	33									
	2	5	05									
	3	+	33									
	4	↓	25									
	5	↑	27									
	6	×	36									
	7	ACC +	60									
	8	2	02									
	9	x⇄y	30									
	a	÷	35									
	b	3	03									
	c	−	34									
	d	e	12									
2	0	÷	35									
	1	2	02									
	2	÷	35									
	3	+	33									
	4	e	12									
	5	÷	35									
	6	7	07									
	7	÷	35	CALCULATE $P(\nu) = -\nu + \frac{1}{12\nu} - \frac{1}{360\nu^3} + \frac{1}{1260\nu^5} - \frac{1}{1680\nu^7} + \frac{1}{2520\nu^9}$								
	8	1	01	$= -\nu \frac{1}{60\nu}\left(5 + \frac{1}{6\nu^2}\left(-1 + \frac{1}{7\nu^2}\left(2 + \frac{1}{2\nu^2}\left(-3 + \frac{2}{\nu^2}\right)\right)\right)\right)$								
	9	−	34									
	a	e	12									
	b	÷	35									
	c	6	06									
	d	÷	35									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	5	05									
1	+	33									
2	6	06									
3	0	00									
4	÷	35									
5	f	15									
6	÷	35									
7	−	34									
8	↑	27									
9	·	21									
a	5	05	CALCULATE $(\nu + \frac{1}{2}) \ln \nu$								
b	+	33									
c	f	15									
d	ln x	65									
4 0	×	36									
1	↓	25									
2	+	33									
3	π	56									
4	↑	27									
5	+	33	CALCULATE $\ln\sqrt{2\pi}$								
6	↓	25									
7	ln x	65									
8	↑	27									
9	2	02									
a	÷	35									
b	↓	25									
c	+	33									
d	↓	25									
5 0	e^x	74									
1	↑	27									
2	f	15									
3 (FROM 6-6)	↑	27									
4 (FROM 7-0)	d	17									
5	IF $x>y$	53									
6	6	06									
7	0	00									
8	IF $x<y$	52									
9	6	06									
a	7	07									
b	GO TO ( )( )	44									
c	7	07	BRANCH TO DISPLAY								
d	1	01									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 (FROM 5-7)	0	1	01									
	1	+	33									
	2	↓	25									
	3	×	36									
	4	GO TO ( )( )	44	CALCULATE $\nu!$ FROM $\bar{\nu}!$								
	5	5	05									
	6	3	03									
	7 (FROM 5-a)	↓	25									
	8	÷	35									
	9	↑	27									
	a	1	01									
	b	−	34									
	c	GO TO ( )( )	44									
	d	5	05									
7	0	4	04									
	1 (FROM 5-d)	1	01									
	2	$x \rightleftarrows y$	30									
	3	d	17	RECALL AND POSITION DISPLAY								
	4	+	33									
	5	CLEAR x	37									
	6	.	21									
	7	END	46	ν	$\Gamma(\nu)$	0	DISPLAY					
	8											
	9											
	a											
	b											
	c											
	d											
8	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

PROGRAM 2 PAGE 4 PART NO 09100-70024

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	STOP	41	ν	0	0	ENTER ν → X					
2	↑	27									
3	1	01									
4	−	34									
5	y→( )	40									
6	d	17									
7	1	01									
8	7	07									
9	IF x<y	52	OMIT LAST TERMS OF P(ν) IF ν > 17								
a	3	03									
b	4	04									
c	d	17	SET FLAG IF ν IS AN INTEGER								
d	int x	64									
1 0	IF x=y	50									
1	SET FLAG	54									
2	SET FLAG	54									
3	−	34	REPLACE ν WITH $\bar{\nu}$ IF ν ≤ 17								
4	CLEAR x	37									
5	IF x>y	53									
6	CLEAR x	37									
7	1	01									
8	+	33									
9	5	05									
a	+	33									
b	↓	25	BEGIN CALCULATION OF P(ν)								
c	↑	27									
d	×	36									
2 0	ACC +	60									
1	2	02									
2	x⇄y	30									
3	÷	35									
4	3	03									
5	−	34									
6	e	12									
7	÷	35									
8	2	02									
9	÷	35									
a	+	33									
b	e	12									
c	÷	35									
d	7	07									

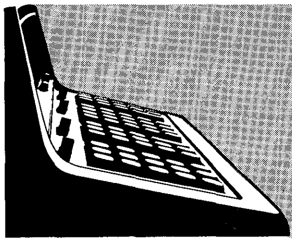
Step		Key	Code	Display x	y	z	Storage f	e	d	c	b	a
3	0	÷	35									
	1	GO TO ( )( )	44									
	2	3	03									
	3	9	11									
	4	d	17	FROM 0-b								
	5	×	36									
	6	ACC +	60	PUT ν, ν^2 IN f, e IF $\nu > 17$								
	7	CLEAR x	37									
	8	$x \rightleftarrows y$	30									
	9	1	01									
	a	−	34									
	b	e	12									
	c	÷	35									
	d	6	06									
4	0	÷	35									
	1	5	05									
	2	+	33	FINISH CALCULATION OF P (ν)								
	3	6	06									
	4	0	00									
	5	÷	35									
	6	f	15									
	7	÷	35									
	8	−	34									
	9	↑	27									
	a	.	21									
	b	5	05	CALCULATE $(\nu + \frac{1}{2}) \ln \nu$								
	c	+	33									
	d	f	15									
5	0	ln x	65									
	1	×	36									
	2	↓	25									
	3	+	33									
	4	π	56									
	5	↑	27									
	6	+	33									
	7	↓	25	CALCULATE $\ln\sqrt{2\pi}$								
	8	ln x	65									
	9	↑	27									
	a	2	02									
	b	÷	35									
	c	CLEAR x	37									
	d	ROLL ↓	31									

Step		Key	Code	Comments
6	0	+	33	DON'T GO THRU. STEPS TO CONVERT $\bar{\nu}!$ TO $\nu!$ IF $\nu>7$ AND DON'T TAKE e^x OF $\ln\nu$ WITHOUT REMOVING CHARACTERISTIC FIRST (steps 6-0 to 6-5)
	1	3	03	
	2	4	04	
	3	IF $x<y$	52	
	4	a	13	
	5	4	04	
	6	↓	25	
	7	e^x	74	
	8	↑	27	
	9	f	15	
	a	↑	27	
	b	d	17	
	c	IF $x>y$	53	CALCULATE $\nu!$ FROM $\bar{\nu}!$ (steps 6-c to 8-7)
	d	7	07	
7	0	7	07	
	1	IF $x<y$	52	
	2	8	10	
	3	0	00	
	4	GO TO ( )( )	44	BRANCH TO REMOVE INSIGNIFICANT ZEROS (steps 7-4 to 7-6)
	5	8	10	
	6	8	10	
	7	1	01	
	8	+	33	
	9	↓	25	
	a	×	36	
	b	GO TO ( )( )	44	
	c	6	06	
	d	a	13	
8	0	↓	25	
	1	÷	35	
	2	↑	27	
	3	1	01	
	4	−	34	
	5	GO TO ( )( )	44	
	6	6	06	
	7	b	14	
	8	CLEAR x	37	REMOVE INSIGNIFICANT ZEROS (steps 8-8 to 8-c)
	9	ROLL ↑	22	
	a	IF FLAG	43	
	b	int x	64	
	c	int x	64	
	d	↑	27	FROM a-3

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	1	01	CALCULATE P_{INT}								
	1	0	00									
	2	IF $x>y$	53									
	3	b	14									
	4	4	04									
	5	.	21									
	6	1	01									
	7	×	36									
	8	1	01									
	9	ROLL ↑	22									
	a	$x \rightleftarrows y$	30									
	b	+	33									
	c	$x \rightleftarrows y$	30									
	d	ROLL ↓	31									
a	0	ROLL ↓	31									
	1	GO TO ( )( )	44									
	2	8	10									
	3	d	17									
	4	1	01	CONVERT lnν! TO LOG ν!, REMOVE CHARACTERISTIC, CONVERT DECIMAL NUMBER TO ln AND TAKE EXPONENT OF 10								
	5	0	00									
	6	ln x	65									
	7	÷	35									
	8	$x \rightleftarrows y$	30									
	9	↑	27									
	a	int x	64									
	b	−	34									
	c	$x \rightleftarrows y$	30									
	d	ROLL ↑	22									
b	0	×	36									
	1	↓	25									
	2	e^x	74									
	3	↑	27									
	4	$y \rightleftarrows ( )$	24	RECALL AND POSITION DISPLAY								
	5	d	17									
	6	1	01									
	7	+	33									
	8	$x \rightleftarrows y$	30									
	9	$y \rightleftarrows ( )$	24									
	a	d	17									
	b	END	46	ν	DECIMAL NO.	EXP. OF 10	DISPLAY					
	c											
	d											

FROM 6–5 (branch to step a4); step 9-2 branches to step b4; step a3 (GO TO 8d).

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70025

BESSEL FUNCTION

The differential equation:

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)\, y = 0$$

is called Bessel's equation of order n. When n is an integer the Bessel function of the first kind of order n, $J_n(x)$, is a solution where:

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{-x^2}{4}\right)^k}{k!\ (n+k)!} \qquad x \geq 0$$

The computation is performed as follows:

$$J_n(x) = \frac{\left(\frac{x}{2}\right)^n}{n!} \left[1 - \frac{\left(\frac{x^2}{4}\right)}{1(n+1)} \cdot \left\{ 1 - \frac{\left(\frac{x^2}{4}\right)}{2(n+2)} \right\} \cdot \left\{ 1 - \frac{\left(\frac{x^2}{4}\right)}{3(n+3)} \right\} \cdot \left\{ 1 - \ldots \right\} \right]$$

Reference:

Handbook of Mathematical Functions
M. Abramowitz & I. A. Stegum
National Bureau of Standards
Sixth Printing, 1967

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: n → Y, x → X

PRESS: CONTINUE

DISPLAY

$J_n(x)$	———	Z
n	———	Y
x	———	X

To enter new value for n and x

EXAMPLE

(A)

$x = 5$

$n = 9$

$J_n(x) = 5.520283139 \times 10^{-3}$

(B)

$x = 24$

$n = 46$

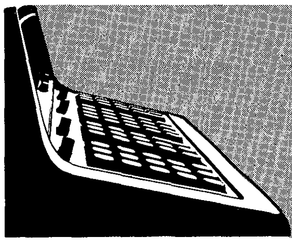
$J_n(x) = 3.347208960 \times 10^{-10}$

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	STOP	41	X	n	O	ENTER DATA					
2	x→( )	23									
3	9	11									
4	x→( )	23	STORE DATA								
5	c	16									
6	y→( )	40									
7	d	17									
8	CLEAR	20									
9	3	03									
a	↑	27									
b	c	16									
c	$\sqrt{x}$	76									
d	$\sqrt{x}$	76									
1 0	÷	35									
1	9	11									
2	+	33									
3	c	16	CALCULATE $\Delta = 1+9X^{\frac{1}{3}} + 3X^{\frac{1}{12}}$ TO BE								
4	ln x	65	ADDED TO LARGER OF n OR X TO FIND M								
5	↑	27									
6	3	03									
7	÷	35									
8	↓	25									
9	e^x	74									
a	+	33									
b	1	01									
c	+	33									
d	2	02									
2 0	↑	27									
1	c	16									
2	÷	35	CALCULATE 2/X AND REPLACE X IN C								
3	y⇄( )	24									
4	c	16									
5	d	17									
6	IF x<y	52	OMIT TEST IF X ≥ n								
7	4	04									
8	3	03									
9	÷	35									
a	2	02									
b	÷	35									
c	↓	25									
d	↑	27									

Step	Key	Code	Display / Comments
30	×	36	
31	ln x	65	
32	−	34	
33	d	17	CALCULATE $q = x^2/4n + (n+1/2)\cdot\ln(n) - n\cdot\ln(x/2) - n$
34	×	36	EXPRESSED AS $q = n\left[(x/2n)^2 - \ln(x/2n)\right]$
35	−	34	$-n + \ln(\sqrt{n})$ AND COMPARE WITH $99\ln(10)$
36	√x	76	$-\ln(\sqrt{2\pi})$, EXIT AND DISPLAY ZERO FOR
37	ln x	65	$J_n(X)$ IF LATTER IS LESS
38	+	33	
39	2	02	
3a	2	02	
3b	7	07	
3c	IF x<y	52	
3d	7	07	
40	d	17	
41	d	17	
42	x⇄y	30	
43	CLEAR x	37	ADD GREATER OF n OR X TO Δ
44	ROLL ↓	31	
45	+	33	
46	2	02	
47	÷	35	
48	x⇄y	30	FORCE m TO BE EVEN
49	int x	64	
4a	×	36	
4b	ENTER EXP	26	
4c	CHG SIGN	32	
4d	9	11	
50	9	11	
51	ROLL ↓	31	
52	y→( )	40	
53	b	14	
54	ACC +	60	
55 (FROM 7-C →)	x⇄y	30	
56	d	17	
57	IF x=y	50	
58	↓	25	
59	y→( )	40	
5a	a	13	
5b	CLEAR x	37	
5c	x⇄y	30	
5d	f	15	

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6 0	IF x=y	50									
1	8	10	BRANCH WHEN m=0								
2	1	01									
3	ROLL ↓	31									
4	IF FLAG	43									
5	6	06	SKIP WHEN m IS ODD								
6	9	11									
7	ACC +	60	ACCUMULATE EVEN TERMS								
8	SET FLAG	54									
9	c	16									
a	ROLL ↑	22									
b	×	36									
c	↓	25									
d	×	36	CALCULATE T_{k-1} FROM T_k AND T_{k+1}								
7 0	↓	25									
1	y⇄( )	24									
2	b	14									
3	x⇄y	30									
4	−	34									
5	CLEAR x	37									
6	↑	27									
7	1	01	DECREMENT m (or k)								
8	ACC −	63									
9	f	15									
a	GO TO ( )( )	44									
b	5	05									
c	5	05									
d	CLEAR x	37									
8 0	x→( )	23									
1	a	13									
2	ROLL ↓	31									
3	e	12									
4	+	33									
5	+	33									
6	↓	25									
7	÷	35									
8	↑	27									
9	y⇄( )	24									
a	9	11	RECALL AND POSITION DISPLAY								
b	d	17									
c	x⇄y	30									
d	END	46	X	n	$J_n(X)$	DISPLAY					

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



09100 B ONLY
PART NO.
09100-70401

HYPERGEOMETRIC SERIES EXPANSION

The program determines the series expansion of the hypergeometric function

$$F(a, b, c; X) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \cdot \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\,\Gamma(b+n)}{\Gamma(c+n)\, n!} X^n$$

The program incorporates programs 09100-70001 (N Factorial) and 09100-70024 (Gamma Function) as subroutines. The a, b, and c must be greater than zero. The program can be used for power series expansion or for the solution of Gauss's hypergeometric differential equation.

Restrictions: (c + n), (b + n) and (c + n) must be less than 69.

Reference: Advanced Engineering Mathematics, E. Kreyszig, John Wiley & Sons, 1968
Handbook of Mathematical Functions, U. S. Department of Commerce, Applied Math Series 55

09100 B ONLY
PART NO.
09100-70401

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: c → Z, b → Y, a → X

PRESS: CONTINUE

DISPLAY

0	Z
F_n	Y
n	X

To run another case:

PRESS: END

EXAMPLE

1. The differential equation (Gauss's Hypergeometric)

$$X(1 - X)\,Y'' + \left[c - (a + b + 1)\,X\right] Y' - abY = 0$$

has solutions given by

$$Y_1(X) = F(a, b, c; X) \text{ and}$$

$$Y_2(X) = X^{1-c} F(a - c + 1,\ b - c + 1,\ 2 - c; X)$$

$$|X| < 1 \quad \text{and } c \neq 0, -1, -2, \ldots$$

Thus

$$X(1 - X)\,Y'' + (1 - 1.25X)\,Y' - .375\,Y = 0$$

is a Gauss equation with

$$a = .75,\ b = .50 \text{ and } c = 1$$

It's solutions are

$$Y_1(X) = F(.75, .50, 1; X) = Y_2(X)$$

n	F
0	1
1	.375
2	.246
3	.188
4	.154
5	.132
6	.116

Thus

$$Y_1(X) = 1 + .375\,X + .246\,X^2 + .188\,X^3 + .154\,X^4 + \ldots$$

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	x→( )	23			
2	−	34			
3	d	17		ENTER	
4	STOP	41	a	b	c
5	CONT	47			
6	CONT	47			
7	x→( )	23			
8	a	13			
9	y→( )	40			
a	b	14			
b	↓	25			
c	y→( )	40			
d	c	16			
1 0	GO TO ( )	44			
(+) 1	▲SUB▼	77			
2	8	10			
3	c	16			
4	CONT	47			
5	x→( )	23			
6	−	34			
7	f	15			
8	a	13			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	8	10			
c	c	16			
d	↑	27			
2 0	x←( )	67			
(+) 1	−	34			
2	f	15			
3	×	36			
4	y→( )	40			
5	−	34			
6	f	15			
7	c	16			
8	GO TO ( )	44			
9	▲SUB▼	77			
a	8	10			
b	c	16			
c	↑	27			
d	x←( )	67			
3 0	−	34			
(+) 1	f	15			
2	÷	35			
3	y→( )	40			
4	−	34			
5	f	15			
6	a	13			
7	GO TO ( )	44			
8	▲SUB▼	77			
9	8	10			
a	6	06			
b	x→( )	23			
c	−	34			
d	e	12			
4 0	b	14			
(+) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	8	10			
4	6	06			
5	↑	27			
6	x←( )	67			
7	−	34			
8	e	12			
9	×	36			
a	y→( )	40			
b	−	34			
c	e	12			
d	c	16			
5 0	GO TO ( )	44			
(+) 1	▲SUB▼	77			
2	8	10			
3	6	06			
4	↑	27			
5	x←( )	67			
6	−	34			
7	e	12			
8	x⇄y	30			
9	÷	35			
a	x←( )	67			
b	−	34			
c	f	15			
d	×	36			
6 0	y→( )	40			
(+) 1	−	34			
2	e	12			
3	x←( )	67			
4	−	34			
5	d	17			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	−	34			
9	6	06			
a	c	16			
b	ROLL ↓	31			
c	x←( )	67			
d	−	34			
7 0	e	12			
(+) 1	ROLL ↑	22			
2	÷	35			
3	x←( )	67			
4	−	34			
5	d	17			
6	↑	27			
7	0	00			
8	ROLL ↓	31		DISPLAY	
9	STOP	41	n	F_n	0
a	CONT	47			
b	↑	27			
c	1	01			
d	+	33			

Storage

	+	−
f		
e		
d		n
c	c	
b	b	
a	a	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
8 0	$y \rightarrow ( )$	40			
(+) 1	−	34			
2	d	17			
3	GO TO ( )	44			
4	3	03			
5	6	06			
6	↑	27			
7	$x \leftarrow ( )$	67			
8	−	34			
9	d	17			
a	+	33			
b	↓	25			
c	↑	27			
d	1	01			

Step	Key	Code	Display x	Display y	Display z
9 0	−	34			
(+) 1	$y \rightarrow ( )$	40			
2	d	17			
3	↑	27			
4	↓	25			
5	int x	64			
6	−	34			
7	CLEAR x	37			
8	GO TO ( )	44			
9	−	34			
a	0	00			
b	0	00			
c					
d					

Step	Key	Code	Display x	Display y	Display z
0 0	IF $x > y$	53			
(−) 1	CLEAR x	37			
2	1	01			
3	+	33			
4	5	05			
5	+	33			
6	↓	25			
7	↑	27			
8	×	36			
9	ACC +	60			
a	2	02			
b	$x \rightleftarrows y$	30			
c	÷	35			
d	3	03			

Step	Key	Code	Display x	Display y	Display z
1 0	−	34			
(−) 1	e	12			
2	÷	35			
3	2	02			
4	÷	35			
5	+	33			
6	e	12			
7	÷	35			
8	7	07			
9	÷	35			
a	1	01			
b	−	34			
c	e	12			
d	÷	35			

Step	Key	Code	Display x	Display y	Display z
2 0	6	06			
(−) 1	÷	35			
2	5	05			
3	+	33			
4	6	06			
5	0	00			
6	÷	35			
7	f	15			
8	÷	35			
9	−	34			
a	↑	27			
b	·	21			
c	5	05			
d	+	33			

Step	Key	Code	Display x	Display y	Display z
3 0	f	15			
(−) 1	ln x	65			
2	×	36			
3	↓	25			
4	+	33			
5	π	56			
6	↑	27			
7	+	33			
8	↓	25			
9	ln x	65			
a	↑	27			
b	2	02			
c	÷	35			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
4 0	+	33			
(−) 1	↓	25			
2	e^x	74			
3	↑	27			
4	f	15			
5	↑	27			
6	d	17			
7	IF $x > y$	53			
8	5	05			
9	2	02			
a	IF $x < y$	52			
b	5	05			
c	9	11			
d	GO TO ( )	44			

Step	Key	Code	Display x	Display y	Display z
5 0	6	06			
(−) 1	3	03			
2	1	01			
3	+	33			
4	↓	25			
5	×	36			
6	GO TO ( )	44			
7	4	04			
8	5	05			
9	↓	25			
a	÷	35			
b	↑	27			
c	1	01			
d	−	34			

Storage

Register	Value
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

 PART NO. 09100-70401 **9100 B ONLY**

Step	Key	Code	Display x	Display y	Display z
6 0	GO TO ( )	44			
(−) 1	4	04			
2	6	06			
3	CONT	47			
4	CONT	47			
5	0	00			
6	$x \to (\ )$	23			
7	e	12			
8	$x \to (\ )$	23			
9	f	15			
a	ROLL ↑	22			
b	RETURN	77			
c	↑	27			
d	0	00			
7 0	$x \rightleftarrows y$	30			
(−) 1	IF $x = y$	50			
2	arc ▾	72			
3	1	01			
4	↑	27			
5	↑	27			
6	1	01			
7	−	34			
8	IF $x > y$	53			
9	8	10			
a	3	03			
b	ROLL ↓	31			
c	×	36			
d	ROLL ↑	22			
8 0	GO TO ( )	44			
(−) 1	7	07			
2	7	07			
3	ROLL ↑	22			
4	RETURN	77			
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

(3 x 3) MATRIX MULTIPLICATION

This program calculates the product matrix C of two 3 x 3 matrices A and B of the form:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$$

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix}$$

where $$C_{ij} = \sum_{k=1}^{3} a_{ik} b_{kj} \qquad \begin{cases} i = 1, 2, 3 \\ j = 1, 2, 3 \end{cases}$$

Reference: Elementary Matrix Algebra (1958), Franz E. Hohn

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

→ ENTER: Row 1 of B

$b_{13} \rightarrow Z$, $b_{12} \rightarrow Y$, $b_{11} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER: Row 2 of B

$b_{23} \rightarrow Z$, $b_{22} \rightarrow Y$, $b_{21} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

ENTER: Row 3 of B

$b_{33} \rightarrow Z$ $b_{32} \rightarrow Y$, $b_{31} \rightarrow X$

→ PRESS: CONTINUE

DISPLAY

i	Z
i	Y
i	X

i = 4 indicates C is complete (1)

ENTER: i th Row of A

$A_{i3} \rightarrow Z$, $A_{i2} \rightarrow Y$, $A_{i1} \rightarrow X$

→ PRESS: CONTINUE

DISPLAY

C_{ij}	Z
j	Y
i	X

(4) (3) (2)

USER INSTRUCTIONS (Con't)

(4) (3) (2) NO

j = 3 ? — YES → (3); NO → (2)

(1)

DISPLAY

4	Z
4	Y
4	X

To run another case: → (4)

PRESS: CONTINUE

EXAMPLE

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 4 & 2 & 1 \\ 1 & 4 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 4 \\ 4 & 3 & 1 \\ 1 & 6 & -2 \end{pmatrix}$$

$$C = \begin{pmatrix} 17 & 35 & -1 \\ 13 & 20 & 16 \\ 15 & 2 & 12 \end{pmatrix}$$

Step	Key	Code	Display x	Display y	Display z
0 (+) 0	CLEAR	20			
1	1	01	ENTER		
2	STOP	41	b_{11}	b_{12}	b_{13}
3	x→( )	23			
4	b	14			
5	y→( )	40			
6	a	13			
7	↓	25			
8	y→( )	40			
9	9	11			
a	CLEAR	20			
b	2	02	ENTER		
c	STOP	41	b_{21}	b_{22}	b_{23}
d	x→( )	23			
1 (+) 0	8	10			
1	y→( )	40			
2	7	07			
3	↓	25			
4	y→( )	40			
5	6	06			
6	CLEAR	20			
7	3	03	ENTER		
8	STOP	41	b_{31}	b_{32}	b_{33}
9	x→( )	23			
a	5	05			
b	y→( )	40			
c	4	04			
d	↓	25			
2 (+) 0	y→( )	40			
1	3	03			
2	1	01			
3	x→( )	23			
4	f	15			
5	x→( )	23			
6	−	34			
7	f	15			
8	↑	27			
9	↑	27			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
0 (−) 0	STOP	41	a_{11}	a_{12}	a_{13}
1	GO TO ( )	44	ENTER		
2	▲SUB▼	77			
3	1	01			
4	6	06	ENTER		
5	STOP	41	a_{21}	a_{22}	a_{23}
6	GO TO ( )	44			
7	▲SUB▼	77			
8	1	01			
9	6	06	ENTER		
a	STOP	41	a_{31}	a_{32}	a_{33}
b	GO TO ( )	44			
c	▲SUB▼	77			
d	1	01			
1 (−) 0	6	06	DISPLAY		
1	STOP	41	4	4	4
2	GO TO ( )	44			
3	+	33			
4	0	00			
5	0	00			
6	x→( )	23			
7	d	17			
8	y→( )	40			
9	c	16			
a	ROLL ↑	22			
b	x→( )	23			
c	−	34			
d	e	12			
2 (−) 0	b	14			
1	×	36			
2	e	12			
3	+	33			
4	y→( )	40			
5	e	12			
6	↓	25			
7	x←( )	67			
8	8	10			
9	×	36			
a	e	12			
b	+	33			
c	y→( )	40			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
3 (−) 0	x←( )	67			
1	−	34			
2	e	12			
3	↑	27			
4	x←( )	67			
5	5	05			
6	×	36			
7	e	12			
8	+	33			
9	f	15			
a	↑	27			
b	x←( )	67			
c	−	34			
d	f	15	DISPLAY		
4 (−) 0	STOP	41	i	j	C_{ij}
1	CONT	47			
2	y⇄( )	24			
3	b	14			
4	y⇄( )	24			
5	9	11			
6	y⇄( )	24			
7	a	13			
8	y→( )	40			
9	b	14			
a	y⇄( )	24			
b	8	10			
c	y⇄( )	24			
d	6	06			

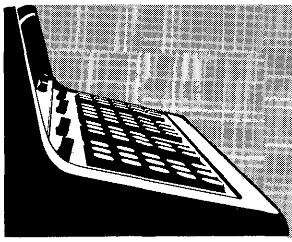
Storage

Register	+			−
f	j			i
e	c_{ij}			a_{i3}
d	a_{i1}			
c	a_{i2}			
b	b_{11}	b_{12}	b_{13}	
a	b_{12}	b_{13}	b_{11}	
9	b_{13}	b_{11}	b_{12}	
8	b_{21}	b_{22}	b_{23}	
7	b_{22}	b_{23}	b_{21}	
6	b_{23}	b_{21}	b_{22}	
5	b_{31}	b_{32}	b_{33}	
4	b_{32}	b_{33}	b_{31}	
3	b_{33}	b_{31}	b_{32}	
2				
1				
0				

Step	Key	Code	Display x	Display y	Display z
5 (−) 0	y⇄( )	24			
1	7	07			
2	y→( )	40			
3	8	10			
4	y⇄( )	24			
5	5	05			
6	y⇄( )	24			
7	3	03			
8	y⇄( )	24			
9	4	04			
a	y→( )	40			
b	5	05			
c	0	00			
d	x→( )	23			
6 (−) 0	e	12			
1	3	03			
2	↑	27			
3	f	15			
4	IF x = y	50			
5	7	07			
6	5	05			
7	↑	27			
8	1	01			
9	+	33			
a	y→( )	40			
b	f	15			
c	c	16			
d	↑	27			
7 (−) 0	d	17			
1	↑	27			
2	GO TO ( )	44			
3	2	02			
4	0	00			
5	1	01			
6	x→( )	23			
7	f	15			
8	↑	27			
9	x←( )	67			
a	−	34			
b	f	15			
c	+	33			
d	y→( )	40			
8 (−) 0	−	34			
1	f	15			
2	↓	25			
3	↑	27			
4	↑	27			
5	RETURN	77			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70403

ROOTS OF 4TH DEGREE POLYNOMIAL

This program determines the real and complex roots of the fourth degree polynomial

$$f(x) = X^4 + a_1X^3 + a_2X^2 + a_3X + a_4 ,$$

where the coefficients a_i are real. The program uses the Lin-Bairstow method which determines a quadratic factor $(X^2 + rX + s)$ such that

$$f(x) = (X^2 + rX + s)(X^2 + b_1X + b_2) + RX + S$$

The variables r and s are obtained by an iteration scheme which reduces the remainder terms R and S to zero. The user can specify the remainder which he can tolerate.

The program applies the following recurrsive relationships:

$$b_1 = a_1 - r \qquad c_1 = b_1 - r$$

$$b_2 = a_2 - rb_1 - s \qquad c_2 = b_2 - rc_1$$

$$b_3 = a_3 - rb_2 - sb_1 \qquad \overline{c_3} = -rc_2 - sc_1$$

$$b_4 = a_4 - rb_3 - sb_2 \qquad R = b_3$$

$$S = b_4 + rb_3$$

These quantities (b_i and c_i) are required for the determination of $\triangle r$ and $\triangle s$ in the equations:

$$c_2 \ \Delta r + c_1 \ \Delta s = b_3$$

$$\overline{c_3} \ \Delta r + c_2 \ \Delta s = b_4$$

The terms r and s are incremented by $\triangle r$ and $\triangle s$ respectively and the remainder terms are tested against the tolerance. If the remainders are small enough to pass the test, then the two quadratics $(X^2 + rX + s)$ and $(X^2 + b_1X + b_2)$ are solved by the quadratic formula. If the remainder is too large, the iteration is repeated and the test repeated.

Locations + (5 - 6) through + (5 - 9) are used for storing the tolerance on $|R|$ and $|S|$.

Reference: Numerical Analysis, Kaiser S. Kunz, 1957, McGraw-Hill Book Company

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
0 —— X

ENTER DATA: $a_3 \rightarrow Z$, $a_2 \rightarrow Y$, $a_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

1 —— Z
1 —— Y
1 —— X

ENTER DATA: $a_4 \rightarrow Z$, $s \rightarrow Y$, $r \rightarrow X$ *

PRESS CONTINUE

DISPLAY

|S| —— Z
|R| —— Y
0 —— X

PRESS: CONTINUE

DISPLAY

s —— Z
r —— Y
1 —— X

PRESS: CONTINUE

Real Roots

0 — Z
R_2 — Y
R_1 — X

or

Complex Roots

$R = a \pm bj$
+ Imaginary Part —— Z
- Imaginary Part —— Y
Real Part of R —— X

PRESS: CONTINUE

* Both r and s must be non-zero

USER INSTRUCTIONS (Con't)

DISPLAY

b_2 —— Z
b_1 —— Y
2 —— X

PRESS: CONTINUE

Real Root

0 — Z
R_4 — Y
R_3 — X

or

Complex Roots

$R = a \pm bj$
+ Imaginary Part —— Z
- Imaginary Part —— Y
Real Part of R —— X

To run another case:

PRESS: END

EXAMPLE

Fourth Order Butterworth Polynomial

$F(s) = s^4 + 2.613\ s^3 + 3.414\ s^2 + 2.613\ s + 1$

$a_1 = 2.613$	$r = 1$
$a_2 = 3.414$	$s = 1$
$a_3 = 2.613$	
$a_4 = 1$	

Tolerance is set as .001

Solution		Actual *
|S|	= 0.00094	
|R|	= 0.00015	
s	= 1.0005	1.0
r	= 0.7661	0.7654
$root_1$	= -.3831 - j.924	
$root_2$	= -.3831 + j.924	
b_2	= 0.9985	1.0
b_1	= 1.8469	1.8478
$root_3$	= -.9234 -j.3818	
$root_4$	= -.9234 +j.3818	

* Network Analysis and Synthesis, Franklin F. Kuo, 1962, John Wiley & Sons

Step	Key	Code	Display x	Display y	Display z
0 (+) 0	CLEAR	20	DISPLAY		
1	STOP	41	a_1	a_2	a_3
2	$x\to()$	23			
3	6	06			
4	$y\to()$	40			
5	7	07			
6	↓	25			
7	$y\to()$	40			
8	8	10			
9	1	01			
a	↑	27			
b	↑	27	ENTER		
c	STOP	41	r	s	a_4
d	ACC +	60			
1 (+) 0	ROLL ↓	31			
1	$y\to()$	40			
2	9	11			
3	ROLL ↑	22			
4	↑	27			
5	$x\leftarrow()$	67			
6	6	06			
7	$x\rightleftarrows y$	30			
8	−	34			
9	$y\to()$	40			
a	d	17			
b	×	36			
c	↓	25			
d	+	33			
2 (+) 0	↑	27			
1	$x\leftarrow()$	67			
2	7	07			
3	ROLL ↑	22			
4	−	34			
5	$y\to()$	40			
6	c	16			
7	f	15			
8	×	36			
9	e	12			
a	↑	27			
b	d	17			
c	×	36			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
3 (+) 0	+	33			
1	↑	27			
2	$x\leftarrow()$	67			
3	8	10			
4	ROLL ↑	22			
5	−	34			
6	$y\to()$	40			
7	b	14			
8	f	15			
9	×	36			
a	e	12			
b	↑	27			
c	c	16			
d	×	36			
4 (+) 0	↓	25			
1	+	33			
2	↑	27			
3	$x\leftarrow()$	67			
4	9	11			
5	ROLL ↑	22			
6	−	34			
7	$y\to()$	40			
8	a	13			
9	b	14			
a	↑	27			
b	f	15			
c	$x\rightleftarrows y$	30			
d	×	36			
5 (+) 0	ROLL ↓	31			
1	+	33			
2	$\lvert y\rvert$	55			
3	↓	25			
4	$\lvert y\rvert$	55			
5	↑	27			
6	.	21			
7	0	00			
8	1	01			
9	0	00			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
0 (−) 0	IF $x<y$	52			
1	2	02			
2	d	17			
3	ROLL ↑	22			
4	IF $x>y$	53			
5	2	02			
6	d	17			
7	ROLL ↓	31			
8	↓	25			
9	$x\rightleftarrows y$	30			
a	↑	27			
b	0	00	DISPLAY		
c	STOP	41	0	$\lvert R\rvert$	$\lvert S\rvert$
d	RCL	61			
1 (−) 0	↑	27			
1	1	01	DISPLAY		
2	STOP	41	1	r	s
3	$x\rightleftarrows y$	30			
4	ROLL ↑	22			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	9	11			
8	2	02			
9	c	16			
a	↑	27			
b	d	17			
c	↑	27			
d	2	02			

Storage

Register	+	−
f	r	r
e	s	s
d	b_1/c_1	
c	b_2/c_2	
b	b_3	
a	b_4	
9	a_4	
8	a_3	
7	a_2	
6	a_1	
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
2 0	STOP	41	2	b_1	b_2
(−) 1	1	01	DISPLAY		
2	x⇄y	30			
3	ROLL ↑	22			
4	GO TO ( )	44			
5	▲SUB▼	77			
6	9	11			
7	2	02			
8	CLEAR	20			
9	STOP	41			
a	CONT	47			
b	CONT	47			
c	CONT	47			
d	d	17			
3 0	↑	27			
(−) 1	f	15			
2	−	34			
3	y→( )	40			
4	d	17			
5	×	36			
6	c	16			
7	x⇄y	30			
8	−	34			
9	y→( )	40			
a	c	16			
b	f	15			
c	×	36			
d	d	17			
4 0	↑	27			
(−) 1	e	12			
2	×	36			
3	↓	25			
4	+	33			
5	↓	25			
6	CHG SIGN	32			
7	↑	27			
8	f	15			
9	x→( )	23			
a	−	34			
b	f	15			
c	e	12			
d	x→( )	23			

Step	Key	Code	Display x	Display y	Display z
5 0	−	34			
(−) 1	e	12			
2	0	00			
3	x→( )	23			
4	e	12			
5	x→( )	23			
6	f	15			
7	c	16			
8	↑	27			
9	a	13			
a	ROLL ↑	22			
b	÷	35			
c	ROLL ↑	22			
d	x⇄y	30			
6 0	÷	35			
(−) 1	↓	25			
2	IF FLAG	43			
3	7	07			
4	1	01			
5	ACC +	60			
6	SET FLAG	54			
7	c	16			
8	↑	27			
9	d	17			
a	↑	27			
b	b	14			
c	GO TO ( )	44			
d	5	05			
7 0	a	13			
(−) 1	ACC −	63			
2	x⇄y	30			
3	y⇄( )	24			
4	e	12			
5	f	15			
6	÷	35			
7	e	12			
8	x⇄y	30			
9	×	36			
a	ROLL ↓	31			
b	−	34			
c	x←( )	67			
d	−	34			

Step	Key	Code	Display x	Display y	Display z
8 0	f	15			
(−) 1	+	33			
2	y→( )	40			
3	f	15			
4	↓	25			
5	x←( )	67			
6	−	34			
7	e	12			
8	+	33			
9	y→( )	40			
a	e	12			
b	f	15			
c	GO TO ( )	44			
d	+	33			
9 0	1	01			
(−) 1	4	04			
2	ROLL ↑	22			
3	÷	35			
4	ROLL ↑	22			
5	x⇄y	30			
6	÷	35			
7	2	02			
8	CHG SIGN	32			
9	÷	35			
a	↓	25			
b	↑	27			
c	×	36			
d	ROLL ↓	31			

Storage

	+	−
f	r	r
e	s	s
d	b_1 / c_1	
c	b_2 / c_2	
b	b_3	
a	b_4	
9	a_4	
8	a_3	
7	a_2	
6	a_1	
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
a (−) 0	x⇄y	30			
1	−	34			
2	CLEAR x	37			
3	IF x=y	50			
4	c	16			
5	3	03			
6	IF x>y	53			
7	b	14			
8	8	10			
9	↓	25			
a	√x	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
b (−) 0	+	33			
1	ROLL ↑	22			
2	+	33			
3	CLEAR x	37			
4	ROLL ↓	31			
5	CONT	47	DISPLAY		
6	STOP	41	ROOTS		
7	RETURN	77			
8	↓	25			
9	CHG SIGN	32			
a	√x	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
c (−) 0	GO TO ( )	44			
1	b	14			
2	5	05			
3	↓	25			
4	CLEAR x	37			
5	ROLL ↓	31			
6	GO TO ( )	44			
7	b	14			
8	5	05			
9	END	46			
a					
b					
c					
d					

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

PROGRAM FOR NUMERICAL SOLUTION OF TWO 1ST ORDER DIFFERENTIAL EQUATIONS

This program may be used to solve a wide variety of pairs of first order differential equations of the form:

$$\frac{dY}{dX} = f(X, Y, Z)$$

$$\frac{dZ}{dX} = g(X, Y, Z)$$

with initial conditions

$$Y_o = Y(X_o)$$
$$Z_o = Z(X_o)$$

The program will calculate successive Y_i's corresponding to equally spaced X_i's with an increment h specified by the operator. Specifically the type of solution involved is a Runge-Kutta Method which uses the equations:

$$Y_{i+1} = Y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$Z_{i+1} = Z_i + \frac{1}{6}(p_1 + 2p_2 + 2p_3 + p_4)$$

where

$$k_1 = h\,f\,(X_i, Y_i, Z_i)$$

$$k_2 = h\,f\,(X_i + \frac{h}{2}, Y_i + \frac{k}{2}1, Z_i + \frac{p}{2}1)$$

$$k_3 = h\,f\,(X_i + \frac{h}{2}, Y_i + \frac{k}{2}2, Z_i + \frac{p}{2}2)$$

$$k_4 = h\,f\,(X_i + h, Y_i + k_3, Z_i + p_3)$$

$$p_1 = h\,g\,(X_i, Y_i, Z_i)$$

$$p_2 = h\,g\,(X_i + \frac{h}{2}, Y_i + \frac{k}{2}1, Z_i + \frac{p}{2}1)$$

$$p_3 = h\,g\,(X_i + \frac{h}{2}, Y_i + \frac{k}{2}2, Z_i + \frac{p}{2}2)$$

$$p_4 = h\,g\,(X_i + h, Y_i + k_3, Z_i + p_3)$$

Reference: An Introduction to Numerical Mathematics, Eduard L. Stiefel, Academic Press, Inc. 1964.

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address is 0-0)

PRESS: GO TO ( )( )

PRESS: -

PRESS: 0

PRESS: 0

SET: PROGRAM

At this time the independent variable X is in both the X and c registers, the variable Y is in both the Y and b registers, and the variable Z is in both the Z and a registers. Starting at -0,0 enter the program steps which take the variables from their respective registers and calculate f(X, Y, Z) and g(X, Y, Z). Place f(X, Y, Z) in the Y register and g(X, Y, Z) in the Z register. The last statement must be RETURN since the main program calls for a subroutine containing the two differential equations.

Note there is a maximum of 14 registers available for storing and positioning f(X, Y, Z) and g(X, Y, Z).

Note also that the contents of the a, b and c registers must be preserved.

SET: [ ]RUN

→ PRESS: END

PRESS: CONTINUE

ENTER DATA: h → X (Increment)

PRESS: CONTINUE

DISPLAY

0 ———	Z
0 ———	Y
1 ———	X

ENTER DATA: Z_o → Z, Y_o → Y, X_o → X

(Initial Conditions)

PRESS: CONTINUE

The calculator will display answers at every increment of the independent variable in the form:

Z_i ———	Z
Y_i ———	Y
X_i ———	X

USER INSTRUCTIONS (Con't)

To hold the solution at the next increment, depress PAUSE until display. To restart, press CONTINUE. To execute program again, enter the new differential equations.

EXAMPLE

$$\frac{dY}{dX} = -\sin X + Z$$

$$\frac{dZ}{dX} = \cos X + 4Y$$

Initial Conditions:

$X_o = 0$

$Y_o = 1$

$Z_o = 2$

Let increment h = .005 SET: RADIANS

The steps that form f(X, Y, Z) and g(X, Y, Z) appear on page 2 of the step pages.

X	Y	Z
.01	1.020	2.050
.02	1.041	2.102
.03	1.062	2.154
.04	1.083	2.207
.05	1.105	2.260
.06	1.127	2.315
.07	1.150	2.370
.08	1.174	2.427
.09	1.197	2.484
.10	1.221	2.543

 PART NO. 09100-70404 9100B ONLY

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20	ENTER		
01	STOP	41	h	0	0
02	↑	27			
03	2	02			
04	÷	35			
05	↓	25			
06	x→( )	23			
07	d	17			
08	1	01	ENTER		
09	STOP	41	X_0	Y_0	Z_0
0a	x→( )	23			
0b	c	16			
0c	ROLL ↓	31			
0d	ACC +	60			
10	x→( )	23			
11	b	14			
12	y→( )	40			
13	a	13			
14	ROLL ↑	22			
15	CLEAR x	37			
16	x→( )	23			
17	−	34			
18	f	15			
19	c	16			
1a	SET FLAG	54	DISPLAY		
1b	PAUSE	57	X_i	Y_i	Z_i
1c	CONT	47			
1d	CONT	47			
20	GO TO ( )	44			
21	▲SUB▼	77			
22	−	34			
23	0	00			
24	0	00			
25	d	17			
26	×	36			
27	ROLL ↑	22			
28	×	36			
29	3	03			
2a	÷	35			
2b	ROLL ↑	22			
2c	x⇄y	30			
2d	÷	35			

Step	Key	Code	Display x	Display y	Display z
30	1	01			
31	ROLL ↓	31			
32	ACC +	60			
33	IF FLAG	43			
34	ACC −	63			
35	CONT	47			
36	ACC +	60			
37	ROLL ↑	22			
38	y⇄( )	24			
39	−	34			
3a	f	15			
3b	+	33			
3c	2	02			
3d	IF x<y	52			
40	6	06			
41	5	05			
42	IF x=y	50			
43	SET FLAG	54			
44	CONT	47			
45	3	03			
46	y⇄( )	24			
47	−	34			
48	f	15			
49	×	36			
4a	ROLL ↑	22			
4b	×	36			
4c	a	13			
4d	+	33			
50	b	14			
51	ROLL ↑	22			
52	+	33			
53	y⇄( )	24			
54	d	17			
55	c	16			
56	x⇄y	30			
57	+	33			
58	IF FLAG	43			
59	arc ▼	72			
5a	−	34			
5b	x⇄y	30			
5c	y⇄( )	24			
5d	d	17			

Step	Key	Code	Display x	Display y	Display z
60	x→( )	23			
61	c	16			
62	GO TO ( )	44			
63	2	02			
64	0	00			
65	3	03			
66	IF x<y	52			
67	6	06			
68	d	17			
69	6	06			
6a	GO TO ( )	44			
6b	4	04			
6c	6	06			
6d	y⇄( )	24			
70	−	34			
71	f	15			
72	ROLL ↓	31			
73	ACC −	63			
74	RCL	61			
75	GO TO ( )	44			
76	1	01			
77	0	00			
78	END	46			
79					
7a					
7b					
7c					
7d					

Storage

Register	Contents
f	Y_0
e	Z_0
d	h/2
c	X_0
b	Y_0
a	Z_0
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

NOTE Do not destroy these registers (f–a) when generating f(X Y Z) and g(X Y Z)

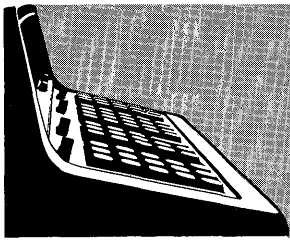
 PART NO. 09100-70404 9100B ONLY

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	4	04			
(−) 1	×	36			
2	c	16			
3	cos x	73			
4	+	33			
5	c	16			
6	sin x	70			
7	CHG SIGN	32			
8	ROLL ↑	22			
9	+	33			
a	RETURN	77			
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



ROOTS OF 6TH DEGREE POLYNOMIAL

This program determines the real and complex roots of the sixth degree polynomial.

$$f(x) = X^6 + a_1X^5 + a_2X^4 + a_3X^3 + a_4X^2 + a_5X + a_6 \quad ,$$

where the coefficients a_i are real. The program uses the Lin-Bairstow method which determines a quadratic factor $(X^2 + rX + s)$ such that

$$f(x) = (x^2 + rX + s)\,(X^4 + b_1X^3 + b_2X^2 + b_3X + b_4) + RX + S$$

The variables r and s are obtained by an iteration scheme which reduces the remainder terms R and S to zero. The user can specify the remainder which he can tolerate.

The program applies the following recurrsive relationships:

$b_1 =$	$a_1 - r$	$c_1 \equiv$	$b_1 - r$
$b_2 =$	$a_2 - rb_1 - s$	$c_2 =$	$b_2 - rc_1 - s$
$b_3 =$	$a_3 - rb_2 - sb_1$	$c_3 =$	$b_3 - rc_2 - sc_1$
$b_4 =$	$a_4 - rb_3 - sb_2$	$c_4 =$	$b_4 - rc_3 - sc_2$
$b_5 =$	$a_5 - rb_4 - sb_3$	$\overline{c}_5 =$	$-rc_4 - sc_3$
$b_6 =$	$a_6 - rb_5 - sb_4$	$R =$	b_5

$$S = b_6 + rb_5$$

These quantities (b_i and c_i) are required for the determination of Δr and Δs in the equations:

$$c_4 \,\Delta r + c_3 \,\Delta s = b_5$$
$$\overline{c}_5 \,\Delta r + c_4 \,\Delta s = b_6$$

The terms r and s are incremented by Δr and Δ s respectively and the remainder terms are tested against the tolerance. If the remainders are small enough to pass the test, then the quadratic factor $(X^2 + rX + s)$ is solved by the quadratic formula and the remaining quartic $(X^4 + b_1X^3 + b_2X^2 + b_3X + b_4)$ is solved using the equations of program 09100-70403 "Roots of 4th Degree Polynomial." If the remainders are too large, the interation and the test are repeated.

Locations — (2 - c) through — (3 - 1) are used for storing the tolerance on $|R|$ and $|S|$ for pass 1 whereas locations (4 - d) through (5 - 8) contain the tolerance for pass 2 (for the reduction of the quartic).

Reference: Numerical Analysis, Kaiser S. Kunz, 1957, McGraw-Hill Book Company

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: s → Y, r → X *

PRESS: CONTINUE

DISPLAY

0	Z
s	Y
1	X

ENTER DATA: a_6 → Z, a_5 → Y, a_4 → X

PRESS: CONTINUE

DISPLAY

a_6	Z
a_6	Y
2	X

ENTER DATA: a_3 → Z, a_2 → Y, a_1 → X

PRESS: CONTINUE

DISPLAY

1	Z
s_1	Y
r_1	X

Terms b_1, b_2, b_3, and b_4, are stored in registers 6, 7, 8, and 9 respectively.

PRESS: GO TO

PRESS: —

PRESS: 0

PRESS: 0

* Both r and s must be non-zero.

USER INSTRUCTIONS (Con't)

ENTER PROGRAM 2 : Side B followed by Side A

PRESS: CONTINUE

DISPLAY

Real Roots or Complex Roots ($R = a \pm jb$)

Real Roots		Complex Roots	
0	Z	+ Imaginary Part	Z
R_2	Y	- Imaginary Part	Y
R_1	X	Real Part of R	X

PRESS: CONTINUE

DISPLAY

2	Z
s_2	Y
r_2	X

PRESS: CONTINUE

DISPLAY

Real Roots or Complex Roots ($R = a \pm jb$)

Real Roots		Complex Roots	
0	Z	+ Imaginary Part	Z
R_2	Y	- Imaginary Part	Y
R_1	X	Real Part of R	X

PRESS: CONTINUE

DISPLAY

3	Z
s_3	Y
r_3	X

PRESS: CONTINUE

DISPLAY

Real Roots or Complex Roots ($R = a \pm jb$)

Real Roots		Complex Roots	
0	Z	+ Imaginary Part	Z
R_2	Y	- Imaginary Part	Y
R_1	X	Real Part of R	X

To run another case, return to the beginning of the USER INSTRUCTIONS

EXAMPLE

Sixth Order Butterworth Polynomial

$$F(s) = s^6 + 3.864\,s^5 + 7.464\,s^4 + 9.141\,s^3 + 7.464\,s^2 + 3.864\,s + 1$$

$a_1 = 3.864$ $\qquad$ $r = 1$
$a_2 = 7.464$ $\qquad$ $s = -1$
$a_3 = 9.141$
$a_4 = 7.464$
$a_5 = 3.864$
$a_6 = 1.$

Tolerance is set at .001

Solution	Actual *
$r_1 = 1.9317$	1.9318
$s_1 = 1.0000$	1.0000
$root_1 = -.9658 - j.2591$	
$root_2 = -.9658 + j.2591$	
$r_2 = 1.4144$	1.4142
$s_2 = 0.9970$	1.0000
$root_3 = -.7072 - j.7049$	
$root_4 = -.7072 + j.7049$	
$r_3 = .5179$	.5176
$s_3 = 1.0019$	1.0000
$root_5 = -.2589 - j.9669$	
$root_6 = -.2589 + j.9669$	

Program 1 Page 1 PART NO. 09100-70405 9100B ONLY

Step	Key	Code	Display x	Display y	Display z
0 (+) 0	CLEAR	20	ENTER		
1	STOP	41	r	s	0
2	ACC +	60			
3	1	01	ENTER		
4	STOP	41	a_4	a_5	a_6
5	x→( )	23			
6	7	07			
7	y→( )	40			
8	8	10			
9	↓	25			
a	y→( )	40			
b	9	11			
c	2	02	ENTER		
d	STOP	41	a_1	a_2	a_3
1 (+) 0	x→( )	23			
1	4	04			
2	y→( )	40			
3	5	05			
4	ROLL ↑	22			
5	x→( )	23			
6	6	06			
7	f	15			
8	−	34			
9	y→( )	40			
a	a	13			
b	×	36			
c	e	12			
d	+	33			
2 (+) 0	↓	25			
1	−	34			
2	y→( )	40			
3	b	14			
4	a	13			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	−	34			
8	d	17			
9	4	04			
a	x←( )	67			
b	6	06			
c	x⇄y	30			
d	−	34			

Step	Key	Code	Display x	Display y	Display z
3 (+) 0	y→( )	40			
1	c	16			
2	b	14			
3	GO TO ( )	44			
4	▲SUB▼	77			
5	−	34			
6	d	17			
7	4	04			
8	x←( )	67			
9	7	07			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			
0 (−) 0	x⇄y	30			
1	−	34			
2	y→( )	40			
3	d	17			
4	c	16			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	d	17			
8	4	04			
9	x←( )	67			
a	8	10			
b	x⇄y	30			
c	−	34			
d	y→( )	40			
1 (−) 0	−	34			
1	e	12			
2	d	17			
3	GO TO ( )	44			
4	▲SUB▼	77			
5	d	17			
6	4	04			
7	x←( )	67			
8	9	11			
9	x⇄y	30			
a	−	34			
b	y→( )	40			
c	−	34			
d	f	15			

Step	Key	Code	Display x	Display y	Display z
2 (−) 0	f	15			
1	↑	27			
2	x←( )	67			
3	−	34			
4	e	12			
5	×	36			
6	ROLL ↓	31			
7	+	33			
8	|y|	55			
9	↓	25			
a	|y|	55			
b	↑	27			
c	•	21			
d	0	00			
3 (−) 0	0	00			
1	1	01			
2	IF x<y	52			
3	5	05			
4	8	10			
5	ROLL ↑	22			
6	IF x>y	53			
7	5	05			
8	8	10			
9	a	13			
a	x→( )	23			
b	6	06			
c	b	14			
d	x→( )	23			

Storage

	+	−
f	r /1	b_6
e	s /1	b_5
d	$b_4/c_4/$ r final	
c	$b_3/c_3/$ s final	
b	$b_2/c_2/$ r	
a	$b_1/c_1/$ s	
9	a_6/b_4	
8	a_5/b_3	
7	a_4/b_2	
6	a_3/b_1	
5	a_2	
4	a_1	
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
40	7	07			
41	c	16			
42	x→( )	23			
43	8	10			
44	d	17			
45	x→( )	23			
46	9	11			
47	RCL	61			
48	↑	27			
49	1	01			
4a	x→( )	23			
4b	e	12			
4c	x→( )	23			
4d	f	15			
50	ROLL ↓	31			
51	x→( )	23			
52	d	17			
53	y→( )	40			
54	c	16	DISPLAY		
55	STOP	41	r_1	s_1	1
56	CONT	47	ENTER		
57	STOP	41	Program 2		
58	a	13			
59	↑	27			
5a	f	15			
5b	—	34			
5c	y→( )	40			
5d	a	13			
60	×	36			
61	e	12			
62	+	33			
63	b	14			
64	x⇄y	30			
65	—	34			
66	y→( )	40			
67	b	14			
68	a	13			
69	GO TO ( )	44			
6a	▲SUB▼	77			
6b	d	17			
6c	4	04			
6d	c	16			

Step	Key	Code	Display x	Display y	Display z
70	x⇄y	30			
71	—	34			
72	y→( )	40			
73	c	16			
74	b	14			
75	GO TO ( )	44			
76	▲SUB▼	77			
77	d	17			
78	4	04			
79	d	17			
7a	x⇄y	30			
7b	—	34			
7c	y→( )	40			
7d	d	17			
80	c	16			
81	GO TO ( )	44			
82	▲SUB▼	77			
83	d	17			
84	4	04			
85	↓	25			
86	CHG SIGN	32			
87	↑	27			
88	f	15			
89	x→( )	23			
8a	b	14			
8b	e	12			
8c	x→( )	23			
8d	a	13			
90	0	00			
91	x→( )	23			
92	e	12			
93	x→( )	23			
94	f	15			
95	d	17			
96	↑	27			
97	x←( )	67			
98	—	34			
99	f	15			
9a	ROLL ↑	22			
9b	÷	35			
9c	ROLL ↑	22			
9d	x⇄y	30			

Step	Key	Code	Display x	Display y	Display z
a0	÷	35			
a1	↓	25			
a2	IF FLAG	43			
a3	b	14			
a4	3	03			
a5	ACC +	60			
a6	SET FLAG	54			
a7	d	17			
a8	↑	27			
a9	c	16			
aa	↑	27			
ab	x←( )	67			
ac	—	34			
ad	e	12			
b0	GO TO ( )	44			
b1	9	11			
b2	a	13			
b3	ACC —	63			
b4	x⇄y	30			
b5	y⇄( )	24			
b6	e	12			
b7	f	15			
b8	÷	35			
b9	x⇄y	30			
ba	×	36			
bb	ROLL ↓	31			
bc	—	34			
bd	b	14			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
c 0	+	33			
(−) 1	y→( )	40			
2	f	15			
3	↓	25			
4	a	13			
5	+	33			
6	y→( )	40			
7	e	12			
8	x←( )	67			
9	5	05			
a	↑	27			
b	x←( )	67			
c	4	04			
d	↑	27			
d 0	GO TO ( )	44			
(−) 1	+	33			
2	1	01			
3	7	07			
4	↑	27			
5	e	12			
6	×	36			
7	ROLL ↑	22			
8	x⇄y	30			
9	f	15			
a	×	36			
b	↓	25			
c	+	33			
d	RETURN	77			

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Program 2 Page 1 PART NO. 09100-70405 9100B ONLY

Step	Key	Code	Display x	Display y	Display z
0 0	1	01			
(+) 1	↑	27			
2	d	17			
3	↑	27			
4	c	16			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	−	34			
8	9	11			
9	2	02			
a	RCL	61			
b	↑	27			
c	x←( )	67			
d	6	06			
1 0	x⇄y	30			
(+) 1	−	34			
2	y→( )	40			
3	d	17			
4	×	36			
5	↓	25			
6	+	33			
7	↑	27			
8	x←( )	67			
9	7	07			
a	ROLL ↑	22			
b	−	34			
c	y→( )	40			
d	c	16			
2 0	f	15			
(+) 1	×	36			
2	e	12			
3	↑	27			
4	d	17			
5	×	36			
6	↓	25			
7	+	33			
8	↑	27			
9	x←( )	67			
a	8	10			
b	ROLL ↑	22			
c	−	34			
d	y→( )	40			

Step	Key	Code	Display x	Display y	Display z
3 0	b	14			
(+) 1	f	15			
2	×	36			
3	e	12			
4	↑	27			
5	c	16			
6	×	36			
7	↓	25			
8	+	33			
9	↑	27			
a	x←( )	67			
b	9	11			
c	ROLL ↑	22			
d	−	34			
4 0	y→( )	40			
(+) 1	a	13			
2	b	14			
3	↑	27			
4	f	15			
5	x⇄y	30			
6	×	36			
7	ROLL ↓	31			
8	+	33			
9	|y|	55			
a	↓	25			
b	|y|	55			
c	↑	27			
d	•	21			
5 0	0	00			
(+) 1	0	00			
2	1	01			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	GO TO ( )	44			
a	−	34			
b	0	00			
c	0	00			
d	END	46			

Step	Key	Code	Display x	Display y	Display z
0 0	IF x<y	52			
(−) 1	2	02			
2	d	17			
3	ROLL ↑	22			
4	IF x>y	53			
5	2	02			
6	d	17			
7	RCL	61			
8	↑	27			
9	2	02			
a	ROLL ↓	31	DISPLAY		
b	STOP	41	r_2	s_2	2
c	CONT	47			
d	ROLL ↑	22			
1 0	1	01			
(−) 1	ROLL ↓	31			
2	x⇄y	30			
3	GO TO ( )	44			
4	▲SUB▼	77			
5	9	11			
6	2	02			
7	c	16			
8	↑	27			
9	d	17			
a	↑	27			
b	3	03			
c	ROLL ↓	31	DISPLAY		
d	STOP	41	r_3	s_3	3

Storage

Register	+	−
f	r(1)	
e	s(1)	
d	r_1 / b_1	
c	s_1 / b_2	
b	b_4	
a	a_4	
9	a_3	
8	a_2	
7	a_1	
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
20	CONT	47			
21	ROLL ↑	22			
22	1	01			
23	ROLL ↓	31			
24	x⇄y	30			
25	GO TO ( )	44			
26	▲SUB▼	77			
27	9	11			
28	2	02			
29	CLEAR	20	DISPLAY		
2a	STOP	41	0	0	0
2b	CONT	47			
2c	CONT	47			
2d	d	17			
30	↑	27			
31	f	15			
32	−	34			
33	y→( )	40			
34	d	17			
35	×	36			
36	c	16			
37	x⇄y	30			
38	−	34			
39	y→( )	40			
3a	c	16			
3b	f	15			
3c	×	36			
3d	d	17			
40	↑	27			
41	e	12			
42	×	36			
43	↓	25			
44	+	33			
45	↓	25			
46	CHG SIGN	32			
47	↑	27			
48	f	15			
49	x→( )	23			
4a	−	34			
4b	f	15			
4c	e	12			
4d	x→( )	23			

Step	Key	Code	Display x	Display y	Display z
50	−	34			
51	e	12			
52	0	00			
53	x→( )	23			
54	e	12			
55	x→( )	23			
56	f	15			
57	c	16			
58	↑	27			
59	a	13			
5a	ROLL ↑	22			
5b	÷	35			
5c	ROLL ↑	22			
5d	x⇄y	30			
60	÷	35			
61	↓	25			
62	IF FLAG	43			
63	7	07			
64	1	01			
65	ACC +	60			
66	SET FLAG	54			
67	c	16			
68	↑	27			
69	d	17			
6a	↑	27			
6b	b	14			
6c	GO TO ( )	44			
6d	5	05			
70	a	13			
71	ACC −	63			
72	x⇄y	30			
73	y⇄( )	24			
74	e	12			
75	f	15			
76	÷	35			
77	e	12			
78	x⇄y	30			
79	×	36			
7a	ROLL ↓	31			
7b	−	34			
7c	x←( )	67			
7d	−	34			

Step	Key	Code	Display x	Display y	Display z
80	f	15			
81	+	33			
82	y→( )	40			
83	f	15			
84	↓	25			
85	x←( )	67			
86	−	34			
87	e	12			
88	+	33			
89	y→( )	40			
8a	e	12			
8b	f	15			
8c	GO TO ( )	44			
8d	+	33			
90	0	00			
91	b	14			
92	ROLL ↑	22			
93	÷	35			
94	ROLL ↑	22			
95	x⇄y	30			
96	÷	35			
97	2	02			
98	CHG SIGN	32			
99	÷	35			
9a	↓	25			
9b	↑	27			
9c	×	36			
9d	ROLL ↓	31			

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
a 0	x⇄y	30			
(-) 1	−	34			
2	CLEAR x	37			
3	IF x=y	50			
4	c	16			
5	3	03			
6	IF x>y	53			
7	b	14			
8	8	10			
9	↓	25			
a	√x	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
b 0	+	33			
(-) 1	ROLL ↑	22			
2	+	33			
3	CLEAR x	37			
4	ROLL ↓	31	DISPLAY		
5	CONT	47	ROOTS		
6	STOP	41			
7	RETURN	77			
8	↓	25			
9	CHG SIGN	32			
a	√x	76			
b	↑	27			
c	CHG SIGN	32			
d	ROLL ↑	22			
c 0	GO TO ( )	44			
(-) 1	b	14			
2	5	05			
3	↓	25			
4	CLEAR x	37			
5	ROLL ↓	31			
6	GO TO ( )	44			
7	b	14			
8	5	05			
9	END	46			
a					
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

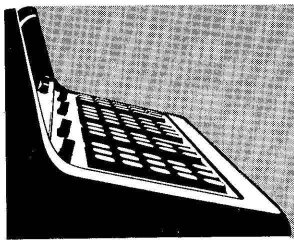
Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70406

CHARACTERISTIC EQUATION OF A 3 X 3 MATRIX AND EIGENVALUE DETERMINATION

This program will calculate the coefficients of the characteristic polynomial of a matrix of the form:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where the a_{ij} are real numbers.

The characteristic polynomial of A is determined by reducing the equation:

$$\text{determinant} \begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{bmatrix} = 0$$

to a polynomial of the form:

$$\lambda^3 + p\lambda^2 + q\lambda + r = 0$$

The eigenvalues are then obtained using the techniques described in Program 09100-70011.

Reference: AN INTRODUCTION TO NUMERICAL ANALYSIS, L. Fox, Oxford University Press, 1965

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $a_{11} \rightarrow Z$, $a_{12} \rightarrow Y$, $a_{13} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	Z

ENTER DATA: $a_{21} \rightarrow Z$, $a_{22} \rightarrow Y$, $a_{23} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

ENTER DATA: $a_{31} \rightarrow Z$, $a_{32} \rightarrow Y$, $a_{33} \rightarrow X$

PRESS: CONTINUE

DISPLAY

r	Z
q	Y
p	X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0	Z
0	Y
1 or 3	X

\# of real roots

PRESS: CONTINUE

DISPLAY

3 Real Roots

λ_3	Z
λ_2	Y
λ_1	X

or

Complex Roots

λ_3	Z
Im 1, 2	Y
Re 1, 2	X

To run program for a new matrix:

PRESS: CONTINUE

EXAMPLE

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & -2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Coefficient Solution:

$r = -17$

$q = -15$

$p = -1$

$$\lambda^3 - 1\,\lambda^2 - 15\,\lambda - 17 = 0$$

Eigenvalues:

$\lambda_3 = 4.832$

$\lambda_2 = -1.524$

$\lambda_1 = -2.308$

Step	Key	Code	Display x	Display y	Display z
0 0 (+)	CLEAR	20			
1	1	01	ENTER		
2	STOP	41	a_{13}	a_{12}	a_{11}
3	CONT	47			
4	x→( )	23			
5	d	17			
6	CLEAR x	37			
7	ROLL ↓	31			
8	ACC +	60			
9	↓	25			
a	2	02	ENTER		
b	STOP	41	a_{23}	a_{22}	a_{21}
c	x→( )	23			
d	c	16			

Step	Key	Code	Display x	Display y	Display z
1 0 (+)	ROLL ↓	31			
1	x→( )	23			
2	b	14			
3	y→( )	40			
4	a	13			
5	f	15			
6	×	36			
7	e	12			
8	↑	27			
9	b	14			
a	+	33			
b	y→( )	40			
c	−	34			
d	f	15			

Step	Key	Code	Display x	Display y	Display z
2 0 (+)	−	34			
1	×	36			
2	CLEAR x	37			
3	ROLL ↓	31			
4	−	34			
5	y→( )	40			
6	−	34			
7	e	12			
8	↓	25			
9	3	03	ENTER		
a	STOP	41	a_{33}	a_{32}	a_{31}
b	y⇄( )	24			
c	−	34			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
3 0 (+)	y→( )	40			
1	−	34			
2	d	17			
3	×	36			
4	y⇄( )	24			
5	−	34			
6	f	15			
7	+	33			
8	y→( )	40			
9	−	34			
a	c	16			
b	−	34			
c	×	36			
d	d	17			

Step	Key	Code	Display x	Display y	Display z
4 0 (+)	ROLL ↑	22			
1	x→( )	23			
2	−	34			
3	b	14			
4	×	36			
5	↓	25			
6	−	34			
7	↓	25			
8	y⇄( )	24			
9	−	34			
a	d	17			
b	x⇄y	30			
c	−	34			
d	x←( )	67			

Step	Key	Code	Display x	Display y	Display z
5 0 (+)	−	34			
1	e	12			
2	↑	27			
3	c	16			
4	×	36			
5	ROLL ↓	31			
6	−	34			
7	y⇄( )	24			
8	e	12			
9	↓	25			
a	×	36			
b	d	17			
c	↑	27			
d	a	13			

Step	Key	Code	Display x	Display y	Display z
6 0 (+)	×	36			
1	↓	25			
2	−	34			
3	↓	25			
4	y⇄( )	24			
5	−	34			
6	e	12			
7	×	36			
8	d	17			
9	ROLL ↓	31			
a	y⇄( )	24			
b	−	34			
c	f	15			
d	+	33			

Step	Key	Code	Display x	Display y	Display z
7 0 (+)	y⇄( )	24			
1	b	14			
2	↓	25			
3	×	36			
4	c	16			
5	↑	27			
6	f	15			
7	×	36			
8	↓	25			
9	−	34			
a	x⇄y	30			
b	y⇄( )	24			
c	−	34			
d	b	14			

Storage

Register	+	−
f	a_{12} /r	
e	a_{11} /q	a_{32}
d	a_{13} /p	
c	a_{22}	
b	a_{22}	
a	a_{21}	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
8 0	×	36			
(+) 1	b	14			
2	+	33			
3	↑	27			
4	y⇄( )	24			
5	—	34			
6	c	16			
7	e	12			
8	x⇄y	30			
9	CHG SIGN	32	DISPLAY		
a	STOP	41	P	q	r
b	CONT	47			
c	ROLL ↓	31			
d	x⇄y	30			
9 0	x→( )	23			
(+) 1	f	15			
2	y→( )	40			
3	e	12			
4	↓	25			
5	y→( )	40			
6	d	17			
7	|y|	55			
8	f	15			
9	x⇄y	30			
a	GO TO ( )	44			
b	—	34			
c	0	00			
d	0	00			
0 0	|y|	55			
(−) 1	+	33			
2	ENTER EXP	26			
3	CHG SIGN	32			
4	2	02			
5	↑	27			
6	1	01			
7	0	00			
8	×	36			
9	↓	25			
a	IF x<y	52			
b	0	00			
c	5	05			
d	x→( )	23			

Step	Key	Code	Display x	Display y	Display z
1 0	c	16			
(−) 1	CLEAR x	37			
2	↑	27			
3	f	15			
4	↑	27			
5	|y|	55			
6	÷	35			
7	y→( )	40			
8	b	14			
9	c	16			
a	x⇄y	30			
b	•	21			
c	1	01			
d	×	36			
2 0	y→( )	40			
(−) 1	c	16			
2	b	14			
3	CHG SIGN	32			
4	x→( )	23			
5	b	14			
6	↓	25			
7	c	16			
8	↑	27			
9	b	14			
a	×	36			
b	↓	25			
c	+	33			
d	↓	25			
3 0	IF x=y	50			
(−) 1	5	05			
2	0	00			
3	↑	27			
4	↑	27			
5	d	17			
6	+	33			
7	↓	25			
8	×	36			
9	e	12			
a	+	33			
b	↓	25			
c	×	36			
d	f	15			

Step	Key	Code	Display x	Display y	Display z
4 0	+	33			
(−) 1	CLEAR x	37			
2	IF x=y	50			
3	5	05			
4	0	00			
5	b	14			
6	×	36			
7	CLEAR x	37			
8	IF x>y	53			
9	2	02			
a	6	06			
b	GO TO ( )	44			
c	1	01			
d	9	11			
5 0	↓	25			
(−) 1	y→( )	40			
2	b	14			
3	d	17			
4	ROLL ↑	22			
5	+	33			
6	↑	27			
7	↓	25			
8	×	36			
9	e	12			
a	+	33			
b	↓	25			
c	CHG SIGN	32			
d	ROLL ↓	31			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
60	2	02			
61	CHG SIGN	32			
62	÷	35			
63	↓	25			
64	↑	27			
65	×	36			
66	ROLL ↓	31			
67	+	33			
68	ENTER EXP	26			
69	CHG SIGN	32			
6a	1	01			
6b	0	00			
6c	ROLL ↑	22			
6d	×	36			
70	|y|	55			
71	ROLL ↓	31			
72	CHG SIGN	32			
73	IF x>y	53			
74	8	10			
75	6	06			
76	SET FLAG	54			
77	CHG SIGN	32			
78	IF x>y	53			
79	CLEAR x	37			
7a	×	36			
7b	↓	25			
7c	√x	76			
7d	−	34			
80	ROLL ↑	22			
81	+	33			
82	↓	25			
83	GO TO ( )	44			
84	8	10			
85	9	11			
86	↓	25			
87	CHG SIGN	32			
88	√x	76			
89	x→( )	23			
8a	e	12			
8b	y→( )	40			
8c	f	15			
8d	CLEAR x	37			
90	↑	27			
91	↑	27			
92	1	01			
93	IF FLAG	43			
94	CLEAR x	37			
95	3	03			
96	STOP	41	1/3	0	0
97	CONT	47			
98	b	14			
99	ROLL ↓	31			
9a	RCL	61			
9b	STOP	41	λ_1 / $R_{1,2}$	λ_2 / $I_{1,2}$	λ_3 / λ_3
9c	CONT	47			
9d	END	46			

DISPLAY (after step 96): 1/3 0 0

DISPLAY (after step 9b): λ_1 λ_2 λ_3 or $R_{1,2}$ $I_{1,2}$ λ_3

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

9100B ONLY
PART NO.
09100-70407

PROGRAM FOR SIMULTANEOUS SOLUTION OF FOUR EQUATIONS IN FOUR UNKNOWNS

This program in its original form was written by Dr. Stefan J. Medwadowski, a Consulting Structural Engineer in San Francisco. One of his hobbies is programming the -hp- 9100A.

4x4 SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

Given a system of linear algebraic equations:

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + a_{14} x_4 = p_1$$

$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + a_{24} x_4 = p_2$$

$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + a_{34} x_4 = p_3$$

$$a_{41} x_1 + a_{42} x_2 + a_{43} x_3 + a_{44} x_4 = p_4$$

or a matrix notation:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

i.e., $\left[a_{ij} \right] \left\{ x_i \right\} = \left\{ p_i \right\}$ with i, j = 1, 2, 3, 4

Such systems occur frequently in the solution of boundary value problems of structural mechanics, such as those which arise in the theory of thin elastic shells or plates.

It is assumed that the solution of the system exists; i.e., that the determinant of the a_{ij} coefficient matrix does not vanish. The coefficients a_{ij} are assumed real.

NOTE: None of the determinates of the leading submatrices may be zero, or

$$\left| a_{11} \right| \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$$

Should one or more of these conditions exist (and should therefore the illegal operation light come on), it may be removed by re-arranging the sequence of the equations within the system. It is always possible to do this as a consequence of the postulated existence of a unique solution.

Method of Solution: Cholewski's Method

Reference: Salvadori and Baron, Numerical Method in Engineering, Prentice-Hall, 1952.

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

→ ENTER DATA: a_{ij} (or p_i) → X

NOTE: Data is entered row by row in the following order with a CONTINUE following each entry:

$a_{11}, a_{12}, a_{13}, a_{14}, \quad p_1$

$a_{21}, a_{22}, a_{23}, a_{24}, \quad p_2$

$a_{31}, a_{32}, a_{33}, a_{34}, \quad p_3$

$a_{41}, a_{42}, a_{43}, a_{44}, \quad p_4$

DISPLAY

0	Z
x_1	Y
x_2	X

PRESS: CONTINUE

DISPLAY

0	Z
x_3	Y
x_4	X

To re-run program,

← PRESS: CONTINUE

EXAMPLE

$$2x_1 + x_2 + 3x_3 + x_4 = 7$$
$$x_1 + 4x_2 + x_3 = 10$$
$$3x_1 - 5x_3 - 2x_4 = 10$$
$$8x_1 - x_2 + 4x_3 + 2x_4 = 22$$

$x_1 = 3$ $\quad$ $x_3 = -1$

$x_2 = 2$ $\quad$ $x_4 = 2$

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20	ENTER		
(+) 1	STOP	41	a_{11}	—	—
2	ROLL ↓	31	ENTER		
3	STOP	41	a_{12}	—	—
4	ROLL ↑	22			
5	÷	35			
6	y→( )	40			
7	f	15			
8	ROLL ↓	31	ENTER		
9	STOP	41	a_{13}	—	—
a	ROLL ↑	22			
b	÷	35			
c	y→( )	40			
d	e	12			
1 0	ROLL ↓	31	ENTER		
(+) 1	STOP	41	a_{14}	—	—
2	ROLL ↑	22			
3	÷	35			
4	y→( )	40			
5	b	14			
6	ROLL ↓	31	ENTER		
7	STOP	41	P_1	—	—
8	ROLL ↑	22			
9	÷	35			
a	y→( )	40			
b	a	13	ENTER		
c	STOP	41	a_{21}	—	—
d	↑	27			
2 0	↑	27			
(+) 1	f	15			
2	×	36	ENTER		
3	STOP	41	a_{22}	—	—
4	x⇄y	30			
5	−	34			
6	y→( )	40			
7	c	16			
8	↓	25			
9	e	12			
a	×	36	ENTER		
b	STOP	41	a_{23}	—	—
c	x⇄y	30			
d	−	34			
3 0	STOP	41	a_{24}	—	—
(+) 1	x→( )	23	ENTER		
2	d	17			
3	c	16			
4	÷	35			
5	y⇄( )	24			
6	d	17			
7	b	14			
8	ROLL ↑	22			
9	×	36			
a	ROLL ↓	31			
b	−	34			
c	c	16			
d	÷	35			
4 0	y⇄( )	24			
(+) 1	a	13			
2	y→( )	40			
3	−	34			
4	f	15			
5	↓	25			
6	×	36	ENTER		
7	STOP	41	P_2	—	—
8	x⇄y	30			
9	−	34			
a	c	16			
b	÷	35	ENTER		
c	STOP	41	a_{31}	—	—
d	ROLL ↑	22			
5 0	f	15			
(+) 1	x⇄y	30			
2	×	36			
3	ROLL ↓	31			
4	y→( )	40			
5	−	34			
6	e	12			
7	x⇄y	30	ENTER		
8	STOP	41	a_{32}	—	—
9	x⇄y	30			
a	−	34			
b	y→( )	40			
c	c	16			
d	d	17			
6 0	×	36			
(+) 1	e	12			
2	ROLL ↑	22			
3	×	36			
4	ROLL ↓	31			
5	+	33	ENTER		
6	STOP	41	a_{33}	—	—
7	x⇄y	30			
8	−	34			
9	y⇄( )	24			
a	a	13			
b	y→( )	40			
c	−	34			
d	d	17			
7 0	c	16			
(+) 1	×	36			
2	b	14			
3	ROLL ↑	22			
4	×	36			
5	ROLL ↓	31			
6	+	33	ENTER		
7	STOP	41	a_{34}	—	—
8	x⇄y	30			
9	−	34			
a	a	13			
b	÷	35			
c	y⇄( )	24			
d	−	34			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	y	z
8 0	e	12			
(+) 1	y→( )	40			
2	−	34			
3	c	16			
4	c	16			
5	×	36			
6	ROLL ↑	22			
7	x⇄y	30			
8	x←( )	67			
9	−	34			
a	f	15			
b	x⇄y	30			
c	×	36			
d	ROLL ↓	31			
9 0	+	33	ENTER		
(+) 1	STOP	41	P_3	—	—
2	x⇄y	30			
3	−	34			
4	a	13			
5	÷	35			
6	y→( )	40			
7	−	34			
8	b	14	ENTER		
9	STOP	41	a_{41}	—	—
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			
0 0	↑	27			
(−) 1	↑	27			
2	f	15			
3	×	36	ENTER		
4	STOP	41	a_{42}	—	—
5	x⇄y	30			
6	−	34			
7	y→( )	40			
8	c	16			
9	d	17			
a	×	36			
b	e	12			
c	ROLL ↑	22			
d	×	36			

Step	Key	Code	Display x	y	z
1 0	ROLL ↓	31			
(−) 1	+	33	ENTER		
2	STOP	41	a_{43}	—	—
3	x⇄y	30			
4	−	34			
5	↓	25			
6	↑	27			
7	y⇄( )	24			
8	−	34			
9	e	12			
a	y→( )	40			
b	a	13			
c	×	36			
d	y⇄( )	24			
2 0	−	34			
(−) 1	d	17			
2	c	16			
3	x⇄y	30			
4	×	36			
5	x⇄y	30			
6	y⇄( )	24			
7	−	34			
8	d	17			
9	+	33			
a	b	14			
b	ROLL ↑	22			
c	×	36			
d	ROLL ↓	31			
3 0	+	33	ENTER		
(−) 1	STOP	41	a_{44}	—	—
2	x⇄y	30			
3	−	34			
4	y⇄( )	24			
5	−	34			
6	f	15	ENTER		
7	STOP	41	P_4	—	—
8	ROLL ↓	31			
9	×	36			
a	ROLL ↓	31			
b	−	34			
c	x⇄y	30			
d	y⇄( )	24			

Step	Key	Code	Display x	y	z
4 0	−	34			
(−) 1	b	14			
2	ROLL ↓	31			
3	y⇄( )	24			
4	−	34			
5	e	12			
6	×	36			
7	ROLL ↓	31			
8	−	34			
9	ROLL ↓	31			
a	y⇄( )	24			
b	−	34			
c	c	16			
d	ROLL ↑	22			
5 0	c	16			
(−) 1	ROLL ↑	22			
2	×	36			
3	ROLL ↓	31			
4	−	34			
5	x←( )	67			
6	−	34			
7	f	15			
8	÷	35			
9	a	13			
a	y→( )	40			
b	c	16			
c	×	36			
d	x⇄y	30			

Storage

Register	Value
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT·PACKARD

Step	Key	Code	Display x	Display y	Display z
6 0	y⇄( )	24			
(–) 1	—	34			
2	c	16			
3	—	34			
4	y→( )	40			
5	a	13			
6	d	17			
7	×	36			
8	ROLL ↓	31			
9	—	34			
a	ROLL ↑	22			
b	y⇄( )	24			
c	—	34			
d	d	17			

Step	Key	Code	Display x	Display y	Display z
7 0	c	16			
(–) 1	×	36			
2	ROLL ↓	31			
3	—	34			
4	y→( )	40			
5	d	17			
6	f	15			
7	×	36			
8	a	13			
9	↑	27			
a	e	12			
b	×	36			
c	↓	25			
d	+	33			

Step	Key	Code	Display x	Display y	Display z
8 0	c	16			
(–) 1	↑	27			
2	b	14			
3	×	36			
4	↓	25			
5	+	33			
6	↓	25			
7	y⇄( )	24			
8	—	34			
9	e	12			
a	—	34			
b	d	17			
c	↑	27			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
9 0	ROLL ↓	31	DISPLAY		
(–) 1	STOP	41	X_2	X_1	0
2	CONT	47			
3	a	13			
4	↑	27			
5	c	16			
6	↑	27			
7	0	00			
8	ROLL ↓	31			
9	CONT	47	DISPLAY		
a	END	46	X_4	X_3	0
b					
c					
d					

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

CONVOLUTION WITH PLOT

This program uses the convolution integral to find the response of a system to an arbitrary forcing function. The forcing function and the impulse response of the system must be known.

Development:

The convolution integral is: $e(t) * h(t) = y(t) = \int_0^t e(\tau)\, h(t-\tau)\, d\tau$

where $e(t)$ is the forcing function

$y(t)$ is the system response due to the forcing function.

$h(t)$ is the impulse response.

It is assumed that the forcing function is zero prior to time equal zero.

NOTE: X_{coef} and Y_{coef} are multiplicative scaling constants applied in the plotting subroutine.

Reference: Transform and State Variable Methods in Linear Systems --
Someshwar C. Gupta
Publisher - John Wiley and Sons, Inc.
Copyright - 1966, 1st Edition

USER INSTRUCTIONS

SET: Decimal Wheel at 6 or less

PRESS: STOP

Using the origin controls, place the pen at an appropriate initial position. A trial run with large Δt will help to determine a proper origin.

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→PRESS: GO TO

PRESS: —

PRESS: 0

PRESS: 0

Enter program steps to form $e(\tau) \cdot h(t - \tau)$ and place this quantity in the Y register. τ is in the X register and in storage register e. t is in storage register d. Registers -0 through -8 are available for generating the product.

The last step must be:

RETURN

SET: [| RUN]

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: Xcoef→X, Ycoef→Y

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: Panels/unit time (N) →Y
Time increment (Δt) →X

→PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

y(t)	Z
t	Y
0	X

NOTE: Each successive depression of the CONTINUE button increments t by the amount Δt and calculates y (t) for the incremented value of t.

y (t) is multiplied by Ycoef, X (t) is multiplied by Xcoef and the scaled values are plotted. To generate the plot free of interruptions replace the STOP at + (8 - 4) with a CONTINUE. *

To run another case

* If no plot is desired, place a RETURN in (-9, 0). To scale the plot axis with AXIS PLOT (09100-76007) use Yshift = 0, Xshift = 0, Yscale = 500/Ycoef, Xscale = 500/Xcoef, Yorigin = 0, and Xorigin = 0.

EXAMPLE

(A) Find the response of a circuit with an impulse response of e^{-t} to a forcing function of u (t) -- for $0 \le t \le \infty$.

Therefore: $e(\tau) = u(\tau) = 1$ for $0 \le t \le \infty$

$$h(t - \tau) = e^{-(t - \tau)} = e^{\tau - t}$$

Y coef = 5000

X coef = 1000

N = 10

Δt = .05

Integrating analytically, the result is:

$$\int_0^t e(\tau) h(t - \tau) d\tau = \int_0^t e^{(\tau - t)} d\tau = 1 - e^{-t} \text{ for } t > 0$$

Results:

t	y (t)
1	.63212
2	.86466
3	.95021
4	.98168
5	.99326

(B) Find the response of a circuit with an impulse response of e^{-t} to a forcing function of sin 2t -- for $0 \leq t \leq \pi$.

Therefore: $e(\tau) = \sin(2\tau)$ for $0 \leq t \leq \pi$

$h(t-\tau) = e^{-(t-\tau)} = e^{(\tau - t)}$

SET: RADIANS

Ycoef = 4000

Xcoef = 1000

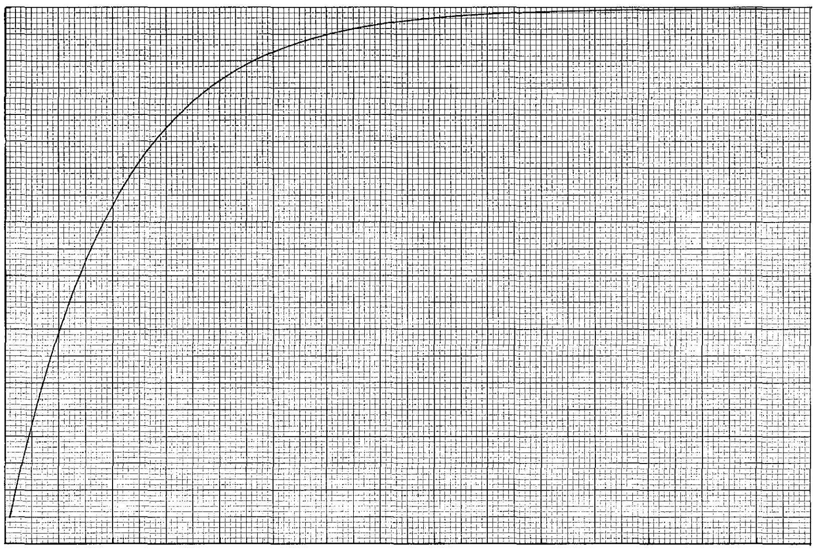
N = 20

$\Delta t = .05$

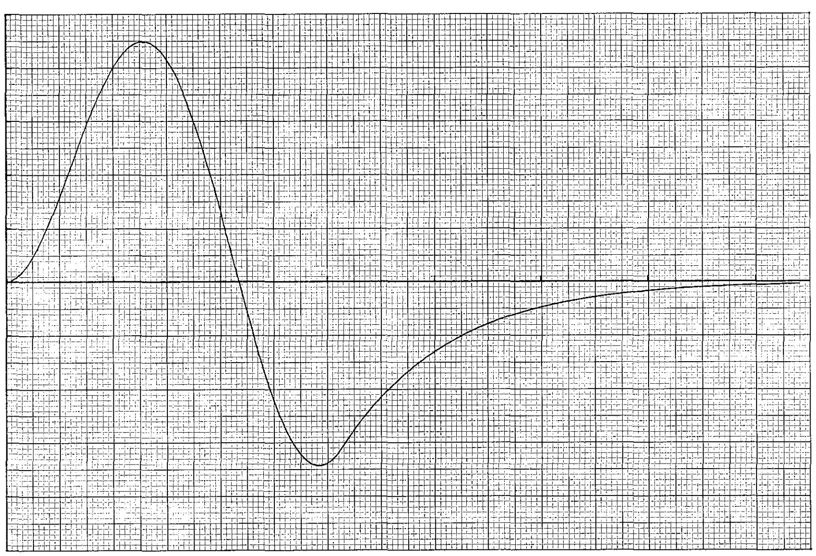
EXAMPLE (Con't)

Results:

t	y(t)
.5	.1948
1.0	.4955
1.5	.5135
2.0	.1642
2.5	-.2724
3.0	-.4200
3.5	-.2672
4.0	-.1626
4.5	-.0983
5.0	-.0596



EXAMPLE A



EXAMPLE B

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	FMT	42			
2	↓	25			
3	1	01	ENTER		
4	STOP	41	X_{coef}	Y_{coef}	0
5	x→( )	23			
6	−	34			
7	f	15			
8	y→( )	40			
9	−	34			
a	e	12			
b	CLEAR	20			
c	2	02	ENTER		
d	STOP	41	Δt	panels	0
1 0	y→( )	40			
(+) 1	b	14			
2	ROLL ↑	22			
3	y→( )	40			
4	a	13			
5	+	33			
6	y→( )	40			
7	d	17			
8	↓	25			
9	×	36			
a	•	21			
b	5	05			
c	+	33			
d	↓	25			
2 0	int x	64			
(+) 1	↑	27			
2	↑	27			
3	2	02			
4	÷	35			
5	↓	25			
6	↑	27			
7	int x	64			
8	IF x<y	52			
9	ROLL ↓	31			
a	1	01			
b	+	33			
c	↑	27			
d	RCL	61			

Step	Key	Code	Display x	Display y	Display z
3 0	ACC −	63			
(+) 1	d	17			
2	x⇄y	30			
3	CLEAR x	37			
4	ROLL ↓	31			
5	x⇄y	30			
6	÷	35			
7	y→( )	40			
8	c	16			
9	ROLL ↑	22			
a	x→( )	23			
b	e	12			
c	GO TO ( )	44			
d	▲SUB▼	77			
4 0	−	34			
(+) 1	0	00			
2	0	00			
3	e	12			
4	↑	27			
5	CLEAR x	37			
6	IF x=y	50			
7	7	07			
8	0	00			
9	d	17			
a	−	34			
b	|y|	55			
c	ENTER EXP	26			
d	9	11			
5 0	CHG SIGN	32			
(+) 1	IF x>y	53			
2	7	07			
3	6	06			
4	e	12			
5	x⇄y	30			
6	d	17			
7	IF x<y	52			
8	0	00			
9	0	00			
a	c	16			
b	ROLL ↑	22			
c	↑	27			
d	IF FLAG	43			

Step	Key	Code	Display x	Display y	Display z
6 0	6	06			
(+) 1	7	07			
2	SET FLAG	54			
3	4	04			
4	GO TO ( )	44			
5	6	06			
6	8	10			
7	2	02			
8	×	36			
9	↓	25			
a	ACC +	60			
b	GO TO ( )	44			
c	3	03			
d	b	14			
7 0	c	16			
(+) 1	ROLL ↑	22			
2	ACC +	60			
3	GO TO ( )	44			
4	3	03			
5	b	14			
6	CLEAR x	37			
7	ROLL ↑	22			
8	ACC +	60			
9	RCL	61			
a	↑	27			
b	c	16			
c	×	36			
d	3	03			

Storage

	+	−
f		X_{coef}
e	τ	Y_{coef}
d	ΣΔt	t
c	ΔX	
b	N	
a	Δt	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
8 (+) 0	÷	35			
1	CLEAR x	37			
2	ROLL ↑	22			
3	x⇄y	30	DISPLAY		
4	STOP	41	0	t	y(t)
5	CONT	47			
6	y→( )	40			
7	—	34			
8	d	17			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	—	34			
c	9	11			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
9 (+) 0	b	14			
1	↑	27			
2	x←( )	67			
3	—	34			
4	d	17			
5	↑	27			
6	IF FLAG	43			
7	1	01			
8	4	04			
9	GO TO ( )	44			
a	0	00			
b	0	00			
c					
d					

Step	Key	Code	Display x	Display y	Display z
0 (−) 0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Registers (-)(0) through (-)(8) available for generation $e(\tau) \cdot h(t-\tau)$

Step	Key	Code	Display x	Display y	Display z
9 (−) 0	x←( )	67			
1	—	34			
2	f	15			
3	×	36			
4	ROLL ↑	22			
5	x⇄y	30			
6	x←( )	67			
7	—	34			
8	e	12			
9	×	36			
a	x⇄y	30			
b	ROLL ↑	22			
c	↑	27			
d	7	07			

Step	Key	Code	Display x	Display y	Display z
a (−) 0	5	05			
1	0	00			
2	0	00			
3	IF x<y	52			
4	y→( )	40			
5	RETURN	77			
6	↓	25			
7	FMT	42			
8	↓	25			
9	RETURN	77			
a					
b					
c					
d					

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

EXAMPLE A

Step	Key	Code	Display x	Display y	Display z
0 0	↑	27			
(–) 1	d	17			
2	−	34			
3	↓	25			
4	e^x	74			
5	↑	27			
6	RETURN	77			

EXAMPLE B

Step	Key	Code	Display x	Display y	Display z
0 0	↑	27			
(–) 1	d	17			
2	−	34			
3	↓	25			
4	e^x	74			
5	↑	27			
6	π	56			
7	↑	27			
8	e	12			
9	IF x>y	53			
a	CLEAR x	37			
b	↑	27			
c	x⇄y	30			
d	2	02			
1 0	×	36			
(–) 1	↓	25			
2	sin x	70			
3	×	36			
4	RETURN	77			

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

NUMERICAL INTEGRATION USING SIMPSON'S RULE WHEN f(x) IS KNOWN

This program evaluates the integral of a known f(x) using Simpson's rule. The equation is:

$$\int_a^b f(X)dX \cong \frac{\Delta X}{3}\left[f(a) + 4f(a+\Delta X) + 2f(a+2\Delta X) + \ldots + 2f\left\{a+(n-2)\Delta X\right\} + 4f\left\{a+(n-1)\Delta X\right\} + f(b)\right]$$

for n panels (n must be even) where $\Delta X = \frac{b-a}{n}$

The specific f(x) is programmed into the calculator by the user as a subroutine and is then used by the general solution to evaluate the integral. Execution time is dependent on the number of panels. Note f(x) should not have any singularities in the integration interval.

Reference:

Numerical Analysis
by
Kaiser S. Kunz

McGraw-Hill Book Company, Inc. (1967)

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: (Starting Address is 0-0)

PRESS: GO TO

PRESS: –

PRESS: 0

PRESS: 0

SET: | PROGRAM | |

Enter the program steps that take X from the X register (X is also in the e register) and calculate f(X). Place f(X) in the Y register and RETURN. The entire (-) page is available for storing and calculating f(X). The flag is unavailable.

SET: | | RUN |

PRESS: END

PRESS: CONTINUE

DISPLAY

0 ————	Z
0 ————	Y
1 ————	X

ENTER DATA: n panels (must be an even number) → Z, b → Y, a → X

PRESS: CONTINUE

DISPLAY

AREA ————	Z
b ————	Y
a ————	X

To calculate the integral of another f(X), repeat the instructions.

PRESS: GO TO

PRESS: –

PRESS: 0

PRESS: 0

and proceed as before.

EXAMPLES

$$\int_{e}^{e^2} \frac{1}{X\,(\ln X)^3}\,dX = \frac{3}{8}$$

1. Let n panels = 30 (must be even)

$b = e^2$

$a = e$

$$\int_{a}^{b} \frac{1}{X\,(\ln X)^3}\,dX = .3750098$$

2. Let n panels = 100

$b = e^2$

$a = e$

$$\int_{a}^{b} \frac{1}{X\,(\ln X)^3}\,dX = .3750001$$

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	1	01		ENTER	
2	STOP	41	a	b	n
3	y→( )	40			
4	d	17			
5	x→( )	23			
6	b	14			
7	−	34			
8	ROLL ↓	31			
9	x⇄y	30			
a	÷	35			
b	y→( )	40			
c	c	16			
d	ROLL ↑	22			
1 0	x→( )	23			
(+) 1	e	12			
2	GO TO ( )	44			
3	▲SUB▼	77			
4	−	34			
5	0	00			
6	0	00			
7	e	12			
8	↑	27			
9	b	14			
a	IF x=y	50			
b	4	04			
c	4	04			
d	d	17			
2 0	−	34			
(+) 1	|y|	55			
2	ENTER EXP	26			
3	9	11			
4	CHG SIGN	32			
5	IF x>y	53			
6	4	04			
7	a	13			
8	e	12			
9	x⇄y	30			
a	d	17			
b	IF x<y	52			
c	4	04			
d	a	13			

Step	Key	Code	Display x	Display y	Display z
3 0	c	16			
(+) 1	ROLL ↑	22			
2	↑	27			
3	IF FLAG	43			
4	3	03			
5	b	14			
6	SET FLAG	54			
7	4	04			
8	GO TO ( )	44			
9	3	03			
a	c	16			
b	2	02			
c	×	36			
d	↓	25			
4 0	ACC +	60			
(+) 1	GO TO ( )	44			
2	1	01			
3	1	01			
4	c	16			
5	ROLL ↑	22			
6	ACC +	60			
7	GO TO ( )	44			
8	1	01			
9	1	01			
a	CLEAR x	37			
b	ROLL ↑	22			
c	ACC +	60			
d	RCL	61			
5 0	↑	27			
(+) 1	c	16			
2	×	36			
3	3	03			
4	÷	35			
5	b	14			
6	ROLL ↑	22			
7	x⇄y	30		DISPLAY	
8	STOP	41	a	b	$\int_a^b$
9	GO TO ( )	44			
a	0	00			
b	0	00			
c					
d					

Registers (-)(0) through (-)(d) are available for forming f(x)

Step	Key	Code	Display x	Display y	Display z
0 0					
(−) 1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Example

Step	Key	Code	Display x	Display y	Display z
0 0	↑	27			
(−) 1	ln x	65			
2	×	36			
3	×	36			
4	×	36			
5	1	01			
6	x⇄y	30			
7	÷	35			
8	RETURN	77			
9					
a					
b					
c					
d					

Storage

Register	+
f	
e	
d	b
c	ΔX
b	a
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

9100B ONLY
PART NO.
09100-70410

INTEGRAL OF THE FORM: $F(x) = \int_a^x f(u)\,du$ WITH PLOT

This program evaluates the integral of a function f(u) between any lower limit a and a successively incremented upper limit.

A modification of Simpson's one third rule is used to perform the integration. The following equations are used:

$$\text{For } j = 0, \quad \int_a^{x_j} f(u)\,du = h/6 \left[1/3\ f(a) + 4/3\ f(x_{j+1}) + 1/3\ f(x_{j+2}) \right]$$

$$\text{For } j \geq 3, \quad \int_{x_{j-1}}^{x_j} f(u)\,du = h/24 \left[f(x_{j-3}) + 5f(x_{j-2}) + 19\ f(x_{j-1}) + 9f(x_j) \right]$$

Notes:

Due to the manner in which the program is written, the function will initially be integrated over an area formed by twice the increment; thereafter, integration will proceed by one increment at a time.

To integrate with some constant (a) as a lower limit (other than zero), in the program steps in the f(u) subroutine add a to u to form u' and form f(u'). For a lower limit of zero simply form f(u). In each case the upper limit will exceed the lower by some multiple of the increment depending on the number of times x is incremented by depression of the CONTINUE key.

To integrate from some lower limit to a specified upper limit without continually depressing the CONTINUE key, enter a Pause at step (5) (7). Then just before the incremented upper limit (shown in a flashing display) reaches the desired upper limit, depress PAUSE--this will stop the program after the next integration.

If a plotter is available a plot of the function being integrated may be obtained by inserting a plot subroutine at (-9) (0). This plot subroutine scales the calculated X and F(X) by multiplying them by X_{coef} and Y_{coef} respectively.

Reference: Numerical Analysis
by
Faiser S. Kunz
McGraw-Hill Book Company, Inc.

Using origin controls, position pen at (0, Y) *

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: GO TO

PRESS: –

PRESS: 0

PRESS: 0

SET: [PROGRAM |]

Starting at (-0) (0) enter the program steps to form the function f(u) to be integrated; u is located in the Y register. After forming f(u) place it in the Y register. The last step of the subroutine must be:

RETURN

NOTE: Registers (-0) thru (-8) are available for programming and storage of f(u).

SET: [| RUN]

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $Y_{coef} \rightarrow Y$, $X_{coef} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: Increment h $\rightarrow$ X

PRESS: CONTINUE

DISPLAY

$\int_a^x f(u)\,du$	Z
(X - a)	Y
0	X

USER INSTRUCTIONS

Each successive "CONTINUE" increments the upper limit and evaluates the integral using the increment upper limit. The integral is plotted after each x incrementation.

If no plot is desired, place a RETURN in (-9)(0) and CONTINUES in (0)(1) and (0)(2).

EXAMPLE

Obtain the cumulative distribution function of the normally distributed random variable X of variance 1 and mean value of 0

The probability density function of X is

$$P(X) = \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}$$

The cumulative distribution function is given by

$$Q(X) = \int_{-\infty}^{x} P(u)\,du$$

The lower limit ($-\infty$) can be replaced by a = -4 without loss of accuracy.

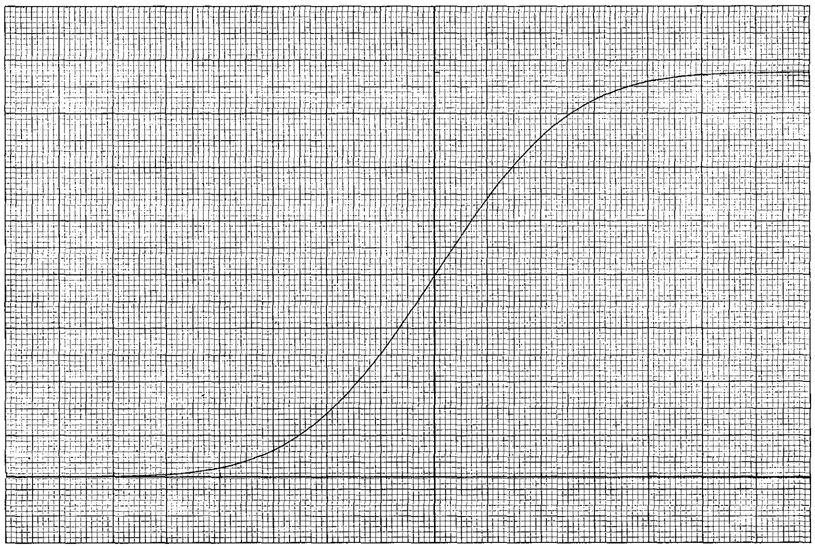
The program steps to generate P(u) are given on page 2 .

Data: Ycoef = 3760, Xcoef = 1000, h = .1

Results

X + 4	$\int_{-4}^{x} P(u)du$	X
0.5	.000201	-3.5
1.0	.001318	-3.0
1.5	.006178	-2.5
2.0	.022719	-2.0
2.5	.066775	-1.5
3.0	.158622	-1.0
3.5	.308504	-0.5
4.0	.499968	-0.0
4.5	.691432	0.5
5.0	.841314	1.0
5.5	.933162	1.5
6.0	.977218	2.0
6.5	.993758	2.5
7.0	.998618	3.0
7.5	.999736	3.5
8.0	.999937	4.0

*The Y (inches) must be determined by trial.



Step	Key	Code	Display x	Display y	Display z
00 (+)	CLEAR	20			
01	FMT	42			
02	↓	25			
03	1	01	ENTER		
04	STOP	41	X_{coef}	Y_{coef}	0
05	x→( )	23			
06	−	34			
07	f	15			
08	y→( )	40			
09	−	34			
0a	e	12			
0b	CLEAR	20			
0c	2	02	ENTER		
0d	STOP	41	h	0	0
10 (+)	x→( )	23			
11	d	17			
12	GO TO ( )	44			
13	▲SUB▼	77			
14	−	34			
15	0	00			
16	0	00			
17	2	02			
18	4	04			
19	÷	35			
1a	e	12			
1b	↑	27			
1c	0	00			
1d	IF x=y	50			
20 (+)	6	06			
21	3	03			
22	1	01			
23	IF x=y	50			
24	7	07			
25	0	00			
26	2	02			
27	IF x=y	50			
28	7	07			
29	8	10			
2a	0	00			
2b	x⇄y	30			
2c	9	11			
2d	ROLL ↑	22			
30 (+)	×	36			
31	ROLL ↓	31			
32	ACC +	60			
33	1	01			
34	9	11			
35	ROLL ↓	31			
36	y⇄( )	24			
37	a	13			
38	ROLL ↓	31			
39	×	36			
3a	ROLL ↓	31			
3b	ACC +	60			
3c	5	05			
3d	ROLL ↓	31			
40 (+)	y⇄( )	24			
41	b	14			
42	ROLL ↓	31			
43	×	36			
44	ROLL ↓	31			
45	ACC −	63			
46	↓	25			
47	1	01			
48	y⇄( )	24			
49	c	16			
4a	x⇄y	30			
4b	ACC +	60			
4c	RCL	61			
4d	↑	27			
50 (+)	d	17			
51	×	36			
52	ROLL ↑	22			
53	×	36			
54	d	17			
55	−	34			
56	CLEAR x	37	DISPLAY		
57	STOP	41	0	X-a	$\int_a^x$
58	CONT	47			
59	GO TO ( )	44			
5a	▲SUB▼	77			
5b	−	34			
5c	9	11			
5d	0	00			
60 (+)	GO TO ( )	44			
61	8	10			
62	3	03			
63	↓	25			
64	y→( )	40			
65	c	16			
66	8	10			
67	×	36			
68	1	01			
69	x⇄y	30			
6a	ACC +	60			
6b	GO TO ( )	44			
6c	8	10			
6d	3	03			
70 (+)	↓	25			
71	y→( )	40			
72	b	14			
73	3	03			
74	2	02			
75	GO TO ( )	44			
76	6	06			
77	7	07			
78	↓	25			
79	y→( )	40			
7a	a	13			
7b	8	10			
7c	×	36			
7d	1	01			

Storage

Register	+	−
f	$f(x_j)$	X_{coef}
e	j	Y_{coef}
d	h	
c		
b		
a		
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
8 0	GO TO ( )	44				9 0	$x\leftarrow()$	67				0 0	4	04			
(+) 1	4	04				(−) 1	−	34				(−) 1	CHG SIGN	32			
2	a	13				2	f	15				2	+	33			
3	e	12				3	×	36				3	↓	25			
4	↑	27				4	ROLL ↑	22				4	↑	27			
5	d	17				5	$x \rightleftarrows y$	30				5	×	36			
6	×	36				6	$x\leftarrow()$	67				6	2	02			
7	GO TO ( )	44				7	−	34				7	÷	35			
8	1	01				8	e	12				8	↓	25			
9	2	02				9	×	36				9	CHG SIGN	32			
a						a	$x \rightleftarrows y$	30				a	e^x	74	EXAMPLE		
b						b	ROLL ↑	22				b	↑	27			
c						c	FMT	42				c	π	56			
d						d	↓	25				d	↑	27			
0						a 0	RETURN	77				1 0	2	02			
1						(−) 1						(−) 1	×	36			
2						2						2	↓	25			
3						3						3	$\sqrt{x}$	76			
4						4						4	÷	35			
5						5						5	RETURN	77			
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

MAXIMUM AND MINIMUM OF Z = Z(X, Y)

This program will calculate the approximate maximum and minimum of a function of two independent variables over specified ranges of these two variables. The method involved is a direct evaluation of the function at a specified number of points matrixly distributed about the X - Y grid.

The program was written mainly for use in conjunction with program 09100-70412 but may also be used for such purposes as obtaining upper and lower bounds within a particular area.

It is recommended that the program be run at least twice within one range (with the sampling number increased the second time) to verify the results of the first run.

USER INSTRUCTIONS

ENTER PROGRAM: Starting Address is (-0) (0)

PRESS: GO TO (+) (3) (a)

SET: [PROGRAM|]

Enter the program steps which take X from the +e register and Y from the +9 register and generate Z(X, Y) in the Y register and program a RETURN for the final step. The flag is not available. (Leave +e and +9 unchanged)

NOTE: Locations + (3) (a) through + (8) (d) are available. (74 steps)

SET: [|RUN]

→ PRESS: GO TO (-) (0) (0)

PRESS: CONTINUE

DISPLAY

0 ———— Z
1 ———— Y
1 ———— X

ENTER RANGE OF X: X_{max} → Y, X_{min} → X

PRESS: CONTINUE

DISPLAY

0 ———— Z
2 ———— Y
2 ———— X

ENTER RANGE OF Y: Y_{max} → Y, Y_{min} → X

PRESS: CONTINUE

DISPLAY

0 ———— Z
3 ———— Y
3 ———— X

ENTER NUMBER OF SAMPLES:
Total number of samples to be taken → X

PRESS: CONTINUE

NOTE: The screen may be blanked for several minutes, the actual time depending upon the specified number of samples and the particular function involved.

USER INSTRUCTIONS (Con't)

DISPLAY

0 ———— Z
Z_{max} ———— Y
Z_{min} ———— X

← To re-run this program with different inputs.

EXAMPLE

Find the max - min of $Z = y^2 - x^2$
where

$-2 \leq X \leq 2$

$-2 \leq Y \leq 2$

Allow 100 samples of the function

Solution:

$-4 \leq Z \leq 4$

Program steps to generate function

3 a	x←( )	67			
(+) b	9	11			
c	↑	27			
d	×	36			
4 0	e	12			
(+) 1	↑	27			
2	×	36			
3	↓	25			
4	−	34			
5	RETURN	77			

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
01	1	01			
02	↑	27	ENTER		
03	STOP	41	X_{min}	X_{max}	0
04	x→( )	23			
05	c	16			
06	y→( )	40			
07	d	17			
08	CLEAR	20			
09	2	02			
0a	↑	27	ENTER		
0b	STOP	41	Y_{min}	Y_{max}	0
0c	x→( )	23			
0d	9	11			
10	y→( )	40			
11	b	14			
12	CLEAR	20			
13	3	03			
14	↑	27	ENTER		
15	STOP	41	N	0	0
16	√x	76			
17	int x	64			
18	↑	27			
19	d	17			
1a	↑	27			
1b	c	16			
1c	−	34			
1d	ROLL ↓	31			
20	x⇄y	30			
21	÷	35			
22	ROLL ↓	31			
23	−	34			
24	ACC +	60			
25	b	14			
26	x⇄y	30			
27	x←( )	67			
28	9	11			
29	−	34			
2a	↓	25			
2b	x⇄y	30			
2c	÷	35			
2d	y→( )	40			

Step	Key	Code	Display x	Display y	Display z
30	a	13			
31	ENTER EXP	26			
32	9	11			
33	9	11			
34	x→( )	23			
35	−	34			
36	e	12			
37	CHG SIGN	32			
38	x→( )	23			
39	−	34			
3a	f	15			
3b	RCL	61			
3c	ACC −	63			
3d	+	33			
40	ACC +	60			
41	d	17			
42	IF x<y	52			
43	5	05			
44	d	17			
45	GO TO ( )	44			
46	▲SUB▼	77			
47	+	33			
48	3	03			
49	a	13			
4a	x←( )	67			
4b	−	34			
4c	e	12			
4d	IF x>y	53			
50	y→( )	40			
51	−	34			
52	e	12			
53	x←( )	67			
54	−	34			
55	f	15			
56	IF x<y	52			
57	y→( )	40			
58	−	34			
59	f	15			
5a	GO TO ( )	44			
5b	3	03			
5c	b	14			
5d	y⇄( )	24			

Step	Key	Code	Display x	Display y	Display z
60	9	11			
61	a	13			
62	+	33			
63	b	14			
64	IF x<y	52			
65	7	07			
66	5	05			
67	y→( )	40			
68	9	11			
69	RCL	61			
6a	ACC −	63			
6b	x⇄y	30	DISPLAY		
6c	c	16	Z_{min}	Z_{max}	0
6d	x⇄y	30			
70	−	34			
71	ACC +	60			
72	GO TO ( )	44			
73	3	03			
74	b	14			
75	CLEAR	20			
76	y⇄( )	24			
77	−	34			
78	f	15			
79	x←( )	67			
7a	−	34			
7b	e	12			
7c	END	46			
7d					

Storage

Register	+	−
f	ΔX	Z max
e	X	Z min
d	X max	
c	X min	
b	Y max	
a	ΔY	
9	Y min	
8		
7		
6		
5		
4		
3		
2		
1		
0		

HEWLETT·PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

Step	Key	Code	x	y	z	Step	Key	Code	x	y	z
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

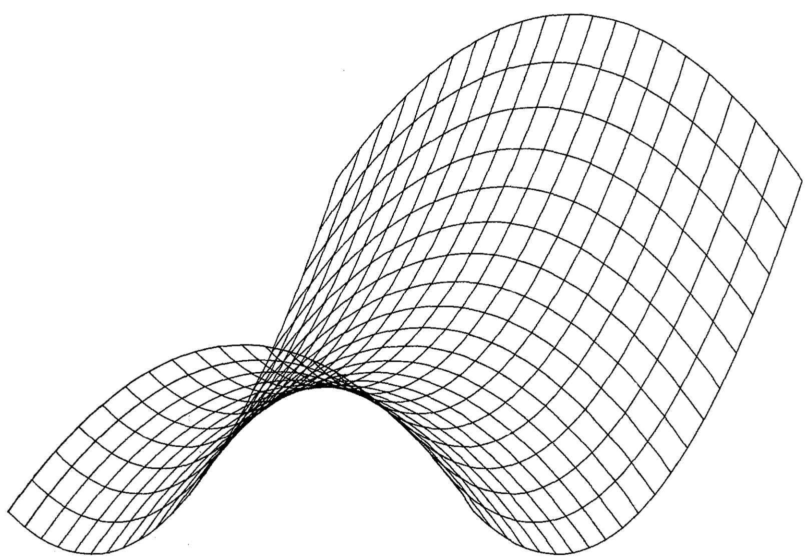
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

PLOT OF Z = Z(X, Y)

This program will plot in three dimensions a wide variety of functions of two independent variables. The user specifies the ranges of the three variables X, Y and Z. If the range of Z is unknown, program 09100-70411, Maximum and Minimum of Z = Z(X, Y) may be used to determine its range for particular ranges of X and Y.

The algorithm incorporated forms an X - Y grid of the independent variables, scales the grid and rotates it in the X - Y plane. It then proceeds to plot the evaluation of the function as incremented lines of elevation at scaled distances above the grid. The function is plotted in a cross hatched pattern, holding one variable as a parameter and incrementing the other variable then reversing these roles to plot in an orthogonal direction.

The angle of rotation and degree of resolution are built into the program and the three necessary scale factors are generated before the plot begins. The instructions for changing any of these to obtain a different perspective of the plot are given at the end of the user instructions.



9100B ONLY
PART NO.
09100-70412

USER INSTRUCTIONS

PRESS: STOP

Using the origin controls, locate the pen in the lower left corner of the paper.

SET: Decimal Wheel at 6 or less

SET: | DEGREES | |

PRESS: END

ENTER PROGRAM: Side A followed by Side B

Has the Max - Min of Z = Z(X, Y) program just been run? YES → go to PRESS: END; NO ↓

PRESS: GO TO (+) (3) (a)

SET: | PROGRAM | |

Enter the program steps which take X from the +e register and Y from the +9 register and generate Z(X, Y). These variables must be left in their respective registers (+e and +9). Leave the programmed value of Z(X, Y) in the Y register and program a RETURN for the final step. The flag is not available.

NOTE: Locations + (3) (a) through + (8) (d) are available. (74 steps)

SET: | |RUN|

→ PRESS: END

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
1 ———— X

ENTER RANGE OF X: X_{max}→Y, X_{min}→X

PRESS: CONTINUE

DISPLAY

0 ———— Z
0 ———— Y
2 ———— X

ENTER RANGE OF Y: Y_{max}→Y, Y_{min}→X

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0 ———— Z
0 ———— Y
3 ———— X

ENTER RANGE OF Z: Z_{max}→Y, Z_{min}→X

PRESS: CONTINUE

The function will now be plotted. It is self-terminating.

NOTE: Execution of this program destroys the −0 and the −1 registers of program steps. To re-run the program, side B must be re-entered at location −(0)(0).

EXAMPLE

Plot the saddle function $Z = y^2 - x^2$ where

$-2 \leq X \leq 2$

$-2 \leq Y \leq 2$

The range of Z is

$-4 \leq Z \leq 4$

Program steps to generate function

(+) 3 a	x←( )	67			
b	9	11			
c	↑	27			
d	×	36			
(+) 4 0	e	12			
1	↑	27			
2	×	36			
3	↓	25			
4	−	34			
5	RETURN	77			

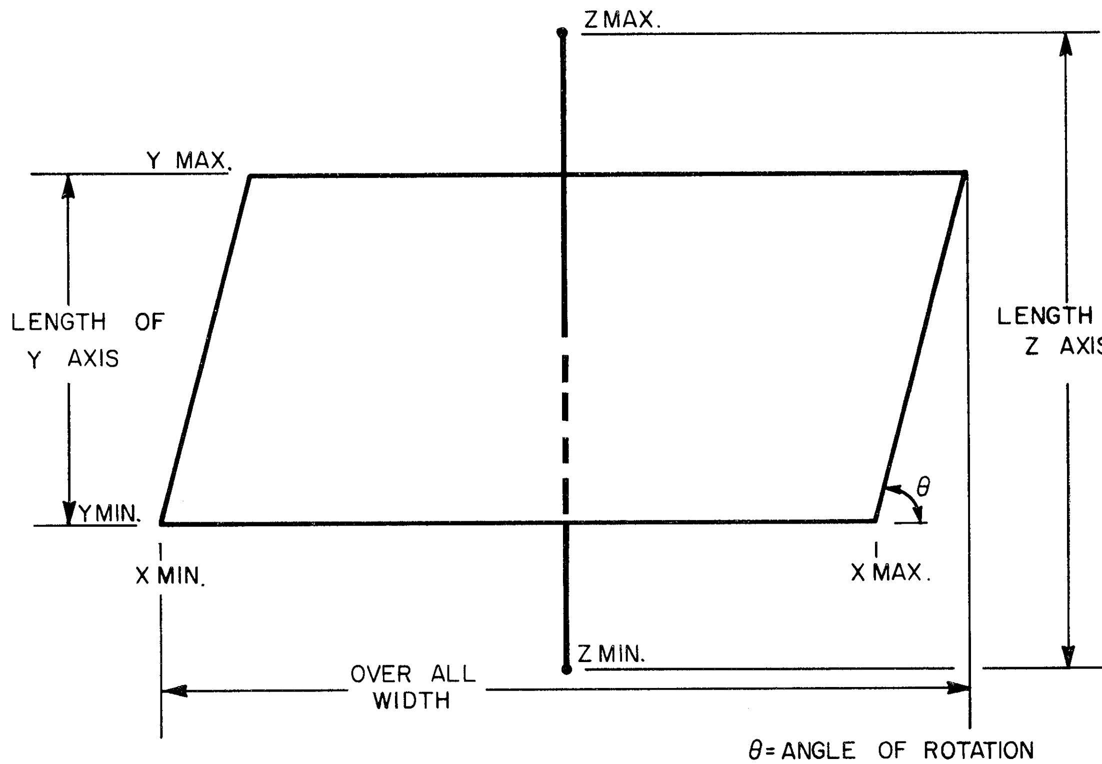


FIGURE 1

The instructions below explain how the user may modify the perspective of a plot. (Refer to Figure 1 for a geometrical representation of the terms.)

1) LENGTH OF Z AXIS: The distance from the minimum value of Z to the maximum value of Z of the plot is specified in plotter variables in locations (–)(2)(3) through (–)(2)(6). The present value is 2500 representing $\frac{2500}{500} = 5$ inches $\left(\frac{2500}{200} = 12.5 \text{ cm}\right)$ on the plot surface.

2) LENGTH OF Y AXIS: The distance from the minimum value of Y to the maximum value of Y of the plot is specified in plotter variables in locations (+)(1)(b) through (+)(2)(0). The present value is 2500 representing $\frac{2500}{500} = 5$ inches $\left(\frac{2500}{200} = 12.5 \text{ cm}\right)$ on the plot surface.

3) OVERALL WIDTH OF PLOT: The overall width of the plot is specified in plotter variables in locations (–)(1)(1) through (–)(1)(4). The present value is 6000 representing $\frac{6000}{500} = 12$ inches $\left(\frac{6000}{200} = 30\text{cm}\right)$ on the plot surface.

4) NUMBER OF GRID LINES: The algorithm plots the function in two directions – across the X axis and across the Y axis. The number of grid lines plotted in each direction is specified by the number in storage (–)(b)(8)and(–)(b)(9). This programmed number is actually one less than the desired number. At present, a 20 is stored giving 21 grid lines in each direction.

5) RESOLUTION: The algorithm plots each grid line in a specified number of increments. This number is stored in (–)(3)(8)and(–)(3)(9). The present programmed increment factor is 40. This should be increased for higher resolution.

6) ANGLE OF ROTATION: The plot must be rotated to gain a useful view of the function. This angle is specified in storage locations (+)(2)(a) through (+)(2)(c). The present angle is 45°.

 PART NO. 09100-70412 9100B ONLY

Step	Key	Code	Display x	Display y	Display z
0 (+) 0	CLEAR	20			
1	1	01		ENTER	
2	STOP	41	X_{min}	X_{max}	0
3	x→( )	23			
4	c	16			
5	y→( )	40			
6	d	17			
7	CLEAR	20			
8	2	02		ENTER	
9	STOP	41	Y_{min}	Y_{max}	0
a	x→( )	23			
b	a	13			
c	x→( )	23			
d	9	11			
1 (+) 0	y→( )	40			
1	b	14			
2	−	34			
3	↑	27			
4	↓	25			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	2	02			
8	a	13			
9	sin x	70			
a	×	36			
b	2	02			
c	5	05			
d	0	00			
2 (+) 0	0	00			
1	x⇄y	30			
2	÷	35			
3	y→( )	40			
4	−	34			
5	e	12			
6	GO TO ( )	44			
7	−	34			
8	0	00			
9	0	00			
a	CONT	47			
b	CONT	47			
c	4	04			
d	5	05			
3 (+) 0	RETURN	77			
1	SET FLAG	54			
2	y⇄( )	24			
3	9	11			
4	y⇄( )	24			
5	e	12			
6	y⇄( )	24			
7	9	11			
8	RETURN	77			

Storage

	+	−
f	ΔX/ΔY	X Grid
e	X/Y	Y Grid
d	X max	
c	X min	
b	Y max	
a	Y min	
9	Xmin/Ymin	
8		
7		
6		
5		
4		
3		
2		
1		Z Grid
0		Z min

Step	Key	Code	Display x	Display y	Display z
0 0	CONT	47			
(−) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	+	33			
4	2	02			
5	a	13			
6	cos x	73			
7	ROLL ↑	22			
8	×	36			
9	d	17			
a	↑	27			
b	c	16			
c	−	34			
d	↓	25			
1 0	+	33			
(−) 1	6	06			
2	0	00			
3	0	00			
4	0	00			
5	x⇄y	30			
6	÷	35			
7	y→( )	40			
8	−	34			
9	f	15			
a	CLEAR	20			
b	3	03		ENTER	
c	STOP	41	Z_{min}	Z_{max}	0
d	x→( )	23			
2 0	−	34			
(−) 1	0	00			
2	−	34			
3	2	02			
4	5	05			
5	0	00			
6	0	00			
7	x⇄y	30			
8	÷	35			
9	y→( )	40			
a	−	34			
b	1	01			
c	d	17			
d	IF FLAG	43			

Step	Key	Code	Display x	Display y	Display z
3 0	SET FLAG	54			
(−) 1	b	14			
2	↑	27			
3	c	16			
4	IF FLAG	43			
5	SET FLAG	54			
6	a	13			
7	−	34			
8	4	04			
9	0	00			
a	÷	35			
b	c	16			
c	IF FLAG	43			
d	SET FLAG	54			
4 0	a	13			
(−) 1	x⇄y	30			
2	−	34			
3	ACC +	60			
4	RCL	61			
5	ACC −	63			
6	+	33			
7	ACC +	60			
8	d	17			
9	IF FLAG	43			
a	SET FLAG	54			
b	b	14			
c	IF x<y	52			
d	a	13			
5 0	4	04			
(−) 1	IF FLAG	43			
2	GO TO ( )	44			
3	▲SUB▼	77			
4	+	33			
5	3	03			
6	1	01			
7	GO TO ( )	44			
8	▲SUB▼	77			
9	+	33			
a	3	03			
b	a	13			
c	x←( )	67			
d	−	34			

Step	Key	Code	Display x	Display y	Display z
6 0	0	00			
(−) 1	−	34			
2	x←( )	67			
3	−	34			
4	1	01			
5	×	36			
6	x←( )	67			
7	9	11			
8	↑	27			
9	a	13			
a	−	34			
b	x←( )	67			
c	−	34			
d	f	15			
7 0	×	36			
(−) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	+	33			
4	2	02			
5	a	13			
6	sin x	70			
7	×	36			
8	ROLL ↓	31			
9	+	33			
a	ROLL ↑	22			
b	÷	35			
c	arc ▾	72			
d	sin x	70			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

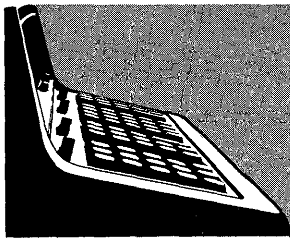
Step	Key	Code	Step	Key	Code
80	cos x	73	b0	SET FLAG	54
81	×	36	b1	d	17
82	y⇄( )	24	b2	↑	27
83	e	12	b3	a	13
84	c	16	b4	IF FLAG	43
85	x⇄y	30	b5	SET FLAG	54
86	−	34	b6	c	16
87	x⇄y	30	b7	−	34
88	y⇄( )	24	b8	2	02
89	e	12	b9	0	00
8a	y⇄( )	24	ba	÷	35
8b	−	34	bb	↓	25
8c	f	15	bc	+	33
8d	x⇄y	30	bd	b	14
90	×	36	c0	IF FLAG	43
91	x⇄y	30	c1	SET FLAG	54
92	y⇄( )	24	c2	d	17
93	−	34	c3	IF $x<y$	52
94	e	12	c4	c	16
95	−	34	c5	d	17
96	↓	25	c6	y→( )	40
97	FMT	42	c7	9	11
98	↓	25	c8	RCL	61
99	IF FLAG	43	c9	ACC −	63
9a	GO TO ( )	44	ca	GO TO ( )	44
9b	▲SUB▼	77	cb	2	02
9c	+	33	cc	c	16
9d	3	03	cd	IF FLAG	43
a0	1	01	d0	d	17
a1	GO TO ( )	44	d1	a	13
a2	4	04	d2	c	16
a3	4	04	d3	x→( )	23
a4	2	02	d4	9	11
a5	ENTER EXP	26	d5	CLEAR	20
a6	3	03	d6	SET FLAG	54
a7	↑	27	d7	GO TO ( )	44
a8	FMT	42	d8	2	02
a9	↑	27	d9	c	16
aa	y⇄( )	24	da	CLEAR	20
ab	9	11	db	FMT	42
ac	b	14	dc	↑	27
ad	IF FLAG	43	dd	END	46

Storage

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B STATISTICS PROGRAM LISTING

70801 - MEAN AND STANDARD DEVIATION
Calculates the mean and standard deviation of n data points.

70802 - STANDARD DEVIATION AND MEAN OF GROUPED DATA
Calculates the mean and standard deviation of data points of certain frequencies.

70803 - LINEAR REGRESSION
Calculates the best fit of a set of data points to the line $y = ax + b$, i.e., the program computes the estimates $\hat{a}$ and $\hat{b}$. It also gives the correlation coefficient r.

70804 - NORMAL PROBABILITY INTEGRAL
Evaluates the integral of the normal density function.

70805 - χ^2 - CHI SQUARE DISTRIBUTION
Calculates the integral of the Chi Square distribution from 0 up to a value of χ^2 for a given number of degrees of freedom.

70806 - χ^2 - CHI SQUARE EVALUATION EXPECTED VALUES EQUAL ($E_i = E$)
Chi square calculation where the expected value of each observation is equal.

70808 - χ^2 - CHI SQUARE EVALUATION EXPECTED VALUES UNEQUAL ($E_i \neq E_j$)
Chi square calculation where the expected values of the observations are not necessarily equal.

70811 - LEAST SQUARES FIT-POWER CURVE
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form: $y = ax^b$

70812 - LEAST SQUARES FIT-EXPONENTIAL
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form: $y = ae^{bx}$

70813 - POISSON DENSITY
Calculates the various summations associated with the Poisson density to give a probability based on an input parameter and summation endpoints.

70816 - RANDOM NUMBER GENERATOR
Random numbers (RN) in the range $0 \leq RN \leq 1$ are calculated; more than 10,000 random numbers may be generated before any previous value is repeated.

70904 - NORMAL (GAUSSIAN) CURVE PLOT
Given mean (M) and variance σ^2, this program generates a normal curve given by

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(X - M)^2}{2\sigma^2}}$$

This program can be used with Program 70905, Histogram Generation.

70907 - ONE WAY ANALYSIS OF VARIANCE
This program separates the total variance in a table of data into that due to chance and that due to differences between the population means underlying each column of sample data.

9100B ONLY

70901 - MULTIPLE LINEAR REGRESSIONS
Given a set of data points (X_i, Y_i, Z_i), this program determines the coefficients of the linear equation

$$Z = a_0 + a_1X + a_2Y$$

70902 - WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA
Calculates the parameters for the Weibull distribution and thus estimates of times to failure percentages may be made.

70903 - NON-LINEAR REGRESSION - LEAST SQUARES PARABOLA
Calculates coefficients fitting data points (x_i, y_i) to an equation of the form:

$$y = a_0 + a_1 x + a_2 x^2$$

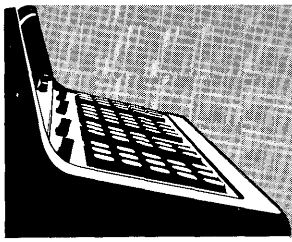
70905 - HISTOGRAM GENERATION WITH PLOT
This program generates and plots a histogram of ten windows given a set of positive numbers. The mean M and variance σ^2 of the data set are computed and stored for use by Program 70904, NORMAL CURVE PLOT.

70906 - HISTOGRAM GENERATION
This program generates a histogram table of ten windows given a data set of positive numbers. In addition it determines the mean M and the variance σ^2 of the data set.

70908 - F DISTRIBUTION
This program evaluates the F distribution density function for given values of F, V_1, and V_2.

70909 - TWO WAY ANALYSIS OF VARIANCE (m X 4)
This program analyses the total statistical variance in a table of data by separating the total variance into two parts, the variance among rows, and the variance between columns. These variances are then compared to the variance due to random influence.

70910 - TWO WAY ANALYSIS OF VARIANCE WITH REPLICATES
This program analyses the total statistical variance of a table of data by separating the total variance into three parts; the variance among rows, the variance between columns, and the variance due to interaction.



PART NO.
09100-70801

MEAN AND STANDARD DEVIATION

This program calculates the mean, $\overline{X}$, and standard deviation, S, of a set of data points by the equations

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

and

$$S = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}}$$

Reference: Introduction to the Theory of Statistics
by Mood and Graybill

McGraw - Hill 1963

USER INSTRUCTIONS	EXAMPLES

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) or END

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
i —— X

(i indicates point to be entered)

ENTER DATA: $X_i \rightarrow X$

PRESS: CONTINUE

Last Entry?

NO

YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S —— Z
$\overline{X}$ —— Y
n —— X

S = Standard deviation

$\overline{X}$ = Mean

n = Number of data points

SAMPLE DATA

5.036	
5.085	
4.991	
4.935	S = .0493
4.999	$\overline{X}$ = 5.0145
5.031	n = 10
5.064	
4.942	
5.051	
5.011	

SAMPLE DATA

4	
5	
6	S = 1.7512
4	$\overline{X}$ = 4.6667
7	n = 6
2	

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	x→( )	23									
	3	d	17									
	4	STOP	41	X_i	0	0	ENTER X_i					
	5	IF FLAG	43	BRANCH AFTER LAST ENTRY								
	6	1	01									
	7	7	07									
	8	↑	27	CALCULATE ΣX_i AND ΣX_i^2								
	9	×	36									
	a	ACC +	60									
	b	d	17	INCREMENT i								
	c	x⇄y	30									
	d	1	01									
1	0	+	33									
	1	y→( )	40									
	2	d	17									
	3	↓	25									
	4	GO TO ( )( )	44									
	5	0	00									
	6	4	04									
	7	d	17	CHANGE n +1 TO n								
	8	↑	27									
	9	1	01									
	a	−	34									
	b	y→( )	40									
	c	d	17									
	d	↑	27									
2	0	RECALL	61	CALCULATE $\overline{X}$								
	1	ROLL ↑	22									
	2	÷	35									
	3	y→( )	40									
	4	f	15									
	5 ▲	×	36	CALCULATE S								
	6	f	15									
	7	×	36									
	8	↓	25									
	9	−	34									
	a	d	17									
	b	↑	27									
	c	1	01									
	d	−	34									

▲ Denotes Revision

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	f	15									
	1	ROLL ↓	31									
	2	÷	35									
	3	d	17									
	4	ROLL ↓	31									
	5	$\sqrt{x}$	76									
	6	$x \rightleftarrows y$	30									
	7	ROLL ↑	22									
	8	END	46	n	$\overline{X}$	S	DISPLAY					
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

MEAN AND STANDARD DEVIATION OF GROUPED DATA

This program calculates the mean, $\overline{X}$, and standard deviation, S, of a set of data points X_1, X_2, · · · , X_K with frequencies f_1, f_2, · · · , f_K respectively. The equations used are

$$\overline{X} = \frac{\sum_{i=1}^{K} f_i X_i}{\sum_{i=1}^{K} f_i}$$

$$S = \sqrt{\frac{\sum_{i=1}^{K} f_i (X_i - \overline{X})^2}{\sum_{i=1}^{K} f_i - 1}}$$

Reference: Introduction to the Theory of Statistics
by Mood and Graybill

McGraw - Hill 1963

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
i	X

(i indicates pair of points to be entered)

ENTER DATA: $f_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

Last Entry ?

NO

YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

S	Z
$\overline{X}$	Y
n	X

S = Standard Deviation

$\overline{X}$ = Mean

n = Number of Data Points

EXAMPLES

SAMPLE DATA

X_i	f_i
1	3
2	3
3	1
4	2
5	1

S = 1.4337

$\overline{X}$ = 2.5

n = 10

SAMPLE DATA

X_i	f_i
41	3
38	5
37	2
39	18
40	22

S = .9010

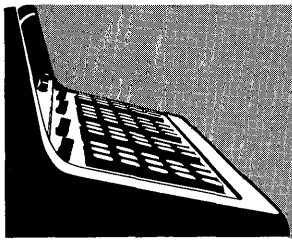
$\overline{X}$ = 39.38

n = 50

 PART NO. 09100-70802

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0	0	CLEAR	20									
	1	$x \to (\ )$	23									
	2	c	16									
	3	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	4	$x \to (\ )$	23									
	5	d	17									
	6	STOP	41	X_i	f_i	0	ENTER X_i AND f_i					
	7	IF FLAG	43	BRANCH AFTER LAST ENTRY								
	8	2	02									
	9	5	05									
	a	ROLL ↑	22	CALCULATE Σf_i								
	b	c	16									
	c	ROLL ↑	22									
	d	+	33									
1	0	$y \to (\ )$	40									
	1	c	16									
	2	ROLL ↑	22	CALCULATE $\Sigma f_i X_i$ AND $\Sigma f_i X_i^2$								
	3	×	36									
	4	$x \rightleftarrows y$	30									
	5	×	36									
	6	ACC +	60									
	7	0	00	CLEAR AND RECALL i								
	8	↑	27									
	9	d	17									
	a	↑	27	INCREMENT i								
	b	1	01									
	c	+	33									
	d	$y \to (\ )$	40									
2	0	d	17									
	1	↓	25									
	2	GO TO ( )( )	44									
	3	0	00									
	4	6	06									
	5	$y \rightleftarrows (\ )$	24	CALCULATE $\overline{X}$								
	6	f	15									
	7	↑	27									
	8	↓	25									
	9	c	16									
	a	÷	35									
	b	ROLL ↓	31									
	c	×	36									
	d	$x \to (\ )$	23									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	f	15									
	1	1	01									
	2	ROLL ↑	22									
	3	−	34									
	4	↓	25									
	5	÷	35									
	6	y⇄( )	24	CALCULATE S								
	7	e	12									
	8	CHG SIGN	32									
	9	÷	35									
	a	e	12									
	b	+	33									
	c	↓	25									
	d	$\sqrt{x}$	76									
4	0	↑	27									
	1	f	15									
	2	↑	27									
	3	c	16									
	4	END	46	n	$\overline{X}$	S	DISPLAY					
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



PART NO.
09100-70803

LINEAR REGRESSION AND CORRELATION COEFFICIENT

This program calculates the equation of the straight line of best fit of a set of data points. The best fit is determined by minimizing the sum of the squares of the deviations of the data points from the line.

The program calculates m and b for the equation

$$Y = mX + b.$$

The program also calculates a correlation coefficient r, an indication of goodness of fit. Note $-1 \leq r \leq 1$ where the sign corresponds to the slope m. If $r = 0$ there is no correlation, and if $r = \pm 1$ there is perfect correlation or a perfect fit.

The defining equations are

$$m = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n} (X_i - \overline{X})^2}$$

$$b = \overline{Y} - m\overline{X}$$

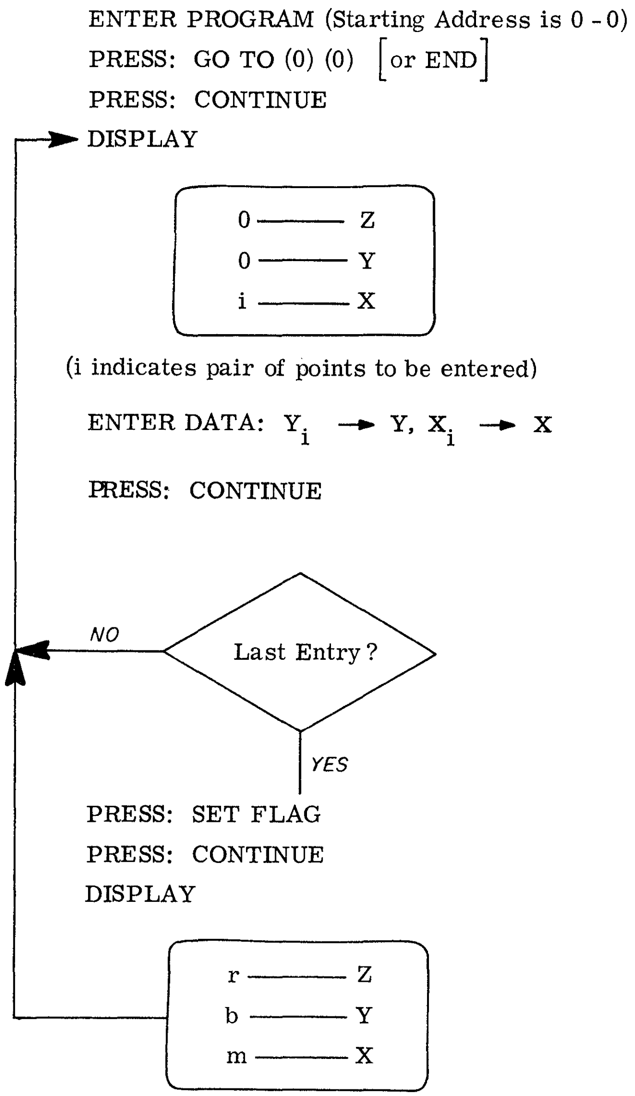
where $\overline{Y} = \frac{\sum_{i=1}^{n} Y_i}{n}$ and $\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$

$$r = \frac{\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \overline{X})^2 \sum (Y_i - \overline{Y})^2}}$$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS



EXAMPLES

X	Y
26	92
30	85
44	78
50	81
62	54
68	51
74	40

r = -.96
b = 121.04
m= -1.03

Y = -1.03X + 121.04

X	Y
0	1
1	3
2	2
3	4
4	5

r = .9
b = 1.2
m= .9

Y = .9X + 1.2

Step		Key	Code	Display x	y	z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	x→( )	23	CLEAR STORAGE REGISTERS								
	2	d	17									
	3	x→( )	23									
	4	c	16									
	5	x→( )	23									
	6	b	14									
	7	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	8	x→( )	23									
	9	a	13									
	a	STOP	41	X_i	Y_i	0	ENTER X_i AND Y_i					
	b	IF FLAG	43	BRANCH AFTER LAST ENTRY								
	c	3	03									
	d	7	07									
1	0	ACC +	60	CALCULATE ΣX_i AND ΣY_i								
	1	↑	27	CALCULATE ΣX_i^2								
	2	×	36									
	3	x⇄y	30									
	4	y⇄( )	24									
	5	d	17									
	6	+	33									
	7	y⇄( )	24	CALCULATE $\Sigma X_i Y_i$								
	8	d	17									
	9	↓	25									
	a	×	36									
	b	b	14									
	c	+	33									
	d	y→( )	40									
2	0	b	14									
	1	ROLL ↑	22	CALCULATE ΣY_i^2								
	2	↑	27									
	3	×	36									
	4	c	16									
	5	+	33									
	6	y→( )	40									
	7	c	16									
	8	a	13	INCREMENT COUNTER								
	9	↑	27									
	a	1	01									
	b	+	33									
	c	y→( )	40									
	d	a	13									

FROM 3-6

Step	Key	Code	Comment
3 0	0	00	CLEAR DISPLAY REGISTERS AND RECALL COUNTER
1	ROLL ↓	31	
2	x⇄y	30	
3	↓	25	
4	GO TO ( )( )	44	
5	0	00	
6	a	13	
7	a	13	DECREMENT COUNT (FROM 0-d)
8	↑	27	
9	1	01	
a	−	34	
b	↓	25	
c	x→( )	23	
d	a	13	
4 0	e	12	CALCULATE $\overline{Y}$
1	↑	27	
2	a	13	
3	÷	35	
4	y⇄( )	24	CALCULATE $\overline{X}$
5	f	15	
6	÷	35	
7	y→( )	40	CALCULATE $\Sigma(X_i - \overline{X})^2$
8	e	12	
9	×	36	
a	e	12	
b	×	36	
c	d	17	
d	x⇄y	30	
5 0	−	34	
1	y→( )	40	CALCULATE $\Sigma(Y_i - \overline{Y})^2$
2	d	17	
3	c	16	
4	↑	27	
5	f	15	
6	↑	27	
7	×	36	
8	a	13	
9	×	36	
a	↓	25	
b	−	34	
c	y→( )	40	
d	c	16	

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	b	14									
	1	↑	27									
	2	f	15									
	3	↑	27	CALCULATE $\Sigma(X_i - \overline{X})(Y_i - \overline{Y})$								
	4	e	12									
	5	×	36									
	6	a	13									
	7	×	36									
	8	↓	25									
	9	−	34									
	a	d	17									
	b	↑	27									
	c	↓	25									
	d	√x	76	CALCULATE r								
7	0	÷	35									
	1	c	16									
	2	√x	76									
	3	÷	35									
	4	d	17									
	5	ROLL ↑	22	CALCULATE m								
	6	x⇄y	30									
	7	÷	35									
	8	y⇄( )	24									
	9	e	12									
	a	e	12									
	b	×	36	CALCULATE b								
	c	f	15									
	d	x⇄y	30									
8	0	−	34									
	1	e	12									
	2	END	46	m	b	r	DISPLAY					
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

PART NO.
09100-70804

NORMAL PROBABILITY INTEGRAL

This program computes the integral of the standardized normal distribution

$$P(X) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-\frac{1}{2}z^2} dz$$

The following equations are used

$$P(X) = \frac{\operatorname{erf}\left(\frac{X}{\sqrt{2}}\right)}{2} + \frac{1}{2}$$

where

$$\operatorname{erf}(X) = \frac{2}{\sqrt{\pi}} e^{-X^2} \sum_{n=0}^{\infty} \frac{2^n}{1 \cdot 3 \cdots (2n+1)} X^{2n+1}$$

Reference: Handbook of Mathematical Functions
by Abramowitz and Stegan

National Bureau of Standards 1964

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) or END

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
1 ——— X

ENTER DATA: X ——→ X

PRESS: CONTINUE

DISPLAY

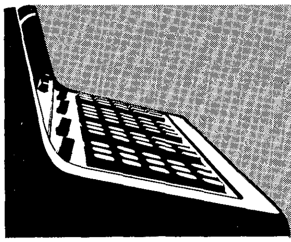
0 ——— Z
P(X) ——— Y
X ——— X

EXAMPLES

X = .3	P(X) = .618
X = -.3	P(X) = .382
X = 0	P(X) = .5
X = 3	P(X) = .999

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST DATA ENTRY								
	2	STOP	41	X	0	0	ENTER X					
	3	IF $x=y$	50	TEST IF X=0								
	4	0	00									
	5	9	11									
	6	GO TO ( )( )	44									
	7	1	01									
	8	1	01									
	9	.	21	ENTER .5 AND BRANCH TO DISPLAY								
	a	5	05									
	b	$x \rightleftarrows y$	30									
	c	GO TO ( )( )	44									
	d	5	05									
1	0	d	17									
	1	$x\to$( )	23	STORE X								
	2	c	16									
	3	IF $x<y$	52	IF X IS NEGATIVE, SET FLAG AND CHANGE SIGN								
	4	SET FLAG	54									
	5	CHG SIGN	32									
	6	↑	27	CALCULATE $\frac{X}{\sqrt{2}} = X'$								
	7	2	02									
	8	$\sqrt{x}$	76									
	9	÷	35									
	a	↓	25	CALCULATE $\frac{2X' e^{-X'^2}}{\sqrt{\pi}}$								
	b	↑	27									
	c	×	36									
	d	↑	27									
2	0	↓	25									
	1	ROLL ↑	22									
	2	e^x	74									
	3	÷	35									
	4	2	02									
	5	×	36									
	6	π	56									
	7	$\sqrt{x}$	76									
	8	÷	35									
	9	1	01	CALCULATE $\frac{1}{X'^2}$								
	a	$y\to$( )	40									
	b	a	13									
	c	ROLL ↑	22									
	d	÷	35									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	y→( )	40									
	1	b	14									
	2	2	02	CALCULATE $\frac{2}{X'^2}$								
	3	÷	35									
	4	ENTER EXP	26									
	5	1	01	ENTER TOLERANCE 10^{-10}								
	6	0	00									
	7	CHG SIGN	32									
	8	ROLL ↑	22									
	9	y⇄( )	24	RECALL $\frac{1}{X'^2}$								
	a	b	14									
	b	ROLL ↓	31									
	c	+	33									
	d	ROLL ↓	31	CALCULATE NEXT TERM OF SERIES			erf (X′)					
4	0	÷	35									
	1	ROLL ↓	31									
	2	y⇄( )	24									
	3	a	13									
	4	+	33	ADD CALCULATED TERM TO SERIES								
	5	y⇄( )	24									
	6	a	13									
	7	y⇄( )	24	RECALL TOLERANCE								
	8	b	14									
	9	IF x > y	53	BRANCH WHEN TERM IS								
	a	3	03	LESS THAN TOLERANCE 10^{-10}								
	b	9	11									
	c	a	13	RECALL erf (X′)								
	d	↑	27									
5	0	2	02									
	1	÷	35									
	2	·	21	CALCULATE P(X) USING erf (X′)								
	3	5	05									
	4	+	33									
	5	1	01									
	6	IF FLAG	43	MODIFY P(X) WHEN X IS NEGATIVE								
	7	x⇄y	30									
	8	−	34									
	9	CLEAR x	37									
	a	ROLL ↓	31									
	b	x⇄y	30									
	c	c	16									
FROM 1-0	d	END	46	X	P(X)	0	DISPLAY					



PART NO.
09100-70805

χ^2– CHI SQUARE DISTRIBUTION

This program evaluates the Chi Square Distribution Integral for a given value of χ^2 and ν degrees of freedom; i.e., the program evaluates

$$P(\chi^2,\nu)= \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})} \int_0^{\chi^2} T^{\nu/2-1} e^{-T/2}\, dT \qquad 0 \leq \chi^2$$

The series approximation used to evaluate the integral is

$$P(\chi^2,\nu)=(\tfrac{1}{2}\chi^2)^{\nu/2} \frac{e^{-\chi^2/2}}{\Gamma(\frac{\nu+2}{2})} \left\{1+\sum_{r=1}^{\infty} \frac{\chi^{2r}}{(\nu+2)(\nu+4)\cdots(\nu+2r)}\right\}$$

Reference: Handbook of Mathematical Functions
by Abramowitz and Stegan

National Bureau of Standards 1964

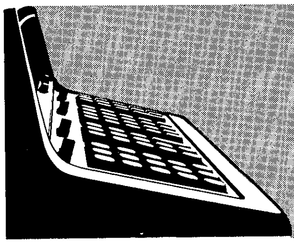
USER INSTRUCTIONS	EXAMPLES
ENTER PROGRAM (Starting Address is 0 - 0)	P(7.88, 1) = .995
PRESS: GO TO (0) (0) [or END]	P(10.6, 2) = .995
→ PRESS: CONTINUE	P(12.8, 17) = .251
ENTER DATA: χ^2 ——→ Y, ν ——→ X	
PRESS: CONTINUE	GENERAL FORM
DISPLAY	$P(\chi^2, \nu)$
$P(\chi^2, \nu)$ ——— Z	
χ^2 ——— Y	
ν ——— X	

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0 0	CLEAR	20									
1	STOP	41	ν	χ^2	0	ENTER ν AND χ^2					
2	$x\to()$	23									
3	d	17									
4	$y\to()$	40	STORE DATA								
5	c	16									
6	2	02									
7	÷	35									
8	$y\to()$	40									
9	f	15									
a	↓	25									
b	ln x	65									
c	↑	27									
d	d	17									
1 0	↑	27									
1	1	01	CALCULATE $\left(\frac{\chi^2}{2}\right)^{\frac{\nu+1}{2}}$								
2	+	33									
3	2	02									
4	÷	35									
5	$y\to()$	40									
6	e	12									
7	↓	25									
8	×	36									
9	↓	25									
a	e^x	74									
b	↑	27									
c	f	15									
d	CHG SIGN	32	CALCULATE $e^{\frac{-\chi^2}{2}}\left(\frac{\chi^2}{2}\right)^{\frac{\nu+1}{2}}$								
2 0	e^x	74									
1	×	36									
2	e	12									
3	↑	27									
4	int x	64									
5	IF $x=y$	50	TRUE IF ν IS ODD								
6	4	04									
7	3	03									
8	2	02	BRANCHES HERE IF ν IS EVEN								
9	ln x	65									
a	×	36									
b	↓	25									
c	e^x	74									
d	×	36									

Step		Key	Code	Display / Comments
3	0	c	16	
	1	$\sqrt{x}$	76	
	2	÷	35	CALCULATE COEFFICIENT $\frac{(1/2\chi^2)^{\frac{\nu}{2}} e^{-\frac{\chi^2}{2}}}{\Gamma(\frac{\nu+2}{2})}$
	3	d	17	
	4	÷	35	
	5	↑	27	DECREMENT ν_i BY 2
	6	2	02	
	7	−	34	
	8	0	00	
	9	IF x=y	50	TEST IF ν_i IS DECREMENTED TO ZERO
	a	6	06	
	b	9	11	
	c	↓	25	
	d	÷	35	
4	0	GO TO ( )( )	44	
	1	3	03	CONTINUE DECREMENTING ν_i AND DIVIDING
	2	5	05	
	3	4	04	BRANCHES HERE IF ν IS ODD (FROM 2-7)
	4	ln x	65	
	5	×	36	
	6	↓	25	
	7	e^x	74	
	8	×	36	
	9	d	17	
	a	↑	27	
	b	2	02	
	c	×	36	
	d	↓	25	
5	0	÷	35	
	1	↑	27	
	2	4	04	DIVIDE BY $(2\nu)\cdots 10\cdot 6\cdot 2\cdot 1$
	3	−	34	
	4	IF x<y	52	
	5	4	04	
	6	d	17	
	7	2	02	
	8	IF x>y	53	CALCULATE COEFFICIENT $\frac{(1/2\chi^2)^{\frac{\nu}{2}} e^{-\frac{\chi^2}{2}}}{\Gamma(\frac{\nu+2}{2})}$
	9	6	06	
	a	0	00	
	b	↓	25	
	c	÷	35	
	d	↑	27	

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	Remarks
6 0	c	16										
1	|y|	55										
2	÷	35										
3	π	56										
4	÷	35										
5	↓	25										
6	$\sqrt{x}$	76										
7	×	36										
8	↑	27										
9	d	17										FROM 3-b
a	x→( )	23										
b	f	15										
c	0	00										
d	x→( )	23										
7 0	e	12										
1	1	01										
2	x⇄y	30										
3	2	02										(from 8-0)
4	ACC +	60										CALCULATE $\left\{1+\sum \frac{\chi^{2r}}{(\nu+2)(\nu+4)\cdots(\nu+2r)}\right\}$ AND $\nu+2r$
5	c	16										
6	×	36										
7	f	15										CALCULATE $\frac{\chi^2}{\nu_i+2}$ (steps 7-7 to 7-8)
8	÷	35										
9	ENTER EXP	26										COMPARE LAST TERM IN SERIES TO 10^{-9} AND STOP SUMMATION IF SMALLER (steps 7-9 to 7-d)
a	9	11										
b	CHG SIGN	32										
c	IF x<y	52										
d	7	07										
8 0	3	03										(to 7-3)
1	e	12										CALCULATE $P(\chi^2,\nu)$ (steps 8-1 to 8-4)
2	+	33										
3	↓	25										
4	×	36										
5	c	16										RECALL χ^2 AND ν (steps 8-5 to 8-7)
6	↑	27										
7	d	17										
8	END	46	ν	χ^2	$P(\chi^2,\nu)$							DISPLAY
9												
a												
b												
c												
d												

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70806

χ^2 CHI SQUARE EVALUATION
EXPECTED VALUES EQUAL ($E_i = E$)

This program calculates the value of χ^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E)^2}{E}$$

where

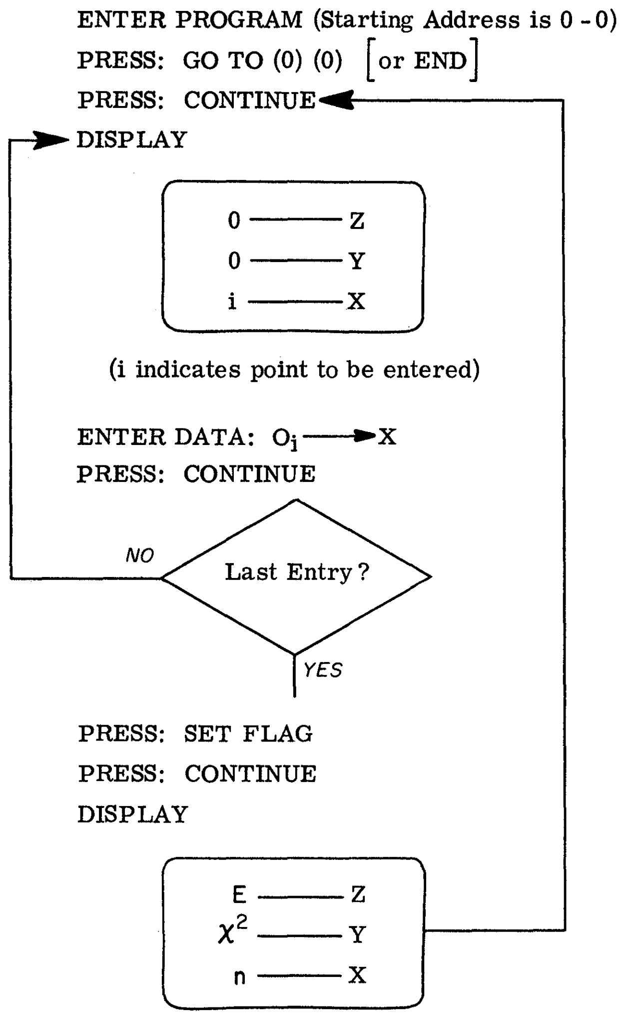
O_i — observed frequency

E — expected frequency of O_i is $E = \frac{\sum_{i=1}^{n} O_i}{n}$

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

USER INSTRUCTIONS



EXAMPLES

The table shows the observed and expected frequencies in tossing a die 120 times. Calculate χ^2 for testing if the die is fair.

FACE	i	1	2	3	4	5	6
O_i Observed Frequency		25	17	15	23	24	16
E_i Expected Frequency		20	20	20	20	20	20

$E = 20$

$\chi^2 = 5.0$

$n = 6$

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	$x \rightarrow ( )$	23									
	3	d	17									
	4	STOP	41	O_i	0	0						
	5	IF FLAG	43									
	6	1	01	BRANCH AFTER LAST ENTRY								
	7	9	11									
	8	↑	27									
	9	×	36	CALCULATE $\sum O_i$ AND $\sum O_i^2$								
	a	ACC +	60									
	b	0	00									
	c	↑	27									
	d	d	17									
1	0	↑	27									
	1	1	01	CLEAR AND INCREMENT n_i								
	2	+	33									
	3	$y \rightarrow ( )$	40									
	4	d	17									
	5	↓	25									
	6	GO TO ( )( )	44									
	7	0	00									
	8	4	04									
	9	$y \rightleftarrows ( )$	24									
	a	d	17									
	b	1	01	DECREMENT n								
	c	−	34									
	d	$y \rightarrow ( )$	40									
2	0	d	17									
	1	RCL	61									
	2	↑	27									
	3	d	17	CALCULATE E								
	4	÷	35									
	5	$y \rightarrow ( )$	40									
	6	f	15									
	7	×	36									
	8	f	15									
	9	×	36									
	a	ROLL ↓	31	CALCULATE χ^2								
	b	−	34									
	c	f	15									
	d	÷	35									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	d	17									
1	END	46	n	χ^2	E	DISPLAY					
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

June 1969
PART NO.
09100-70808

χ^2 CHI SQUARE EVALUATION
▲ EXPECTED VALUES UNEQUAL ($E_i \neq E_j$)

This program calculates the value of χ^2 by the equation

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

where O_i — observed frequency

E_i — expected frequency of O_i

Reference: Mathematical Statistics
by John E. Freund

Prentice - Hall 1962

▲ Denotes Revision

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
i —— X

(i indicates points to be entered)

ENTER DATA: $O_i \rightarrow Y$, $E_i \rightarrow X$

PRESS: CONTINUE

Last Entry? NO → DISPLAY; YES ↓

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0 —— Z
χ^2 —— Y
n —— X

EXAMPLES

The table shows the observed and expected frequencies of some numbers. Calculate χ^2.

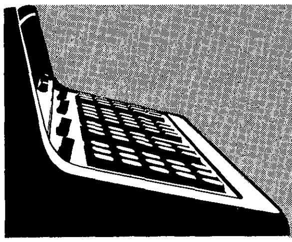
i	1	2	3	4	5	6
O_i - Observed Frequency	8	50	47	56	5	14
E_i - Expected Frequency	9.6	46.75	51.85	54.4	8.25	9.15

χ^2 = 4.844

n = 6

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
2	ACC +	60									
3	f	15	RECALL i TO INDICATE NEXT ENTRY								
4	STOP	41	Ei	Oi	0	ENTER EXPECTED VALUE AND OBSERVED VALUE					
5	IF FLAG	43									
6	1	01	BRANCH AFTER LAST ENTRY								
7	9	11									
8	−	34									
9	$x \rightleftarrows y$	30									
a	↑	27									
b	×	36									
c	↓	25	CALCULATE χ^2 AND INCREMENT i								
d	$x \rightleftarrows y$	30									
1 0	÷	35									
1	1	01									
2	ACC +	60									
3	0	00									
4	↑	27									
5	↑	27	CLEAR AND BRANCH FOR NEXT ENTRY								
6	GO TO ( )( )	44									
7	0	00									
8	3	03									
9	RECALL	61									
a	↑	27									
b	1	01	RECALL χ^2, CLEAR AND CALCULATE n								
c	−	34									
d	0	00									
2 0	ROLL ↓	31									
1	END	46	n	χ^2	0	DISPLAY					
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

Step	Key	Code	Display			Storage					
			x	y	z	F	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70811

LEAST SQUARES FIT - POWER CURVE

This program computes the least squares fit and correlation coefficient of N pairs of data points for a power curve of the form:

$$Y = aX^{b}$$

The equation is linearized into $\ln Y = b \ln X + \ln a$

where

$$b = \frac{N\sum(\ln X \ln Y) - \sum \ln X \sum \ln Y}{N\sum(\ln X)^2 - (\sum \ln X)^2}$$

and

$$r = \frac{N\sum \ln X \ln Y - (\sum \ln X)(\sum \ln Y)}{\sqrt{\left[N\sum(\ln X)^2 - (\sum \ln X)^2\right]\left[N\sum(\ln Y)^2 - (\sum \ln Y)^2\right]}}$$

$$\ln a = \frac{\sum \ln Y}{N} - \frac{\sum \ln X}{N}\, b$$

Note: $X_i > 0$ and $Y_i > 0$, $i = 1, \cdots, N$

Reference: Statistical Theory and Methodology in Science and Engineering by K.A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 —— Z
0 —— Y
i —— X

(i indicates pair of points to be entered)

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

Last Entry? — NO: return to DISPLAY; YES: continue

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r —— Z
b —— Y
a —— X

EXAMPLES

$Y = aX^b$

X	Y
1.0001	25.58
3.16	14.55
10	9.26
31.6	5.63
100	3.48
316	2.12
1000	1.7

r = -0.9964
b = -0.4022
a = 23.5871

$Y = 23.5871X^{-.4022}$

X	Y
1	3
2	4.2574
3	5.2248
4	6.0417
5	6.7624

r = 1.000
b = .505
a = 3.000

$Y = 3X^{.505}$

Step		Key	Code	Display x	Display y	Display z	Storage / Remarks
0	0	CLEAR	20	CLEAR STORAGE AND DISPLAY 1 TO INDICATE FIRST ENTRY (steps 0-0 to 0-9)			
	1	x→( )	23				
	2	d	17				
	3	x→( )	23				
	4	c	16				
	5	x→( )	23				
	6	b	14				
	7	1	01				
	8	x→( )	23				
	9	a	13				
	a	STOP	41	X_i	Y_i	0	ENTER X_i AND Y_i
	b	IF FLAG	43	BRANCH AFTER ALL DATA HAS BEEN ENTERED (steps 0-b to 0-d)			
	c	3	03				
	d	b	14				
1	0	ln x	65				
	1	x⇄y	30				
	2	ln x	65				
	3	x⇄y	30				
	4	ACC +	60	CALCULATE $\sum \ln X_i$, $\sum \ln Y_i$			
	5	↑	27	CALCULATE $\sum (\ln X_i)^2$ (steps 1-5 to 1-a)			
	6	×	36				
	7	x⇄y	30				
	8	y⇄( )	24				
	9	d	17				
	a	+	33				
	b	y⇄( )	24	CALCULATE $\sum (\ln X_i)(\ln Y_i)$ (steps 1-b to 2-4)			
	c	d	17				
	d	↓	25				
2	0	×	36				
	1	b	14				
	2	+	33				
	3	y→( )	40				
	4	b	14				
	5	ROLL ↑	22	CALCULATE $\sum (\ln Y_i)^2$ (steps 2-5 to 2-b)			
	6	↑	27				
	7	×	36				
	8	c	16				
	9	+	33				
	a	y→( )	40				
	b	c	16				
	c	a	13				
	d	↑	27				

FROM 3-a (→ 0-a)

JUNE 1969
PART NO. 09100-70811

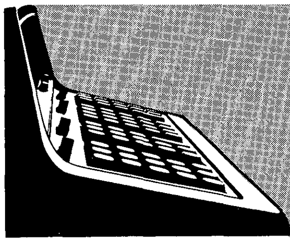
Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	1	01									
	1	+	33	INCREMENT COUNTER								
	2	y→( )	40									
	3	a	13									
	4	CLEAR x	37									
	5	ROLL ↓	31									
	6	x⇄y	30									
	7	↓	25	CLEAR DISPLAY AND RECALL COUNTER								
	8	GO TO ( )( )	44									
	9	0	00									
	a	a	13									
	b (FROM 0-d)	a	13									
	c	↑	27									
	d	1	01	DECREMENT COUNTER								
4	0	−	34									
	1	y→( )	40									
	2	a	13									
	3	e	12									
	4	↑	27	CALCULATE $\Sigma \frac{\ln Y_i}{N}$								
	5	a	13									
	6	÷	35									
	7	y⇄( )	24									
	8	f	15	CALCULATE $\Sigma \frac{\ln X_i}{N}$								
	9	÷	35									
	a	y→( )	40									
	b	e	12									
	c	×	36									
▲	d	e	12	CALCULATE $\Sigma(\ln x)^2 - \frac{(\Sigma \ln x)^2}{N}$								
5	0	×	36									
	1	d	17									
	2	x⇄y	30									
	3	−	34									
	4	y→( )	40									
	5	d	17									
	6	c	16									
	7	↑	27									
	8	f	15									
▲	9	↑	27	CALCULATE $\Sigma(\ln y)^2 - \frac{(\Sigma \ln y)^2}{N}$								
	a	×	36									
	b	a	13									
	c	×	36									
	d	↓	25									

▲ Denotes Revision

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	−	34									
	1	y→( )	40									
	2	c	16									
	3	b	14									
	4	↑	27									
	5	f	15									
	6	↑	27									
	7	e	12	CALCULATE $\Sigma(\ln x \ln y) - \frac{(\Sigma \ln x)(\Sigma \ln y)}{N}$								
	8	×	36									
	9	a	13									
	a	×	36									
	b	↓	25									
	c	−	34									
	d	d	17									
7	0	↑	27									
	1	↓	25									
	2	√x	76	CALCULATE r								
	3	÷	35									
	4	c	16									
	5	√x	76									
	6	÷	35									
	7	d	17									
	8	ROLL ↑	22	CALCULATE b								
	9	x⇄y	30									
	a	÷	35									
	b	y⇄( )	24									
	c	e	12									
	d	e	12									
8	0	×	36	CALCULATE ln a								
	1	f	15									
	2	x⇄y	30									
	3	−	34									
	4	e	12									
	5	x⇄y	30									
	6	e^x	74									
	7	END	46	a	b	r	DISPLAY					
	8											
	9											
	a											
	b											
	c											
	d											

▲ Denotes Revision

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70812

LEAST SQUARES FIT - EXPONENTIAL

This program computes the least squares fit and a correlation coefficient of n pairs of data points for an exponential function of the form:

$$y = ae^{bx}$$

The equation is linearized into

$$\ln y = \ln a + bx$$

or

$$Y = A + bx$$

Using a linear regression method,

$$b = \frac{n\sum xY - \sum x \sum Y}{n\sum x^2 - (\sum x)^2}$$

$$A = \frac{\sum Y - b\sum x}{n}$$

$$a = e^A$$

the correlation coefficient is given by

$$r = \frac{n\sum xY - \sum x \sum Y}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum Y^2 - (\sum Y)^2\right]}}$$

Note: $Y_i > 0 \quad i = 1, \cdots, n$

Reference: Statistical Theory and Methodology in Science and Engineering
by K. A. Brownlee

John Wiley and Sons 1965

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
i	X

(i indicates pair of points to be entered)

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

Last Entry? — NO: return to DISPLAY; YES: continue

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

r	Z
b	Y
a	X

EXAMPLES

GENERAL FORM: $Y = ae^{bX}$

X	Y
.5	7.12
1.2	11.67
3.1	44.75
7.4	935.64

r = 1.000
b = .707
a = 4.998
$Y = 4.998e^{.707X}$

X	Y
.72	2.16
1.31	1.61
1.95	1.16
2.58	.85

r = - 1.000
b = -.503
a = 3.103
$Y = 3.103e^{-.503X}$

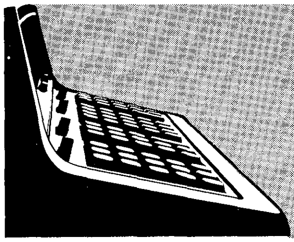
Step	Key	Code	Display x	Display y	Display z	Notes
0 0	CLEAR	20				CLEAR STORAGE AND DISPLAY 1 TO INDICATE FIRST ENTRY
1	x→( )	23				
2	a	13				
3	x→( )	23				
4	b	14				
5	x→( )	23				
6	c	16				
7	1	01				
8	x→( )	23				(FROM 3-5)
9	d	17				
a	STOP	41	x_i	y_i	0	ENTER x_i AND y_i
b	IF FLAG	43				TEST FOR LAST ENTRY
c	3	03				
d	6	06				
1 0	x⇄y	30				
1	ln x	65				CALCULATE $\ln y_i$
2	ACC +	60				CALCULATE, ACCUMULATE AND STORE $\sum x, \sum \ln y, \sum \ln^2 y, \sum x \ln y$
3	↑	27				
4	×	36				
5	x⇄y	30				
6	y⇄( )	24				
7	b	14				
8	+	33				
9	y⇄( )	24				
a	b	14				
b	↓	25				
c	×	36				
d	c	16				
2 0	+	33				
1	y→( )	40				
2	c	16				
3	ROLL ↑	22				CALCULATE, ACCUMULATE AND STORE $\sum x^2$
4	↑	27				
5	×	36				
6	a	13				
7	+	33				
8	y→( )	40				
9	a	13				
a	CLEAR x	37				INCREMENT COUNTER
b	↑	27				
c	d	17				
d	↑	27				

Step		Key	Code	Comment
3	0	1	01	
	1	+	33	
	2	↓	25	
	3	GO TO ( ) ( )	44	
	4	0	00	
	5	8	10	FROM 0-d
	6	d	17	DECREMENT COUNTER
	7	↑	27	
	8	1	01	
	9	−	34	
	a	y→( )	40	
	b	d	17	
	c	c	16	CALCULATE $n\sum x \ln y$
	d	×	36	
4	0	↑	27	
	1	RCL	61	CALCULATE $\sum x \sum \ln y$
	2	×	36	
	3	↓	25	
	4	−	34	CALCULATE AND STORE $n\sum x \ln y - \sum x \sum \ln y$
	5	y→( )	40	
	6	c	16	
	7	d	17	CALCULATE AND STORE $n\sum x^2 - (\sum x)^2$
	8	↑	27	
	9	a	13	
	a	×	36	
	b	e	12	
	c	↑	27	
	d	×	36	
5	0	↓	25	
	1	−	34	
	2	y→( )	40	
	3	a	13	
	4	d	17	CALCULATE AND STORE $n\sum \ln y - (\sum \ln y)^2$
	5	↑	27	
	6	b	14	
	7	×	36	
	8	f	15	
	9	↑	27	
	a	×	36	
	b	↓	25	
	c	−	34	
	d	a	13	

Display: x, y, z — Storage: f, e, d, c, b, a

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	×	36	CALCULATE $\sqrt{\left[n\sum x^2-(\sum x)^2\right]\left[n\sum \ln^2 y-(\sum \ln y)^2\right]}$								
	1	↓	25									
	2	$\sqrt{x}$	76									
	3	↑	27									
	4	c	16	CALCULATE AND STORE r								
	5	x⇄y	30									
	6	÷	35									
	7	y→( )	40									
	8	b	14									
	9	c	16	CALCULATE b								
	a	↑	27									
	b	a	13									
	c	÷	35									
	d	e	12	CALCULATE A								
7	0	↑	27									
	1	↓	25									
	2	×	36									
	3	f	15									
	4	x⇄y	30									
	5	−	34									
	6	d	17									
	7	÷	35									
	8	b	14	RECALL, POSITION, AND CALCULATE a								
	9	ROLL ↓	31									
	a	e^x	74									
	b	END	46	a	b	r	DISPLAY					
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70813

POISSON DENSITY

The Poisson density function is defined by $f(n; \lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$ where λ may be estimated by

$$\hat{\lambda} = \text{the expected value of } X_i \ (n = 0, 1, 2, \dots)$$

$$\sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^{-\lambda} e^{\lambda} = 1$$

The Poisson density function is a descrete density which is used to evaluate such things as component failure probabilities. It is also used to approximate the Binomial Distribution when the number of events (N) is large and the probability that an event will happen (p) is small i.e., a general rule of thumb is for $p \leq .1$ and $\hat{\lambda} = Np \leq 5$.

Reference: Mathematical Statistics
by John E. Freund

1962 Prentice-Hall

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

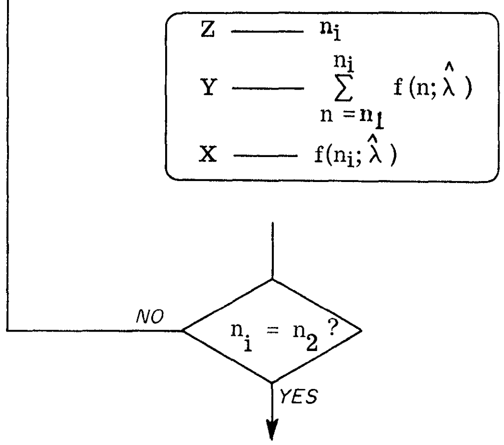
PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

→ ENTER DATA: $n_2 \rightarrow Z$, $n_1 \rightarrow Y$, $\hat{\lambda} \rightarrow X$

→ PRESS: CONTINUE

DISPLAY



PRESS: CONTINUE

EXAMPLES

A Poisson distribution is given by

$$f(n;\ .72) = \frac{(.72)^n e^{-.72}}{n!}$$

Find: a) $f(0;\ .72)$

$\hat{\lambda} = .72$, $n_1 = 0$, $n_2 = 0$

$f(0;\ .72) = .48675226$

b) $f(3;\ .72)$

$\hat{\lambda} = .72$, $n_1 = 3$, $n_2 = 3$

$f(3;\ .72) = .03027988$

If 3% of the electric bulbs manufactured by a company are defective find the probability that in a sample of 100 bulbs

a) Between 1 and 3 bulbs will be defective.

Expected no. of bulbs that are defective $= \hat{\lambda} = (.03)(100) = 3$

$$\sum_{n=1}^{3} f(n;3) = .59744482$$

where $\hat{\lambda} = 3$, $n_1 = 1$, $n_2 = 3$

b) Less than or equal to 2 bulbs are defective

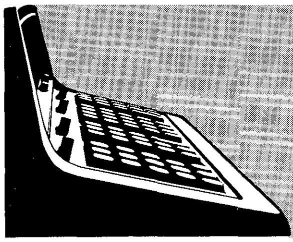
$$\sum_{n=0}^{2} f(n;3) = .42319008$$

where $\hat{\lambda} = 3$, $n_1 = 0$, $n_2 = 2$

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
0 1	STOP	41	$\hat{\lambda}$	n_1	n_2	ENTER DATA; $n_2, n_1, \hat{\lambda}$					
0 2	$x \to (\ )$	23									
0 3	c	16									
0 4	$y \to (\ )$	40									
0 5	b	14	STORE DATA (steps 0 2 – 0 8)								
0 6	ROLL ↑	22									
0 7	$x \to (\ )$	23									
0 8	d	17									
0 9	$x \rightleftarrows y$	30									
0 a	CHG SIGN	32									
0 b	e^x	74									
0 c	↑	27									
0 d	c	16									
1 0	ln x	65									
1 1	↑	27									
1 2	b	14	CALCULATE $e^{-\lambda} \hat{\lambda}^{n_i}$ (steps 0 9 – 1 9)								
1 3	×	36									
1 4	$x \rightleftarrows y$	30									
1 5	e^x	74									
1 6	ROLL ↑	22									
1 7	×	36									
1 8	$y \to (\ )$	40									
1 9	a	13									
1 a	b	14									
1 b	↑	27									
1 c	↑	27	DECREMENT n_i (steps 1 a – 2 0)								
1 d	1	01									
2 0	−	34									
2 1	IF $x > y$	53									
2 2	2	02									
2 3	a	13									
2 4	ROLL ↓	31									
2 5	×	36									
2 6	ROLL ↑	22	CONTINUE CALCULATING $n_i \cdot (n_i - 1) \cdots 1$ (steps 2 4 – 2 9)								
2 7	GO TO ( )( )	44									
2 8	2	02									
2 9	0	00									
2 a	a	13									
2 b	ROLL ↑	22									
2 c	↑	27									
2 d	0	00									

FROM 4-7 (to step 0 0)
FROM 5-2 (to step 0 a)

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	$x \rightleftarrows y$	30									
1	IF $x = y$	50									
2	arc ▾	72	IF $n_i! = 0$, ENTER 1								
3	1	01									
4	$x \rightleftarrows y$	30									
5	↓	25	CALCULATE $\frac{e^{-\hat{\lambda}}\hat{\lambda}^{n_i}}{n_i!}$ AND $\sum_{n=n_1}^{n_i} f(n,\hat{\lambda})$								
6	÷	35									
7	$x \rightleftarrows y$	30									
8	↑	27									
9	ACC +	60									
a	b	14									
b	ROLL ↓	31	POSITION DISPLAY								
c	RCL	61									
d	STOP	41	$\frac{e^{-\hat{\lambda}}\hat{\lambda}^{n_i}}{n_i!}$	$\sum_{n=n_i}^{n_i} f(n,\hat{\lambda})$	0						
4 0	CLEAR x	37									
1	$x \rightarrow ( )$	23	ZERO STORAGE REGISTER f								
2	f	15									
3	d	17									
4	ROLL ↑	22									
5	IF $x = y$	50									
6	0	00	BRANCH TO INITIAL ENTRY WHEN THROUGH								
7	0	00									
8	↑	27									
9	1	01									
a	+	33	INCREMENT n_i								
b	$y \rightarrow ( )$	40									
c	b	14									
d	c	16	RECALL $\hat{\lambda}$ AND CONTINUE SUMMATION								
5 0	GO TO ( )( )	44									
1	0	00									
2	a	13									
3	END	46									
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-70816

RANDOM NUMBER GENERATOR

This program calculates random numbers (RN) in the range $0 \le RN_i \le 1$ using the formula given below:

$$RN_i = \left[\pi + RN_{(i-1)}\right]^8 - \text{Int.} \left\{ \left[\pi + RN_{(i-1)}\right]^8 \right\}$$

RN_i is the current random number and $RN_{(i-1)}$ is the last calculated random number. More than 10,000 random numbers may be generated before values are repeated.

USER INSTRUCTIONS	EXAMPLE
ENTER PROGRAM: (Starting Address is 0 - 0)	RN_1 = .5310
PRESS: GO TO (0) (0) [or END]	RN_2 = .6918
PRESS: CONTINUE	RN_3 = .3712
DISPLAY	RN_4 = .6890
0 ——— Z 0 ——— Y RN_i ——— X	
TO RESET PROBLEM:	

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0	0	CLEAR	20									
	1	↑	27									
	2	π	56									
	3	+	33									
	4	↓	25									
	5	↑	27									
	6	×	36									
	7	↓	25									
	8	↑	27									
	9	×	36	CALCULATE $\left[\pi + RN_{(i-1)}\right]^8 - \text{int.}\left\{\left[\pi + RN_{(i-1)}\right]^8\right\} = RN_i$ (steps 02–12)								
	a	↓	25									
	b	↑	27									
	c	×	36									
	d	↓	25									
1	0	↑	27									
	1	int x	64									
	2	−	34									
	3	↓	25									
	4	STOP	41	RN_i	0	0	DISPLAY					
	5	GO TO ( )( )	44									
	6	0	00	BRANCH TO CALCULATE NEXT RANDOM NUMBER (steps 15–17; to step 01)								
	7	1	01									
	8	END	46									
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

PART NO.
09100-70904

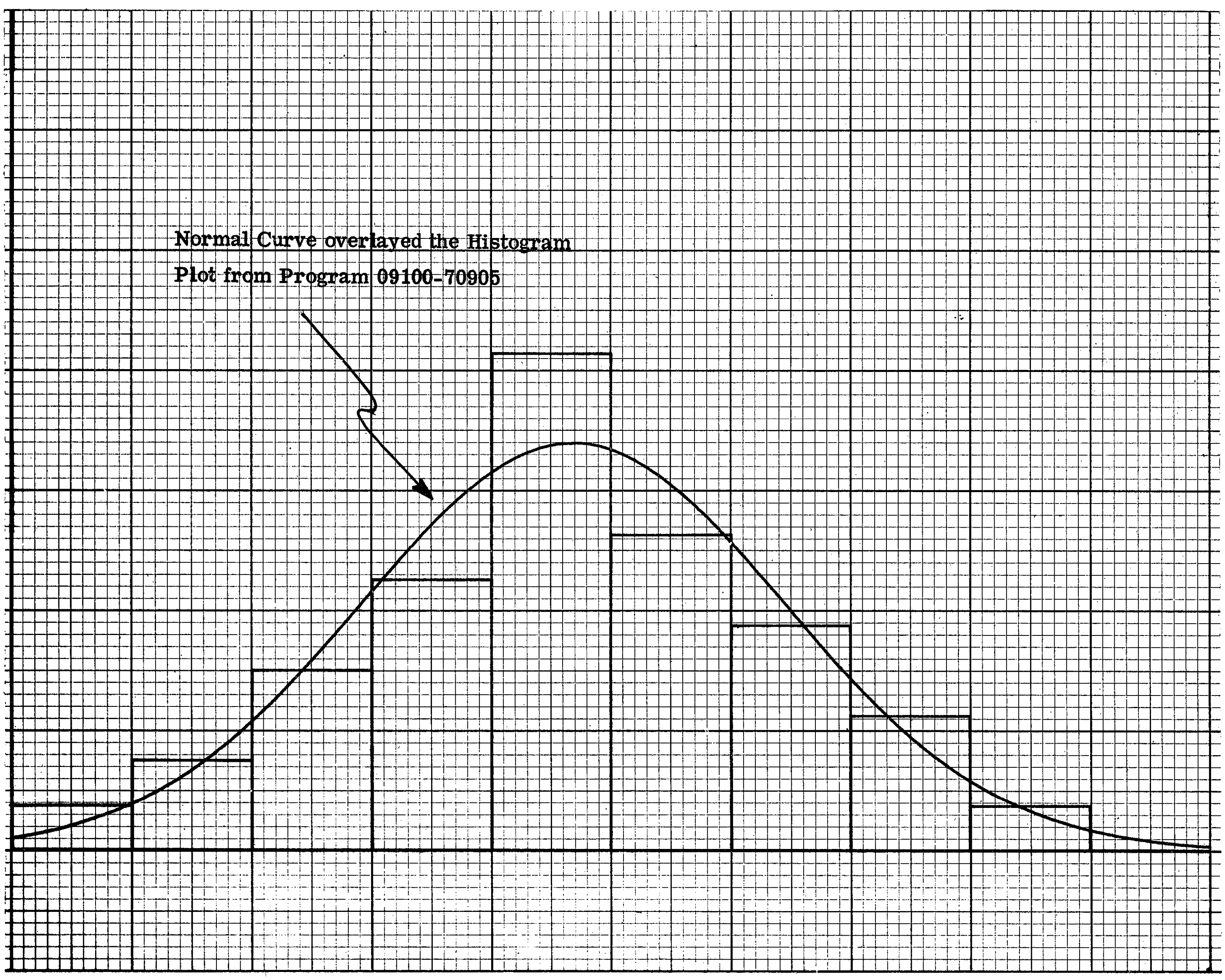
NORMAL CURVE PLOT

This program generates a normal curve given mean M_h and variance σ^2_h. The program determines Y from:

$$Y = \frac{1}{\sigma \sqrt{2\pi}} e^{\frac{-(h - M_h)^2}{2\sigma^2_h}}$$

by varying h from 0 to 10 in increments of 0.1. The program requires that M_h and σ^2_h be stored in the f and e registers respectively prior to execution. This program was intended to be used in conjunction with program 09100-70905 Histogram Generation (with Plot). To plot in units of centimeters, place a 2 in locations (2) (5) and (2) (c).

USER INSTRUCTIONS	EXAMPLE
SET: Decimal Wheel at 6 or less Using the orgin controls, locate the pen at X = 1 in., Y = 1 in. ENTER PROGRAM: (Starting Address is 0-0) PRESS: END PRESS: CONTINUE	$M_h = 4.679$ $\sigma_h^2 = 3.132$ See plot below



HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
00	e	12			
(+) 1	$\sqrt{x}$	76			
2	↑	27			
3	π	56			
4	↑	27			
5	2	02			
6	×	36			
7	↓	25			
8	$\sqrt{x}$	76			
9	×	36			
a	y→( )	40			
b	c	16			
c	0	00			
d	↑	27			
10	y→( )	40			
(+) 1	d	17			
2	f	15			
3	−	34			
4	↓	25			
5	↑	27			
6	×	36			
7	e	12			
8	↑	27			
9	2	02			
a	×	36			
b	↓	25			
c	÷	35			
d	↓	25			
20	CHG SIGN	32			
(+) 1	e^x	74			
2	↑	27			
3	c	16			
4	÷	35			
5	7	07			
6	5	05			
7	0	00			
8	0	00			
9	×	36			
a	d	17			
b	↑	27			
c	5	05			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
30	0	00			
(+) 1	×	36			
2	↓	25			
3	FMT	42			
4	↓	25			
5	d	17			
6	↑	27			
7	•	21			
8	1	01			
9	+	33			
a	1	01			
b	0	00			
c	•	21			
d	1	01			
40	IF x > y	53			
(+) 1	1	01			
2	0	00			
3	CLEAR	20			
4	FMT	42			
5	↑	27			
6	END	46			

Storage

Register	Contents
f	M_h
e	σ_h^2
d	h
c	$\sigma_h\sqrt{2\pi}$
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

PART NO.
09100-70907

ONE WAY ANALYSIS OF VARIANCE m x n

This program separates the total variance in a table of data into a portion due to chance and a portion due to differences between and population means underlying each column of sample data. It then calculates the variance ratio.

$$F = \frac{nm(m-1) \sum_{j=1}^{n} (X_j - \bar{X})}{(n-1) \sum_{i=1}^{m} \sum_{j=1}^{n} (X_{ij} - \bar{X}_j)}$$

with $\nu_1 = n - 1$ degrees of freedom

$\nu_2 = n(m-1)$ degrees of freedom

where

$$\bar{X} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij}$$

$$\bar{X}_j = \frac{1}{n} \sum_{j=1}^{n} X_{ij}$$

The equation used by the program is:

$$F = \frac{nm(m-1) \left\{ \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}{(n-1) \left\{ \sum_{j=1}^{n} \sum_{i=1}^{m} X^2_{ij} - \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 - \frac{1}{m} \sum_{j=1}^{n} \left[\sum_{i=1}^{m} X_{ij} \right]^2 + \frac{1}{mn} \left[\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} \right]^2 \right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A at address 0-0

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: n Columns → Y, m Rows → X

PRESS: CONTINUE

DISPLAY

0	Z	
j	Y	jth Column
i	X	ith Row

Enter data Column by Column

ENTER DATA: X_{ij} → X

PRESS: CONTINUE

Last data entered? NO → return to DISPLAY; YES ↓

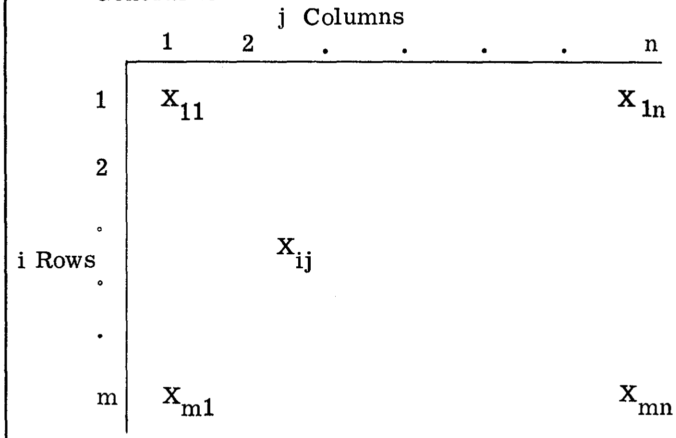
DISPLAY

F	Z	F=variance ratio
ν_1	Y	ν_1=degrees of freedom in numerator
ν_2	X	ν_2=degrees of freedom in denominator

PRESS: CONTINUE for new case

EXAMPLE

General form



Columns

Rows			
	172	203	161
	185	172	149
	165	187	183
	194	183	156
	212	179	144

$F = 5.01$

$\nu_1 = 2$

$\nu_2 = 12$

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
(+) 01	$x\to(\ )$	23			
02	9	11			
03	$x\to(\ )$	23			
04	b	14		ENTER	
05	STOP	41	M	N	0
06	$y\to(\ )$	40			
07	c	16			
08	↑	27			
09	1	01			
0a	+	33			
0b	$y\to(\ )$	40			
0c	d	17			
0d	$x\rightleftarrows y$	30			

Step	Key	Code	Display x	Display y	Display z
10	CLEAR x	37			
(+) 11	ROLL ↓	31			
12	$x\to(\ )$	23			
13	a	13		ENTER	
14	STOP	41	X_{ij}	0	0
15	$y\rightleftarrows(\ )$	24			
16	b	14			
17	+	33			
18	$y\rightleftarrows(\ )$	24			
19	b	14			
1a	↑	27			
1b	×	36			
1c	ACC +	60			
1d	↓	25			

Step	Key	Code	Display x	Display y	Display z
20	a	13			
(+) 21	↑	27			
22	1	01			
23	+	33			
24	d	17			
25	IF $x>y$	53			
26	1	01			
27	0	00			
28	↓	25			
29	b	14			
2a	↑	27			
2b	×	36			
2c	↓	25			
2d	$y\rightleftarrows(\ )$	24			

Step	Key	Code	Display x	Display y	Display z
30	9	11			
(+) 31	+	33			
32	$y\rightleftarrows(\ )$	24			
33	9	11			
34	CLEAR x	37			
35	ROLL ↓	31			
36	1	01			
37	−	34			
38	$x\to(\ )$	23			
39	a	13			
3a	CLEAR x	37			
3b	$x\to(\ )$	23			
3c	b	14			
3d	IF $x<y$	52			

Step	Key	Code	Display x	Display y	Display z
40	1	01			
(+) 41	3	03			
42	d	17			
43	↑	27			
44	1	01			
45	−	34			
46	$y\to(\ )$	40			
47	d	17			
48	RCL	61			
49	↑	27			
4a	×	36			
4b	c	16			
4c	÷	35			
4d	d	17			

Step	Key	Code	Display x	Display y	Display z
50	÷	35			
(+) 51	↓	25			
52	−	34			
53	$y\to(\ )$	40			
54	e	12			
55	ROLL ↓	31			
56	$y\rightleftarrows(\ )$	24			
57	9	11			
58	d	17			
59	÷	35			
5a	↓	25			
5b	$x\rightleftarrows y$	30			
5c	−	34			
5d	e	12			

Step	Key	Code	Display x	Display y	Display z
60	$x\rightleftarrows y$	30			
(+) 61	−	34			
62	$x\rightleftarrows y$	30			
63	÷	35			
64	c	16			
65	×	36			
66	↑	27			
67	1	01			
68	−	34			
69	$y\to(\ )$	40			
6a	f	15			
6b	↓	25			
6c	÷	35			
6d	d	17			

Step	Key	Code	Display x	Display y	Display z
70	↑	27			
(+) 71	1	01			
72	−	34			
73	↓	25			
74	×	36			
75	↑	27			
76	c	16			
77	×	36			
78	f	15			
79	$x\rightleftarrows y$	30			
7a	CONT	47		DISPLAY	
7b	END	46	V_2	V_1	F
7c					
7d					

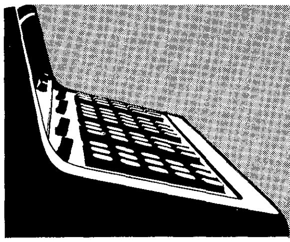
Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70901

MULTIPLE LINEAR REGRESSION

This program fits any number of data points (X_i, Y_i, Z_i) to a linear, two variable equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

where X and Y are the independent variables.

Development:

The constants a_o, a_1, and a_2 of the equation may be found by solving simultaneously the following normal equations which represent the least square plane (approximating plane) formed by the data points.

$$\sum Z = a_o n + a_1 \sum X + a_2 \sum Y$$

$$\sum XZ = a_o \sum X + a_1 \sum X^2 + a_2 \sum XY$$

$$\sum YZ = a_o \sum Y + a_1 \sum XY + a_2 \sum Y^2$$

In the program the constant a_2 is found from solving the equations by matrix algebra. Therefore,

$$a_2 = \frac{n(\sum X^2 \sum YZ - \sum XZ \sum XY) - \sum X(\sum X \sum YZ - \sum Y \sum XZ) + \sum Z(\sum X \sum XY - \sum Y \sum X^2)}{D}$$

Where D (the determinant) = $\begin{vmatrix} n & \sum X & \sum Y \\ \sum X & \sum X^2 & \sum XY \\ \sum Y & \sum XY & \sum Y^2 \end{vmatrix}$

After finding a_2, the solution is reduced to two equations in two unknowns which are:

$$M = a_o n + a_1 \sum X$$

$$N = a_o \sum X + a_1 \sum X^2$$

where $M = (\sum Z - a_2 \sum Y)$ $\qquad N = (\sum XZ - a_2 \sum XY)$

These two equations are then solved for a_o and a_1.

Reference:

Introduction to the Theory of Statistics
Mood and Graybill
McGraw-Hill, 1963

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

→ DISPLAY

i	Z
0	Y
0	X

ENTER DATA: $Z_i \rightarrow Z$, $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

Has Last set of points been entered ? (NO: return to DISPLAY; YES: continue)

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

a_2	Z
a_1	Y
a_0	X

To reset problem, PRESS: CONTINUE (return to PRESS: CONTINUE)

EXAMPLES

(A) Equation of the form:

$$Z = a_o + a_1 X + a_2 Y$$

Input data:

X	Y	Z
1	0	3
0	1	4
1	1	6
3	4	19
2	2	11

Solution:

$$Z = 1 + 2X + 3Y$$

(B) Equation of the form :

$$Z = a_o + a_1 (\log X) + a_2 (\log Y)$$

Note to enter data ;
ENTER: $Z_i \rightarrow X$
PRESS: ↑
ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow X$
PRESS: log X

Input data:

X	Y	Z
1	2	4.6505
1	1	4.5
4	3.63	5.9841
10	5	6.8495
8	16	6.9082
13	7	7.1504
3	10	5.9542

Solution:

$$Z = 4.5 + 2 (\log X) + .4999 (\log Y)$$

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
01	x→( )	23			
02	a	13			
03	x→( )	23			
04	b	14			
05	x→( )	23			
06	c	16			
07	x→( )	23			
08	9	11			
09	x→( )	23			
0a	−	34			
0b	f	15			
0c	x→( )	23			
0d	−	34			
10	e	12			
11	1	01			
12	x→( )	23			
13	d	17			
14	ROLL ↓	31	ENTER		
15	STOP	41	X_i	Y_i	Z_i
16	IF FLAG	43			
17	6	06			
18	5	05			
19	y→( )	40			
1a	−	34			
1b	d	17			
1c	×	36			
1d	x⇄y	30			
20	y⇄( )	24			
21	a	13			
22	+	33			
23	y⇄( )	24			
24	a	13			
25	ROLL ↓	31			
26	ACC +	60			
27	↑	27			
28	×	36			
29	x⇄y	30			
2a	y⇄( )	24			
2b	−	34			
2c	e	12			
2d	+	33			
30	CONT	47			
31	y⇄( )	24			
32	−	34			
33	e	12			
34	↓	25			
35	×	36			
36	c	16			
37	+	33			
38	y→( )	40			
39	c	16			
3a	ROLL ↑	22			
3b	y⇄( )	24			
3c	−	34			
3d	d	17			
40	x⇄y	30			
41	×	36			
42	↑	27			
43	×	36			
44	ROLL ↑	22			
45	y⇄( )	24			
46	9	11			
47	+	33			
48	y⇄( )	24			
49	9	11			
4a	b	14			
4b	+	33			
4c	y→( )	40			
4d	b	14			
50	ROLL ↑	22			
51	y⇄( )	24			
52	−	34			
53	f	15			
54	+	33			
55	y→( )	40			
56	−	34			
57	f	15			
58	1	01			
59	↑	27			
5a	d	17			
5b	+	33			
5c	y→( )	40			
5d	d	17			
60	CLEAR x	37			
61	↑	27			
62	GO TO ( ) ( )	44			
63	1	01			
64	5	05			
65	y⇄( )	24			
66	−	34			
67	f	15			
68	d	17			
69	↑	27			
6a	1	01			
6b	−	34			
6c	y→( )	40			
6d	d	17			
70	↓	25			
71	f	15			
72	×	36			
73	CHG SIGN	32			
74	×	36			
75	ROLL ↑	22			
76	y⇄( )	24			
77	−	34			
78	e	12			
79	GO TO ( ) ( )	44			
7a	−	34			
7b	0	00			
7c	0	00			
7d					

Storage

Register	+	−
f	ΣX	ΣY^2
e	ΣZ	ΣX^2
d	n	ΣY
c	ΣXZ	
b	ΣY	
a	ΣXY	
9	ΣYZ	
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
0 0	y→( )	40			
(−) 1	−	34			
2	e	12			
3	×	36			
4	d	17			
5	×	36			
6	ROLL ↓	31			
7	+	33			
8	a	13			
9	↑	27			
a	×	36			
b	d	17			
c	×	36			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
1 0	−	34			
(−) 1	↑	27			
2	y⇄( )	24			
3	−	34			
4	e	12			
5	y→( )	40			
6	−	34			
7	e	12			
8	b	14			
9	×	36			
a	×	36			
b	↓	25			
c	−	34			
d	b	14			

Step	Key	Code	Display x	Display y	Display z
2 0	↑	27			
(−) 1	f	15			
2	×	36			
3	a	13			
4	×	36			
5	2	02			
6	×	36			
7	↓	25			
8	+	33			
9	y→( )	40			
a	−	34			
b	f	15			
c	y⇄( )	24			
d	9	11			

Step	Key	Code	Display x	Display y	Display z
3 0	↑	27			
(−) 1	↓	25			
2	f	15			
3	×	36			
4	CHG SIGN	32			
5	×	36			
6	ROLL ↑	22			
7	x⇄y	30			
8	d	17			
9	×	36			
a	x←( )	67			
b	−	34			
c	e	12			
d	×	36			

Step	Key	Code	Display x	Display y	Display z
4 0	ROLL ↓	31			
(−) 1	+	33			
2	ROLL ↑	22			
3	x⇄y	30			
4	b	14			
5	×	36			
6	e	12			
7	×	36			
8	ROLL ↓	31			
9	−	34			
a	f	15			
b	ROLL ↑	22			
c	×	36			
d	a	13			

Step	Key	Code	Display x	Display y	Display z
5 0	×	36			
(−) 1	ROLL ↓	31			
2	+	33			
3	d	17			
4	ROLL ↑	22			
5	×	36			
6	c	16			
7	×	36			
8	ROLL ↓	31			
9	−	34			
a	f	15			
b	ROLL ↑	22			
c	×	36			
d	b	14			

Step	Key	Code	Display x	Display y	Display z
6 0	×	36			
(−) 1	↓	25			
2	+	33			
3	x←( )	67			
4	−	34			
5	f	15			
6	÷	35			
7	y→( )	40			
8	9	11			
9	↑	27			
a	↓	25			
b	b	14			
c	×	36			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
7 0	x⇄y	30			
(−) 1	−	34			
2	a	13			
3	y→( )	40			
4	a	13			
5	ROLL ↑	22			
6	×	36			
7	c	16			
8	x⇄y	30			
9	−	34			
a	y→( )	40			
b	c	16			
c	f	15			
d	↑	27			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
8 0	d	17			
(−) 1	y→( )	40			
2	d	17			
3	y⇄( )	24			
4	−	34			
5	e	12			
6	y→( )	40			
7	−	34			
8	f	15			
9	x→( )	23			
a	b	14			
b	CLEAR	20			
c	y⇄( )	24			
d	−	34			
9 0	f	15			
(−) 1	c	16			
2	↑	27			
3	d	17			
4	÷	35			
5	ROLL ↑	22			
6	x⇄y	30			
7	÷	35			
8	↓	25			
9	IF FLAG	43			
a	a	13			
b	9	11			
c	ACC +	60			
d	d	17			
a 0	y⇄( )	24			
(−) 1	b	14			
2	↑	27			
3	a	13			
4	ROLL ↑	22			
5	SET FLAG	54			
6	GO TO ( ) ( )	44			
7	9	11			
8	4	04			
9	ACC −	63			
a	x⇄y	30			
b	y⇄( )	24			
c	e	12			
d	f	15			

Step	Key	Code	Display x	Display y	Display z
b 0	÷	35			
(−) 1	e	12			
2	x⇄y	30			
3	×	36			
4	ROLL ↓	31			
5	−	34			
6	ROLL ↑	22			
7	y⇄( )	24			
8	9	11			
9	ROLL ↑	22	DISPLAY		
a	STOP	41	a_0	a_1	a_2
b	CONT	47			
c	END	46			
d					

Storage

f, e, d, c, b, a, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0

HEWLETT · PACKARD

HEWLETT · PACKARD

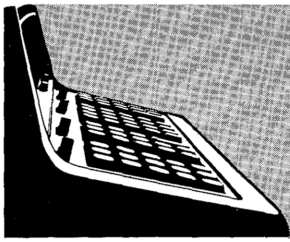
HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z	Step	Key	Code	Display x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70902

WEIBULL DISTRIBUTION PARAMETER CALCULATION FOR FAILURE DATA

The Weibull probability density function is given by

$$f(X) = \frac{bX^{(b-1)}}{\theta^b} e^{-\left(\frac{X}{\theta}\right)^b}$$

and the cummulative distribution function is given by

$$F(X) = 1 - e^{-\left(\frac{X}{\theta}\right)^b}$$

For a set of data, the Weibull parameters b and θ are to be calculated for these functions.

A common application is to use Weibull analysis for failure data where all samples are tested to failure. To use the program, list the items in order of increasing time to failure. The number of items and times to failure are entered. The parameters b , θ , and r are displayed. r is a correlation coefficient indicating goodness of fit. The time required for 10% (B_{10}) to fail is displayed and times to other failure percentages ($B_{\%}$) may be requested.

The Median Rank (M. R.) is calculated by the equation

$$M.R. = \frac{j-.3}{N+4}$$

where j = failure order number

N = number of samples tested

This is an approximation of F(X).

The commulative distribution function is linearized into the form

$$b\,\mathrm{Ln}X - b\,\mathrm{Ln}\theta = \mathrm{Ln}\mathrm{Ln}\left(\frac{1}{1-F(X)}\right)$$

A least squares fit is performed which calculates the slope, intercept, and correlation coefficient. The solution is similar to the linear regression program 09100-70803. Thus estimates of b and θ are obtained.

USER INSTRUCTIONS	EXAMPLES

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

ENTER DATA: N → X

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
j	X

(j indicates point to be entered)

ENTER DATA: t_j → X (Data must be ordered)

Last Entry? — NO / YES

PRESS: CONTINUE

DISPLAY

r	Z
θ	Y
b	X

→ PRESS: CONTINUE

DISPLAY

0	Z
B%	Y
%	X

(first time will be 10%)

ENTER DATA: % → X

TO RESTART A NEW PROBLEM

PRESS: END

TEST DATA

Hours to failure (must be ordered)

34

60

75

95

119

158

N = 6 (number of samples)

r = .999

θ = 104.091

b = 1.953

B_{10} = 32.887

B_{90} = 159.539

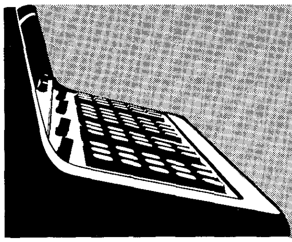
Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
(+) 01	x→( )	23			
02	d	17			
03	x→( )	23			
04	c	16			
05	x→( )	23			
06	b	14	ENTER		
07	STOP	41	N	0	0
08	x→( )	23			
09	8	10			
0a	↑	27			
0b	•	21			
0c	4	04			
0d	+	33			
10	↓	25			
(+) 11	x→( )	23			
12	9	11			
13	1	01			
14	x→( )	23			
15	a	13	ENTER		
16	STOP	41	T_j	0	0
17	ln x	65			
18	↑	27			
19	a	13			
1a	↑	27			
1b	•	21			
1c	3	03			
1d	−	34			
20	x←( )	67			
(+) 21	9	11			
22	÷	35			
23	1	01			
24	x⇄y	30			
25	−	34			
26	1	01			
27	x⇄y	30			
28	÷	35			
29	↓	25			
2a	ln x	65			
2b	ln x	65			
2c	x⇄y	30			
2d	ACC +	60			
30	↑	27			
(+) 31	×	36			
32	x⇄y	30			
33	y⇄( )	24			
34	d	17			
35	+	33			
36	y⇄( )	24			
37	d	17			
38	CONT	47			
39	↓	25			
3a	×	36			
3b	b	14			
3c	+	33			
3d	y→( )	40			
40	b	14			
(+) 41	ROLL ↑	22			
42	↑	27			
43	×	36			
44	c	16			
45	+	33			
46	y→( )	40			
47	c	16			
48	a	13			
49	↑	27			
4a	1	01			
4b	+	33			
4c	x←( )	67			
4d	8	10			
50	x⇄y	30			
(+) 51	↑	27			
52	0	00			
53	ROLL ↓	31			
54	IF x>y	53			
55	6	06			
56	0	00			
57	x→( )	23			
58	a	13			
59	x⇄y	30			
5a	↓	25			
5b	GO TO ( )	44			
5c	1	01			
5d	6	06			
60	e	12			
(+) 61	↑	27			
62	a	13			
63	÷	35			
64	y⇄( )	24			
65	f	15			
66	÷	35			
67	y→( )	40			
68	e	12			
69	×	36			
6a	e	12			
6b	×	36			
6c	d	17			
6d	x⇄y	30			
70	−	34			
(+) 71	y→( )	40			
72	d	17			
73	c	16			
74	↑	27			
75	f	15			
76	↑	27			
77	×	36			
78	a	13			
79	×	36			
7a	GO TO ( )	44			
7b	−	34			
7c	0	00			
7d	0	00			

Storage

Register	+	−
f		
e		
d		
c		
b		
a		
9	N + 4	
8	N	
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
0 0	↓	25			
0 1	−	34			
0 2	y→( )	40			
0 3	c	16			
0 4	b	14			
0 5	↑	27			
0 6	f	15			
0 7	↑	27			
0 8	e	12			
0 9	×	36			
0 a	a	13			
0 b	×	36			
0 c	↓	25			
0 d	−	34			
1 0	d	17			
1 1	↑	27			
1 2	↓	25			
1 3	√x	76			
1 4	÷	35			
1 5	c	16			
1 6	√x	76			
1 7	÷	35			
1 8	d	17			
1 9	ROLL ↑	22			
1 a	x⇄y	30			
1 b	÷	35			
1 c	y⇄( )	24			
1 d	e	12			
2 0	e	12			
2 1	×	36			
2 2	f	15			
2 3	x⇄y	30			
2 4	−	34			
2 5	e	12			
2 6	CHG SIGN	32			
2 7	÷	35			
2 8	x⇄y	30			
2 9	e^x	74			
2 a	x⇄y	30			
2 b	CHG SIGN	32	DISPLAY		
2 c	STOP	41	b	θ	r
2 d	CONT	47			
3 0	↑	27			
3 1	1	01			
3 2	x⇄y	30			
3 3	÷	35			
3 4	y→( )	40			
3 5	f	15			
3 6	ROLL ↑	22			
3 7	ln x	65			
3 8	x→( )	23			
3 9	e	12			
3 a	1	01			
3 b	0	00			
3 c	x→( )	23			
3 d	d	17			
4 0	↑	27			
4 1	ENTER EXP	26			
4 2	2	02			
4 3	÷	35			
4 4	1	01			
4 5	−	34			
4 6	x⇄y	30			
4 7	CHG SIGN	32			
4 8	÷	35			
4 9	↓	25			
4 a	ln x	65			
4 b	ln x	65			
4 c	↑	27			
4 d	f	15			
5 0	×	36			
5 1	e	12			
5 2	+	33			
5 3	0	00			
5 4	ROLL ↓	31			
5 5	e^x	74			
5 6	x⇄y	30			
5 7	d	17	DISPLAY		
5 8	STOP	41	%	B%	0
5 9	CONT	47			
5 a	GO TO ( )	44			
5 b	3	03			
5 c	c	16			
5 d	END	46			

Storage



NON-LINEAR REGRESSION
THE LEAST SQUARE PARABOLA

Development:

The least square parabola approximating the set of points $(X_1, Y_1) \ldots\ldots (X_i, Y_i)$ has the equation:

$$Y = a_o + a_1 x + a_2 x^2$$

where the constants a_o, a_1, and a_2 are determined by solving simultaneously the following normal equations:

$$\Sigma Y = a_o n + a_1 \Sigma X + a_2 \Sigma X^2$$
$$\Sigma XY = a_o \Sigma X + a_1 \Sigma X^2 + a_2 \Sigma X^3$$
$$\Sigma X^2 Y = a_o \Sigma X^2 + a_1 \Sigma X^3 + a_2 \Sigma X^4$$

In the program the constant a_2 is found by matrix algebra; the determinate (D) involved in the solution is:

$$D = \begin{vmatrix} n & \Sigma X & \Sigma X^2 \\ \Sigma X & \Sigma X^2 & \Sigma X^3 \\ \Sigma X^2 & \Sigma X^3 & \Sigma X^4 \end{vmatrix}$$

The equation for a_2 is therefore:

$$a_2 = \frac{\left(n(\Sigma X^2 \Sigma X^2 Y - \Sigma X^3 \Sigma XY) - \Sigma X(\Sigma X \Sigma X^2 Y - \Sigma X^2 \Sigma XY) + \Sigma Y \left[\Sigma X \Sigma X^3 - (\Sigma X^2)^2\right]\right)}{D}$$

After finding a_2 the solution is reduced to two equations in two unknowns which are:

$$N = a_o n + a_1 \Sigma X$$
$$M = a_o \Sigma X + a_1 \Sigma X^2$$

where $M = \Sigma XY - a_2 \Sigma X^3$ and $N = \Sigma Y - a_2 \Sigma X^2$

These equations are then solved for a_o and a_1.

NOTE: Curves with the following equations may also be fitted with this program:

$$Y = a_o + a_1 X \quad (1)$$
$$\log Y = a_o + a_1 X \quad (2)$$
$$\log Y = a_o + a_1 X + a_2 X^2 \quad (3)$$
$$Y = a_o + a_1 \log X \quad (4)$$
$$\log Y = a_o + a_1 \log X \quad (5)$$
$$\log Y = a_o + a_1 (\log X) + a_2 (\log X)^2 \quad (6)$$

An equation of the form of (6) is solved in the examples.

The general form, representing all of these equations, which can be fitted is:

$$f(Y) = a_o + a_1 f(X) + a_2 f^2(X)$$

Reference:

Publisher -- McGraw-Hill
Authors -- Alexander M. Mood & Franklin A. Graybill
Introduction into the Theory of Statistics -- 2nd Edition (1961)

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: END

→ PRESS: CONTINUE

→ DISPLAY

i	———	Z
0	———	Y
0	———	X

(i indicates pair of points to be entered)

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$

PRESS: CONTINUE

Has all the data been entered? NO (return to DISPLAY) / YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

a_2	———	Z
a_1	———	Y
a_0	———	X

To calculate coefficients for new data:

PRESS: END

EXAMPLES

(A) Equation of the form: $Y = a_o + a_1 X + a_2 X^2$

Data:

X	Y
3	29
0	2
5	67
2	16
1.5	11
4	46
1	7

Solution: $Y = 2 + 3X + 2X^2$

(B) Equation of the form:
$\log Y = a_o + a_1 \log X + a_2 (\log X)^2$

Note: Data to be entered is $\log Y_i$, $\log X_i$; therefore to enter data sets:

ENTER: $Y_i \rightarrow X$
PRESS: log X
PRESS: ↑
ENTER: $X_i \rightarrow Y$
PRESS: log X

Data:

X	Y
1	2.7183
2	35.1595
3	245.3746
4	1188.7946
5	4530.5750

Solution: $\log Y = .43 + 3.0 \log X + 2.30 (\log X)^2$

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	$x\to(\ )$	23			
2	c	16			
3	$x\to(\ )$	23			
4	b	14			
5	$x\to(\ )$	23			
6	a	13			
7	$x\to(\ )$	23			
8	−	34			
9	f	15			
a	$x\to(\ )$	23			
b	−	34			
c	e	12			
d	1	01			
1 0	$x\to(\ )$	23			
(+) 1	d	17			
2	ROLL ↓	31	ENTER		
3	STOP	41	X_i	Y_i	i
4	IF FLAG	43			
5	5	05			
6	5	05			
7	ACC +	60			
8	×	36			
9	$x \rightleftarrows y$	30			
a	$y \rightleftarrows (\ )$	24			
b	c	16			
c	+	33			
d	$y \rightleftarrows (\ )$	24			
2 0	c	16			
(+) 1	↑	27			
2	↓	25			
3	×	36			
4	↓	25			
5	$y \rightleftarrows (\ )$	24			
6	−	34			
7	f	15			
8	+	33			
9	$y \rightleftarrows (\ )$	24			
a	−	34			
b	f	15			
c	↓	25			
d	×	36			

Step	Key	Code	Display x	Display y	Display z
3 0	$x \rightleftarrows y$	30			
(+) 1	$y \rightleftarrows (\ )$	24			
2	b	14			
3	+	33			
4	$y \rightleftarrows (\ )$	24			
5	b	14			
6	×	36			
7	a	13			
8	$x \rightleftarrows y$	30			
9	+	33			
a	$y\to(\ )$	40			
b	a	13			
c	ROLL ↑	22			
d	×	36			
4 0	$x \rightleftarrows y$	30			
(+) 1	$y \rightleftarrows (\ )$	24			
2	−	34			
3	e	12			
4	+	33			
5	$y\to(\ )$	40			
6	−	34			
7	e	12			
8	1	01			
9	↑	27			
a	d	17			
b	+	33			
c	$y\to(\ )$	40			
d	d	17			
5 0	CLEAR x	37			
(+) 1	↑	27			
2	GO TO ( )	44			
3	1	01			
4	3	03			
5	$x\leftarrow(\ )$	67			
6	−	34			
7	e	12			
8	↑	27			
9	f	15			
a	↑	27			
b	×	36			
c	b	14			
d	ROLL ↑	22			

Step	Key	Code	Display x	Display y	Display z
6 0	×	36			
(+) 1	ROLL ↑	22			
2	CHG SIGN	32			
3	×	36			
4	$y \rightleftarrows (\ )$	24			
5	d	17			
6	1	01			
7	−	34			
8	$y \rightleftarrows (\ )$	24			
9	d	17			
a	d	17			
b	ROLL ↑	22			
c	×	36			
d	↓	25			
7 0	+	33			
(+) 1	a	13			
2	↑	27			
3	×	36			
4	d	17			
5	×	36			
6	↓	25			
7	−	34			
8	b	14			
9	↑	27			
a	×	36			
b	×	36			
c	ROLL ↓	31			
d	−	34			

Storage

	+	−
f	ΣY	$\Sigma X^2 Y$
e	ΣX	ΣX^4
d	n	
c	ΣXY	
b	ΣX^2	
a	ΣX^3	
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
8 0	f	15			
(+) 1	ROLL ↑	22			
2	×	36			
3	2	02			
4	×	36			
5	a	13			
6	×	36			
7	↓	25			
8	+	33			
9	y⇄( )	24			
a	−	34			
b	f	15			
c	↑	27			
d	↓	25			
9 0	b	14			
(+) 1	×	36			
2	d	17			
3	×	36			
4	f	15			
5	ROLL ↑	22			
6	x⇄y	30			
7	×	36			
8	×	36			
9	ROLL ↓	31			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			
0 0	−	34			
(−) 1	e	12			
2	ROLL ↑	22			
3	×	36			
4	a	13			
5	×	36			
6	ROLL ↓	31			
7	+	33			
8	d	17			
9	ROLL ↑	22			
a	×	36			
b	c	6			
c	×	36			
d	ROLL ↓	31			

Step	Key	Code	Display x	Display y	Display z
1 0	−	34			
(−) 1	f	15			
2	x→( )	23			
3	−	34			
4	e	12			
5	ROLL ↑	22			
6	×	36			
7	b	14			
8	×	36			
9	ROLL ↓	31			
a	+	33			
b	↓	25			
c	ROLL ↓	31			
d	×	36			
2 0	e	12			
(−) 1	×	36			
2	ROLL ↓	31			
3	−	34			
4	ROLL ↑	22			
5	y⇄( )	24			
6	−	34			
7	f	15			
8	ROLL ↓	31			
9	÷	35			
a	b	14			
b	x⇄y	30			
c	×	36			
d	ROLL ↓	31			
3 0	−	34			
(−) 1	y⇄( )	24			
2	c	16			
3	a	13			
4	ROLL ↑	22			
5	x→( )	23			
6	−	34			
7	f	15			
8	×	36			
9	ROLL ↓	31			
a	−	34			
b	y→( )	40			
c	a	13			
d	CLEAR	20			

Step	Key	Code	Display x	Display y	Display z
4 0	x←( )	67			
(−) 1	−	34			
2	e	12			
3	↑	27			
4	c	16			
5	↑	27			
6	d	17			
7	÷	35			
8	ROLL ↑	22			
9	x⇄y	30			
a	÷	35			
b	↓	25			
c	IF FLAG	43			
d	5	05			
5 0	d	17			
(−) 1	ACC +	60			
2	b	14			
3	y⇄( )	24			
4	−	34			
5	e	12			
6	↑	27			
7	a	13			
8	ROLL ↑	22			
9	SET FLAG	54			
a	GO TO ( )	44			
b	4	04			
c	7	07			
d	ACC −	63			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
6 0	x⇄y	30			
(–) 1	y⇄( )	24			
2	e	12			
3	f	15			
4	÷	35			
5	e	12			
6	x⇄y	30			
7	×	36			
8	ROLL ↓	31			
9	–	34			
a	ROLL ↑	22			
b	y⇄( )	24			
c	–	34			
d	f	15			
7 0	ROLL ↑	22			
(–) 1	STOP	41	DISPLAY		
2	CONT	47	a_0	a_1	a_2
3	END	46			
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

HEWLETT · PACKARD

0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HISTOGRAM GENERATION (WITH PLOT)

This program generates and plots a histogram of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance(σ_x^2) of the raw data, and the mean (M_h) and the variance (σ_h^2) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

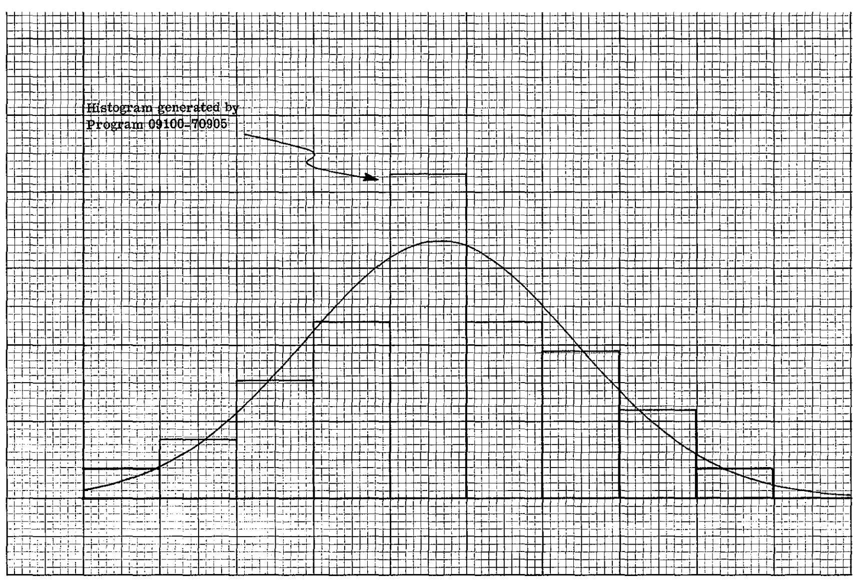
$$M_h = \frac{M_x}{W}$$

$$\sigma_h^2 = \frac{\sigma_x^2}{W^2}$$

where W is the histogram window width (normalizing factor)*. The program plots the histogram and stores M_h and σ_h^2 for use by program 09100-70904 which can be used to plot a normal curve over the histogram.

This program uses Indirect Addressing and is self-destructing of the registers +(0,0) through +(d,d). Thus, to rerun, the A side must be re-entered in the calculator.

NOTE: To generate a histogram with 1 cm. wide windows, place 2's in locations (-)(6)(c), (-)(7)(6), and (-)(8)(5).



* The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0 → 200, a W of of 20 would be proper.

USER INSTRUCTIONS

PRESS: END

Using the orgin controls, locate the pen at X = 1 in., Y = 1 in.

SET: Decimal Wheel at 6 or less

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: W ⟶ X

PRESS: CONTINUE

DISPLAY

N	———	Z
N	———	Y
1	———	X

N is the number of data points entered.

ENTER DATA: X_N ⟶ X

PRESS: CONTINUE

Has all data been entered? NO (return to display) / YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

%	———	Z	(% of data points in Group)
K_A	———	Y	(No. of data points in Group)
Group	———	X	(Group #)

PRESS: CONTINUE

Plot Window

Is Group # less than 10? YES (return to display) / NO

USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2	———	Z
M_x	———	Y
0	———	X

PRESS: CONTINUE

DISPLAY

σ_h^2	———	Z
M_h	———	Y
1	———	X

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%
1	1	2.5
2	2	5.
3	4	10
4	6	15.
5	11	27.5
6	7	17.5
7	5	12.5
8	3	7.5
9	1	2.5
10	0	0

N = 40

σ_x^2 = 1252.644 $\qquad$ σ_h^2 = 3.132

M_x = 93.575 $\qquad$ M_h = 4.679

The histogram plot is given with the normal curve superimposed. The normal curve resulted from running program 09100-70904 following completion of the Histogram Generation program.

0 (+)

Step	Key	Code	Display x	Display y	Display z
0	CLEAR	20			
1	x→( )	23			
2	9	11			
3	x→( )	23			
4	8	10			
5	x→( )	23			
6	7	07			
7	x→( )	23			
8	6	06			
9	x→( )	23			
a	5	05			
b	x→( )	23			
c	4	04			
d	x→( )	23			

c (+)

Step	Key	Code	Display x	Display y	Display z
0	x→( )	23			
1	0	00			
2	x→( )	23			
3	1	01			
4	1	01			
5	CHG SIGN	32			
6	ROLL ↓	31			
7	↓	25			
8	1	01			
9	+	33			
a	↑	27			
b	↓	25			
c	CONT	47	ENTER		
d	STOP	41	X	N	N

1 (−)

Step	Key	Code	Display x	Display y	Display z
0	0	00			
1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	0	00			

1 (+)

Step	Key	Code	Display x	Display y	Display z
0	3	03	ENTER		
1	STOP	41	W	0	0
2	x→( )	23			
3	a	13			
4	CLEAR x	37			
5	x→( )	23			
6	2	02			
7	GO TO ( )	44			
8	c	16			
9	0	00			
a	CONT	47			
b	CONT	47			
c	CONT	47			
d	CONT	47			

d (+)

Step	Key	Code	Display x	Display y	Display z
0	IF FLAG	43			
1	d	17			
2	d	17			
3	↑	27			
4	×	36			
5	ACC +	60			
6	x⇄y	30			
7	a	13			
8	÷	35			
9	GO TO ( )	44			
a	−	34			
b	0	00			
c	2	02			
d	GO TO ( )	44			

2 (−)

Step	Key	Code	Display x	Display y	Display z
0	IF FLAG	43			
1	2	02			
2	a	13			
3	CLEAR x	37			
4	1	01			
5	+	33			
6	SET FLAG	54			
7	GO TO ( )	44			
8	0	00			
9	d	17			
a	GO TO ( )	44			
b	+	33			
c	c	16			
d	7	07			

2 (+)

Place Continue's in registers 2 thru b

0 (−)

Step	Key	Code	Display x	Display y	Display z
0	3	03			
1	0	00			
2	↓	25			
3	int x	64			
4	↑	27			
5	ENTER EXP	26			
6	9	11			
7	+	33			
8	y→( )	40			
9	−	34			
a	1	01			
b	CLEAR x	37			
c	CLEAR x	37			
d	y⇄( )	24			

Storage

Register	+	−
f	$\Sigma X / M_x / M_h$	
e	$\Sigma X^2 / \sigma_h^2$	
d		
c		
b	N	
a	W	
9	K_9	
8	K_8	
7	K_7	
6	K_6	
5	K_5	
4	K_4	
3	K_3	
2	K_2	
1	K_1	K_A
0	K_0	

K_0 – K_9: 10 HISTOGRAM WINDOWS

Step	Key	Code	Display x	Display y	Display z
30	y→( )	40			
31	b	14			
32	CLEAR x	37			
33	↑	27			
34	ENTER EXP	26			
35	9	11			
36	+	33			
37	y→( )	40			
38	−	34			
39	4	04			
3a	−	34			
3b	0	00			
3c	↑	27			
3d	y⇄( )	24			
40	0	00			
41	0	00			
42	0	00			
43	0	00			
44	0	00			
45	0	00			
46	0	00			
47	0	00			
48	0	00			
49	0	00			
4a	0	00			
4b	0	00			
4c	0	00			
4d	0	00			
50	y→( )	40			
51	−	34			
52	1	01			
53	b	14			
54	÷	35			
55	ENTER EXP	26			
56	2	02			
57	×	36			
58	1	01			
59	ROLL ↑	22			
5a	+	33			
5b	x←( )	67			
5c	−	34			
5d	1	01			

Step	Key	Code	Display x	Display y	Display z
60	x⇄y	30	DISPLAY		
61	STOP	41	GROUP	K_A	%
62	CONT	47			
63	ROLL ↑	22			
64	↑	27			
65	7	07			
66	5	05			
67	×	36			
68	ROLL ↑	22			
69	x⇄y	30			
6a	1	01			
6b	−	34			
6c	5	05			
6d	0	00			
70	0	00			
71	×	36			
72	↓	25			
73	FMT	42			
74	↓	25			
75	↑	27			
76	5	05			
77	0	00			
78	0	00			
79	+	33			
7a	↓	25			
7b	FMT	42			
7c	↓	25			
7d	x⇄y	30			
80	0	00			
81	x⇄y	30			
82	FMT	42			
83	↓	25			
84	↑	27			
85	5	05			
86	0	00			
87	0	00			
88	÷	35			
89	1	01			
8a	0	00			
8b	IF x>y	53			
8c	3	03			
8d	4	04			

Step	Key	Code	Display x	Display y	Display z
90	0	00			
91	↑	27			
92	FMT	42			
93	↓	25			
94	5	05			
95	ENTER EXP	26			
96	3	03			
97	x⇄y	30			
98	FMT	42			
99	↓	25			
9a	↑	27			
9b	FMT	42			
9c	↓	25			
9d	RCL	61			
a0	↑	27			
a1	b	14			
a2	÷	35			
a3	y→( )	40			
a4	f	15			
a5	×	36			
a6	f	15			
a7	×	36			
a8	↓	25			
a9	−	34			
aa	b	14			
ab	÷	35			
ac	f	15			
ad	↑	27			

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
b 0	0	00	DISPLAY		
(−) 1	STOP	41	0	M_x	σ_x^2
2	CONT	47			
3	a	13			
4	÷	35			
5	x⇄y	30			
6	ROLL ↓	31			
7	÷	35			
8	÷	35			
9	↓	25			
a	x⇄y	30			
b	x→( )	23			
c	f	15			
d	y→( )	40			
c 0	e	12			
(−) 1	↑	27			
2	1	01	DISPLAY		
3	STOP	41	1	M_h	σ_h^2
4	CONT	47			
5	END	46			

Storage

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

Step	Key	Code	x	y	z	Step	Key	Code	x	y	z
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HISTOGRAM GENERATION (WITHOUT PLOT)

This program generates a histogram table of ten windows given a data set of positive numbers. In addition, it determines the mean (M_x) and the variance ($\sigma_x 2$) of the raw data, and the mean (M_h) and variance ($\sigma_h 2$) of the normalized histogram data. Since the raw data is normalized by the program to values $0 \leq h \leq 10$, the new mean and variance are given by

$$M_h = \frac{M_x}{W}$$

$$\sigma_h^2 = \frac{\sigma_x^2}{W^2}$$

where W is the histogram window width (normalization factor). The window width W is chosen such that all normalized data entries X/W will lie between 0 and 10. Thus, if the data ranges from 0 → 200, a W of 20 would be proper.

This program uses Indirect Addressing. The (+) registers are used for storage whereas the (—) registers are used for program steps.

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address is (-0)(0))

PRESS: GO TO (-) (0) (0)

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: W → X

PRESS: CONTINUE

→ DISPLAY

N	Z
N	Y
1	X

ENTER DATA: X → X
PRESS: CONTINUE

Has all data been entered? NO (return to DISPLAY) / YES

PRESS: SET FLAG

PRESS: CONTINUE

→ DISPLAY

%	Z	(% of data points in Group)
K_A	Y	(Number of data points in Group)
Group	X	(Group #)

PRESS: CONTINUE

Is Group less than 10? YES (return to DISPLAY) / NO

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

σ_x^2	Z
M_x	Y
0	X

PRESS: CONTINUE

DISPLAY

σ_h^2	Z
M_h	Y
1	X

PRESS: GO TO
PRESS: —
PRESS: 0
PRESS: 0

} To consider another set of data.

EXAMPLE

The data set is:

104, 92, 83, 78, 58, 135, 146, 24, 74, 85, 81, 128, 140, 113, 79, 78, 53, 42, 34, 85, 96, 110, 133, 158, 171, 108, 84, 90, 73, 11, 51, 118, 68, 139, 92, 109, 89, 124, 91, 116.

The data varies between 0 and 200 so W is chosen to be 20.

Result

Group	K_A	%
1	1	2.5
2	2	5
3	4	10
4	6	15
5	11	27.5
6	7	17.5
7	5	12.5
8	3	7.5
9	1	2.5
10	0	0

N = 40

$\sigma_x^2 = 1252.644$ $\quad \sigma_h^2 = 3.132$

$M_x = 93.575$ $\quad M_h = 4.679$

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(–) 1	x→( )	23			
2	0	00			
3	x→( )	23			
4	1	01			
5	x→( )	23			
6	2	02			
7	x→( )	23			
8	3	03			
9	x→( )	23			
a	4	04			
b	x→( )	23			
c	5	05			
d	x→( )	23			
1 0	6	06			
(–) 1	x→( )	23			
2	7	07			
3	x→( )	23			
4	8	10			
5	x→( )	23			
6	9	11	ENTER		
7	STOP	41	W	0	0
8	x→( )	23			
9	a	13			
a	1	01			
b	CHG SIGN	32			
c	ROLL ↓	31			
d	↓	25			
2 0	1	01			
(–) 1	+	33			
2	↑	27			
3	↓	25			
4	CONT	47	ENTER		
5	STOP	41	X	N	N
6	IF FLAG	43			
7	5	05			
8	a	13			
9	↑	27			
a	×	36			
b	ACC +	60			
c	x⇄y	30			
d	a	13			

Step	Key	Code	Display x	Display y	Display z
3 0	÷	35			
(–) 1	↓	25			
2	int x	64			
3	↑	27			
4	ENTER EXP	26			
5	9	11			
6	+	33			
7	y→( )	40			
8	–	34			
9	4	04			
a	CLEAR x	37			
b	CLEAR x	37			
c	CLEAR x	37			
d	y⇄( )	24			
4 0	0	00			
(–) 1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	0	00			
5 0	IF FLAG	43			
(–) 1	1	01			
2	d	17			
3	CLEAR x	37			
4	1	01			
5	+	33			
6	SET FLAG	54			
7	GO TO ( )	44			
8	3	03			
9	d	17			
a	y→( )	40			
b	b	14			
c	CLEAR x	37			
d	↑	27			

Step	Key	Code	Display x	Display y	Display z
6 0	ENTER EXP	26			
(–) 1	9	11			
2	+	33			
3	y→( )	40			
4	–	34			
5	7	07			
6	–	34			
7	0	00			
8	↑	27			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	y⇄( )	24			
7 0	0	00			
(–) 1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	0	00			
a	0	00			
b	0	00			
c	0	00			
d	0	00			

Storage

Register	+	–
f	ΣX	
e	ΣX^2	
d		
c	K_A	
b	N	
a	W	
9	K_9	
8	K_8	
7	K_7	
6	K_6	
5	K_5	
4	K_4	
3	K_3	
2	K_2	
1	K_1	
0	K_0	

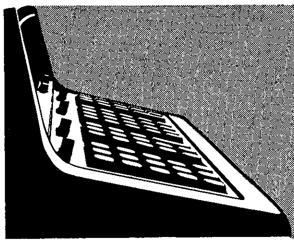
Step	Key	Code	Display x	Display y	Display z
8 0	y→( )	40			
(–) 1	c	16			
2	b	14			
3	÷	35			
4	ENTER EXP	26			
5	2	02			
6	×	36			
7	1	01			
8	ROLL ↑	22			
9	+	33			
a	x←( )	67			
b	c	16			
c	x⇄y	30	DISPLAY		
d	STOP	41	GROUP	K	%
9 0	CONT	47			
(–) 1	↑	27			
2	1	01			
3	0	00			
4	IF x>y	53			
5	6	06			
6	0	00			
7	RCL	61			
8	↑	27			
9	b	14			
a	÷	35			
b	y→( )	40			
c	f	15			
d	×	36			
a 0	f	15			
(–) 1	×	36			
2	↓	25			
3	−	34			
4	b	14			
5	÷	35			
6	f	15			
7	↑	27			
8	0	00	DISPLAY		
9	STOP	41	0	M_x	σ_x^2
a	CONT	47			
b	a	13			
c	÷	35			
d	x⇄y	30			

Step	Key	Code	Display x	Display y	Display z
b 0	ROLL ↓	31			
(–) 1	÷	35			
2	÷	35			
3	↓	25			
4	x⇄y	30			
5	↑	27			
6	1	01	DISPLAY		
7	STOP	41	1	M_h	σ_h^2
8	CONT	47			
9	GO TO ( )	44			
a	−	34			
b	0	00			
c	0	00			
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



F-DISTRIBUTION

This program evaluates the integral of the F distribution density function

$$Q = \int_F^{\infty} \frac{\Gamma\left(\frac{V_1 + V_2}{2}\right) \quad X^{V_1/2 - 1} \left(\frac{V_1}{V_2}\right)^{V_1/2}}{\Gamma\left(\frac{V_1}{2}\right) \quad \Gamma\left(\frac{V_2}{2}\right)\left(1 + \frac{V_1}{V_2} X\right)^{\frac{V_1+V_2}{2}}} \, dx$$

for given values of F, V_1, V_2 .

The integral is evaluated by means of the following series:

V_2 EVEN

$$Q \ (F / V_1, V_2) = 1 - (1 - X)^{V_1/2} \left[1 + \frac{V_1 X}{2} + \ldots + \frac{V_1 \ (V_1 + 2) \ldots (V_2 + V_1 - 4)}{2 \cdot 4 \ldots (V_2 - 2)} X^{\frac{V_2 - 2}{2}} \right]$$

V_2 ODD

$$Q \ (F / V_1, V_2) = X^{V_2/2} \left[1 + \frac{V_2}{2} (1 - X) + \ldots + \frac{V_2(V_2 + 2) \ldots (V_2 + V_1 - 4)}{2 \cdot 4 \ldots (V_1 - 2)} (1 - X)^{\frac{V_1 - 2}{2}} \right]$$

V_1 and V_2 both odd

$$Q(F/V_1, V_2) = 1 - A + B$$

$$A = \begin{cases} \frac{2}{\pi} \left\{ \theta + \sin\theta \cos\theta \left[1 + 2/3 \cos^2\theta + \ldots + \frac{2 \cdot 4 \ldots (V_2 - 3)}{3 \cdot 5 \ldots (V_2 - 2)} \cos^{V_2 - 2} \theta \right] \right\} & V_2 > 1 \\ \frac{2\theta}{\pi} & V_2 = 1 \end{cases}$$

$$\theta = \text{Arc Tan} \sqrt{\frac{F}{V_2}}$$

$$B = \begin{cases} \frac{2 \left(\frac{V_2 - 1}{2}\right)!}{\sqrt{\pi} \left(\frac{V_2 - 2}{2}\right)!} \sin\theta_1 \cdot \cos^{V_2} \theta_1 \left\{ 1 + \frac{V_2 + 1}{3} \sin^2\theta_1 + \ldots + \frac{(V_2 + 1)(V_2 + 3) \ldots (V_2 + V_2 - 4)}{(3)(5) \ldots (V_1 - 2)} \cdot \sin^{V_2 - 3} \theta_1 \right\} & V_1 > 1 \\ = 0 & V_1 = 1 \end{cases}$$

$$\theta_1 = \text{Arc Tan} \sqrt{\frac{V_1 F}{V_2}}$$

Reference: Handbook of Mathematical Functions, Abromowitz and Stegun, National Bureau of Standards (1964)

USER INSTRUCTIONS	EXAMPLE

SET: ☐ RADIANS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: END

PRESS: CONTINUE

ENTER DATA: F ratio → Z
V_1 for numerator → Y
V_2 for denominator → X

PRESS: CONTINUE

DISPLAY

$Q(F/V_1, V_2)$	Z
0	Y
0	X

To calculate significance level for new data.

General Form $Q(F/V_1, V_2)$

$Q(4.21/7, 6) = 0.05$

$Q(11.4/4, 5) = 0.01$

$Q(3.79/7, 7) = .05$

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20	ENTER		
1	STOP	41	V_2	V_1	F
2	x→( )	23			
3	f	15			
4	y→( )	40			
5	e	12			
6	ROLL ↓	31			
7	y→( )	40			
8	b	14			
9	×	36			
a	↓	25			
b	+	33			
c	↓	25			
d	÷	35			
1 0	f	15			
1	↑	27			
2	1	01			
3	x→( )	23			
4	c	16			
5	2	02			
6	÷	35			
7	↓	25			
8	↑	27			
9	int x	64			
a	IF x = y	50			
b	3	03			
c	4	04			
d	e	12			
2 0	x⇄y	30			
1	2	02			
2	÷	35			
3	↓	25			
4	↑	27			
5	int x	64			
6	IF x < y	52			
7	7	07			
8	d	17			
9	SET FLAG	54			
a	RCL	61			
b	ACC −	63			
c	x⇄y	30			
d	ACC +	60			

Step	Key	Code	Display x	Display y	Display z
3 0	1	01			
1	ROLL ↑	22			
2	−	34			
3	↑	27			
4	↓	25			
5	y→( )	40			
6	d	17			
7	e	12			
8	↑	27			
9	4	04			
a	−	34			
b	f	15			
c	+	33			
d	↑	27			
4 0	2	02			
1	−	34			
2	CLEAR x	37			
3	IF x = y	50			
4	5	05			
5	b	14			
6	y→( )	40			
7	f	15			
8	↓	25			
9	÷	35			
a	d	17			
b	×	36			
c	c	16			
d	×	36			
5 0	1	01			
1	+	33			
2	y→( )	40			
3	c	16			
4	↓	25			
5	2	02			
6	−	34			
7	f	15			
8	GO TO ( )	44			
9	3	03			
a	d	17			
b	c	16			
c	↑	27			
d	1	01			

Step	Key	Code	Display x	Display y	Display z
6 0	↑	27			
1	d	17			
2	CHG SIGN	32			
3	+	33			
4	↓	25			
5	ln x	65			
6	↑	27			
7	e	12			
8	×	36			
9	2	02			
a	÷	35			
b	↓	25			
c	e^x	74			
d	×	36			
7 0	IF FLAG	43			
1	7	07			
2	6	06			
3	1	01			
4	x⇄y	30			
5	−	34			
6	CLEAR x	37			
7	↑	27	DISPLAY		
8	STOP	41	0	0	Q
9	CONT	47			
a	GO TO ( )	44			
b	0	00			
c	0	00			
d	f	15			

Storage

Register	+	−
f	V_2	
e	V_1	
d		
c		
b	F	
a		
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
8 0	↑	27			
(+) 1	2	02			
2	÷	35			
3	↓	25			
4	int x	64			
5	↑	27			
6	↑	27			
7	2	02			
8	×	36			
9	ln x	65			
a	ROLL ↑	22			
b	×	36			
c	↓	25			
d	e^x	74			
9 0	x→( )	23			
(+) 1	c	16			
2	1	01			
3	−	34			
4	↑	27			
5	↓	25			
6	2	02			
7	−	34			
8	ROLL ↓	31			
9	×	36			
a	|y|	55			
b	ROLL ↑	22			
c	IF x<y	52			
d	9	11			
a 0	7	07			
(+) 1	↓	25			
2	π	56			
3	×	36			
4	c	16			
5	÷	35			
6	2	02			
7	÷	35			
8	y→( )	40			
9	c	16			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			

Step	Key	Code	Display x	Display y	Display z
0 0	y⇄( )	24			
(−) 1	b	14			
2	f	15			
3	÷	35			
4	e	12			
5	×	36			
6	ROLL ↓	31			
7	$\sqrt{x}$	76			
8	arc ▾	72			
9	tan x	71			
a	↑	27			
b	sin x	70			
c	CONT	47			
d	y→( )	40			
1 0	−	34			
(−) 1	f	15			
2	↑	27			
3	×	36			
4	y→( )	40			
5	d	17			
6	RCL	61			
7	1	01			
8	IF x=y	50			
9	7	07			
a	3	03			
b	x→( )	23			
c	b	14			
d	f	15			
2 0	+	33			
(−) 1	4	04			
2	−	34			
3	e	12			
4	↑	27			
5	2	02			
6	−	34			
7	IF x>y	53			
8	4	04			
9	5	05			
a	y→( )	40			
b	−	34			
c	e	12			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
3 0	÷	35			
(−) 1	d	17			
2	×	36			
3	b	14			
4	×	36			
5	1	01			
6	+	33			
7	y→( )	40			
8	b	14			
9	↓	25			
a	2	02			
b	−	34			
c	↑	27			
d	y⇄( )	24			
4 0	−	34			
(−) 1	e	12			
2	GO TO ( )	44			
3	2	02			
4	6	06			
5	IF FLAG	43			
6	9	11			
7	5	05			
8	ROLL ↑	22			
9	x⇄y	30			
a	3	03			
b	IF x>y	53			
c	5	05			
d	c	16			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
5 0	2	02				8 0	↑	27			
(–) 1	÷	35				(–) 1	×	36			
2	↑	27				2	y→( )	40			
3	↓	25				3	d	17			
4	1	01				4	f	15			
5	—	34				5	↑	27			
6	ROLL ↓	31				6	1	01			
7	×	36				7	y→( )	40			
8	ROLL ↑	22				8	—	34			
9	IF x<y	52				9	e	12			
a	5	05				a	x→( )	23			
b	5	05				b	b	14			
c	ROLL ↓	31				c	SET FLAG	54			
d	c	16				d	—	34			
6 0	÷	35				9 0	IF x<y	52			
(–) 1	1	01				(–) 1	3	03			
2	↑	27				2	a	13			
3	d	17				3	CLEAR x	37			
4	—	34				4	x→( )	23			
5	√x	76				5	b	14			
6	ROLL ↑	22				6	↑	27			
7	×	36				7	d	17			
8	ROLL ↑	22				8	√x	76			
9	√x	76				9	×	36			
a	ln x	65				a	e	12			
b	x⇄y	30				b	sin x	70			
c	f	15				c	×	36			
d	×	36				d	e	12			
7 0	ROLL ↓	31				a 0	+	33			
(–) 1	e^x	74				(–) 1	2	02			
2	×	36				2	×	36			
3	b	14				3	π	56			
4	×	36				4	÷	35			
5	y→( )	40				5	c	16			
6	c	16				6	x⇄y	30			
7	y⇄( )	24				7	—	34			
8	—	34				8	1	01			
9	f	15				9	+	33			
a	y→( )	40				a	CLEAR x	37			
b	e	12				b	↑	27			
c	↓	25				c	STOP	41			
d	cos x	73				d	END	46			

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

HEWLETT · PACKARD

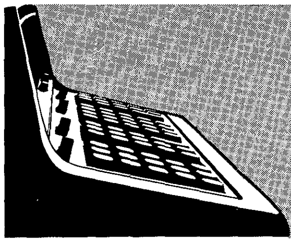
HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	



9100B ONLY
PART NO.
09100-70909

TWO WAY ANALYSIS OF VARIANCE (m x 4)

This program analyzes the total statistical variance in a table of data by separating the total variance into two parts, the variance among rows of data, and the variance between columns of data, and comparing each to the variance due to random influence. In a table of four columns and m rows it calculates the variance ratio between columns.

$$F_c = \frac{m \sum_{j=1}^{4} (\bar{X}_j - \bar{X})^2 \Big/ 3}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{X})^2 \Big/ (m-1)(3)}$$

with V_1 = 3 degrees of freedom

and V_2 = 3 (m - 1) degrees of freedom

and the variance ratio between rows:

$$F_r = \frac{4 \sum_{i=1}^{m} (\bar{X}_i - \bar{X})^2 \Big/ (m-1)}{\sum_{j=1}^{4} \sum_{i=1}^{m} (X_{ij} - \bar{X}_j - \bar{X}_i + \bar{X})^2 \Big/ 3(m-1)}$$

with V_1 = m - 1 degrees of freedom
V_2 = 3 (m - 1) degrees of freedom

where:

$$\bar{X}_j = \frac{1}{m} \sum_{i=1}^{m} X_{ij} \qquad \bar{X}_i = \frac{1}{4} \sum_{j=1}^{4} X_{ij}$$

$$\bar{X} = \frac{1}{4m} \sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}$$

V_1 = degrees of freedom in numerator
V_2 = degrees of freedom in denominator

The equations used by the program are:

$$F_r = \frac{4(m-1)\left\{\frac{1}{4} \sum_{i=1}^{m} \left[\sum_{j=1}^{4} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}{3\left\{\sum_{j=1}^{4} \sum_{i=1}^{m} X^2_{ij} - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m} \sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4} \sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

$$F_c = \frac{3m\left\{\frac{1}{m} \sum_{j=1}^{4} \left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}{(m-1)\left\{\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}^2 - \frac{1}{4m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{m} \sum_{j=1}^{4}\left[\sum_{i=1}^{m} X_{ij}\right]^2 - \frac{1}{4} \sum_{i=1}^{m}\left[\sum_{j=1}^{4} X_{ij}\right]^2 + \frac{1}{2m}\left[\sum_{j=1}^{4} \sum_{i=1}^{m} X_{ij}\right]^2\right\}}$$

Reference: Freund, John E., Mathematical Statistics, Prentice Hall (1962)

USER INSTRUCTIONS

	j Columns 1	2	3	4
i Rows 1				
2				
.	DATA TABLE			
.				
.				
m				

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: END

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: m → X number of rows in data table

PRESS: CONTINUE

→ DISPLAY

0	Z
i	Y
j	X

ENTER DATA: X_{ij} → X j = column, i = row (data entered row by row)

PRESS: CONTINUE

X_{14} entered?

NO → (return to DISPLAY)

YES ↓

USER INSTRUCTIONS (Con't)

DISPLAY

F_c	Z	F ratio between columns
V_1	Y	V_1 degrees of freedom in numerator
V_2	X	V_2 degrees of freedom in denominator

PRESS: CONTINUE

DISPLAY

F_r	Z	F ratio between rows
V_1	Y	V_1 degrees of freedom in numerator
V_2	X	V_2 degrees of freedom in denominator

To re-run program:

← PRESS: END

EXAMPLE

General form

	j Columns 1	2	3	4
i Rows 1	X_{11}	. . .		X_{14}
2	.			
.	.	X_{ij}		
.	.			
.				
m	X_{m1}	. . .		X_{m4}

	Columns			
Rows	58.2	49.1	60.1	75.8
	56.2	54.1	70.9	58.2
	65.3	51.6	39.2	48.7

$F_c = 0.43$ $V_1 = 3$ $V_2 = 6$

$F_r = 0.92$ $V_1 = 2$ $V_2 = 6$

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	x→( )	23			
2	a	13			
3	x→( )	23			
4	b	14			
5	x→( )	23			
6	c	16			
7	x→( )	23			
8	9	11			
9	x→( )	23			
a	8	10	ENTER		
b	STOP	41	m	0	0
c	ROLL ↑	22			
d	y→( )	40			
1 0	d	17			
(+) 1	1	01	ENTER		
2	STOP	41	X_{1j}	0	0
3	x→( )	23			
4	7	07			
5	y⇄( )	24			
6	a	13			
7	+	33			
8	y⇄( )	24			
9	a	13			
a	↑	27			
b	×	36			
c	ACC +	60			
d	CLEAR x	37			
2 0	ROLL ↓	31			
(+) 1	2	02	ENTER		
2	STOP	41	X_{2j}	0	0
3	y⇄( )	24			
4	7	07			
5	+	33			
6	y⇄( )	24			
7	7	07			
8	y⇄( )	24			
9	b	14			
a	+	33			
b	y⇄( )	24			
c	b	14			
d	↑	27			
3 0	×	36			
(+) 1	ACC +	60			
2	CLEAR x	37			
3	ROLL ↓	31			
4	3	03	ENTER		
5	STOP	41	X_{3j}	0	0
6	y⇄( )	24			
7	7	07			
8	+	33			
9	y⇄( )	24			
a	7	07			
b	y⇄( )	24			
c	c	16			
d	+	33			
4 0	y⇄( )	24			
(+) 1	c	16			
2	↑	27			
3	×	36			
4	ACC +	60			
5	CLEAR x	37			
6	ROLL ↓	31			
7	4	04	ENTER		
8	STOP	41	X_{4j}	0	0
9	y⇄( )	24			
a	7	07			
b	+	33			
c	y⇄( )	24			
d	7	07			
5 0	y⇄( )	24			
(+) 1	8	10			
2	+	33			
3	y⇄( )	24			
4	8	10			
5	↑	27			
6	×	36			
7	ACC +	60			
8	y⇄( )	24			
9	7	07			
a	↓	25			
b	y⇄( )	24			
c	9	11			
d	↑	27			
6 0	×	36			
(+) 1	CLEAR x	37			
2	ROLL ↓	31			
3	+	33			
4	y⇄( )	24			
5	9	11			
6	1	01			
7	−	34			
8	CLEAR x	37			
9	GO TO ( )	44			
a	−	34			
b	0	00			
c	0	00			
d					
0 0	IF x<y	52			
(−) 1	6	06			
2	3	03			
3	RCL	61			
4	ACC −	63			
5	↑	27			
6	×	36			
7	4	04			
8	÷	35			
9	d	17			
a	÷	35			
b	↓	25			
c	−	34			
d	ACC +	60			

Storage

Register	Contents
f	
e	
d	M
c	
b	
a	ΣX_{ij}
9	
8	
7	ΣX_{ij}
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
10	a	13			
(−) 11	↑	27			
12	×	36			
13	b	14			
14	↑	27			
15	×	36			
16	↓	25			
17	+	33			
18	c	16			
19	↑	27			
1a	×	36			
1b	↓	25			
1c	+	33			
1d	↑	27			
20	y⇄( )	24			
(−) 21	8	10			
22	↓	25			
23	↑	27			
24	×	36			
25	↓	25			
26	+	33			
27	d	17			
28	÷	35			
29	f	15			
2a	−	34			
2b	↑	27			
2c	y⇄( )	24			
2d	9	11			
30	4	04			
(−) 31	÷	35			
32	f	15			
33	−	34			
34	y→( )	40			
35	a	13			
36	e	12			
37	ROLL ↑	22			
38	−	34			
39	x⇄y	30			
3a	ROLL ↑	22			
3b	−	34			
3c	y→( )	40			
3d	b	14			
40	↓	25			
(−) 41	÷	35			
42	d	17			
43	↑	27			
44	1	01			
45	−	34			
46	↓	25			
47	x→( )	23			
48	d	17			
49	×	36			
4a	↑	27			
4b	3	03			
4c	×	36			
4d	x⇄y	30	DISPLAY		
50	STOP	41	V_2	V_1	$F_{col.}$
(−) 51	CONT	47			
52	a	13			
53	↑	27			
54	b	14			
55	÷	35			
56	3	03			
57	×	36			
58	↑	27			
59	d	17			
5a	×	36			
5b	x⇄y	30	DISPLAY		
5c	STOP	41	V_2	V_1	F_{row}
5d	CONT	47			
60	GO TO ( )	44			
(−) 61	6	06			
62	7	07			
63	GO TO ( )	44			
64	+	33			
65	1	01			
66	1	01			
67	END	46			

Storage

TWO WAY ANALYSIS OF VARIANCE WITH REPLICATES
(THREE WAY)

This program analyzes the total statistical variance in a table of data by separating the total variance into three parts, the variance among rows, the variance between columns, and the variance due to interactions.

Computational **Equations**

$$SST = \sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr}^2 - \frac{1}{knm}\left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr}\right]^2$$

$$SSA = \frac{1}{nm} \sum_{i}^{k} \left[\sum_{j}^{n} \sum_{r}^{m} X_{ijr}\right]^2 - \frac{1}{knm}\left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr}\right]^2$$

$$SSB = \frac{1}{km} \sum_{j}^{n} \left[\sum_{r}^{m} \sum_{i}^{k} X_{ijr}\right]^2 - \frac{1}{knm}\left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr}\right]^2$$

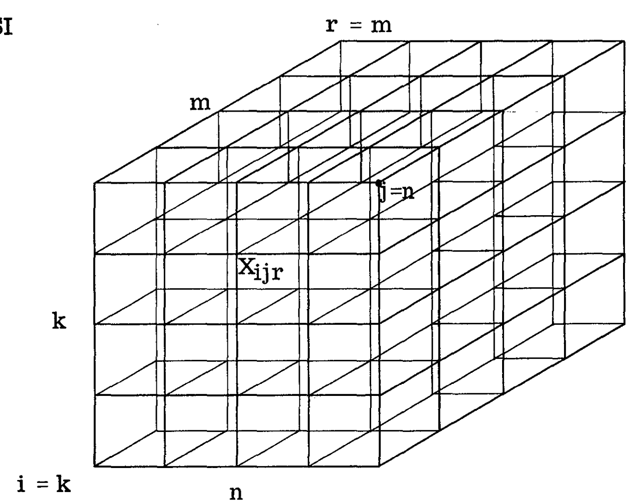
$$SSI = \frac{1}{m} \sum_{i}^{k} \sum_{j}^{n} \left[\sum_{r}^{m} X_{ijr}\right]^2 - \frac{1}{nm} \sum_{i}^{k} \left[\sum_{j}^{n} \sum_{r}^{m} X_{ijr}\right]^2 - \frac{1}{km} \sum_{j}^{n}\left[\sum_{i}^{k} \sum_{r}^{m} X_{ijr}\right]^2 + \frac{1}{kmn}\left[\sum_{i}^{k} \sum_{j}^{n} \sum_{r}^{m} X_{ijr}\right]^2$$

$$F_R = \frac{SSA}{(k-1)} \quad \frac{kn(m-1)}{SSE}$$

$$F_C = \frac{SSB}{(n-1)} \quad \frac{kn(m-1)}{SSE}$$

$$F_I = \frac{SSI}{(k-1)(n-1)} \cdot \frac{kn(m-1)}{SSE}$$

$$SSE = SST - SSA - SSB - SSI$$



Reference: Mathematical Statistics, John E. Freund, Prentice Hall, 1963

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM 1: Side A followed by Side B

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: rows k → Z, columns n → Y, repetitions m → X

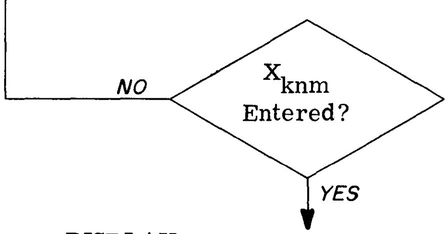
PRESS: CONTINUE

→ DISPLAY

i	Z	row
j	Y	column
r	X	repitition

ENTER DATA: X_{ijr} → X

PRESS: CONTINUE



DISPLAY

?*)(?	Z
0	Y
0	X

ENTER PROGRAM 2: Starting Address is (–)(0)(0)

PRESS: GO TO

PRESS: —

PRESS: 0

PRESS: 0

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

F_{row}	Z
V_1	Y
V_2	X

PRESS: CONTINUE

DISPLAY

F_{column}	Z
V_1	Y
V_2	X

PRESS: CONTINUE

DISPLAY

$F_{interaction}$	Z
V_1	Y
V_2	X

EXAMPLE

n = 3 columns; k = 4 rows; m = 2 repitions

Row 1	58.2	56.2	65.3
	52.6	41.2	60.8
Row 2	49.1	54.1	51.6
	42.8	50.5	48.4
Row 3	60.1	70.9	39.2
	58.3	73.2	40.7
Row 4	75.8	58.2	48.7
	71.5	51.0	41.4

Results:

F_{row} = 4.42 V_1 = 3 V_2 = 12

F_{column} = 9.39 V_1 = 2 V_2 = 12

$F_{interaction}$ = 14.93 V_1 = 6 V_2 = 12

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20	ENTER		
(+) 01	STOP	41	k	n	m
02	x→( )	23			
03	d	17			
04	y→( )	40			
05	c	16			
06	↓	25			
07	y→( )	40			
08	b	14			
09	CLEAR	20			
0a	x→( )	23			
0b	a	13			
0c	x→( )	23			
0d	9	11			
10	x→( )	23			
(+) 11	8	10			
12	x→( )	23			
13	7	07			
14	x→( )	23			
15	6	06			
16	x→( )	23			
17	5	05			
18	x→( )	23			
19	4	04			
1a	x→( )	23			
1b	3	03			
1c	x→( )	23			
1d	−	34			
20	f	15			
(+) 21	x→( )	23			
22	−	34			
23	e	12			
24	x→( )	23			
25	1	01			
26	x→( )	23			
27	0	00			
28	GO TO ( )	44			
29	−	34			
2a	0	00			
2b	0	00			
2c					
2d					

Step	Key	Code	Display x	Display y	Display z
00	x→( )	23			
(−) 01	2	02			
02	1	01			
03	+	33			
04	y→( )	40			
05	9	11			
06	GO TO ( )	44			
07	▲SUB▼	77			
08	9	11			
09	d	17			
0a	y⇄( )	24			
0b	5	05			
0c	+	33			
0d	y⇄( )	24			
10	5	05			
(−) 11	GO TO ( )	44			
12	▲SUB▼	77			
13	9	11			
14	d	17			
15	y⇄( )	24			
16	4	04			
17	+	33			
18	y⇄( )	24			
19	4	04			
1a	GO TO ( )	44			
1b	▲SUB▼	77			
1c	9	11			
1d	d	17			
20	y⇄( )	24			
(−) 21	3	03			
22	+	33			
23	y⇄( )	24			
24	3	03			
25	GO TO ( )	44			
26	▲SUB▼	77			
27	9	11			
28	d	17			
29	y⇄( )	24			
2a	2	02			
2b	+	33			
2c	y⇄( )	24			
2d	2	02			

Step	Key	Code	Display x	Display y	Display z
30	GO TO ( )	44			
(−) 31	▲SUB▼	77			
32	9	11			
33	d	17			
34	y⇄( )	24			
35	8	10			
36	+	33			
37	y→( )	40			
38	8	10			
39	GO TO ( )	44			
3a	▲SUB▼	77			
3b	9	11			
3c	d	17			
3d	y⇄( )	24			
40	−	34			
(−) 41	e	12			
42	+	33			
43	y⇄( )	24			
44	−	34			
45	e	12			
46	GO TO ( )	44			
47	▲SUB▼	77			
48	9	11			
49	d	17			
4a	y⇄( )	24			
4b	−	34			
4c	f	15			
4d	+	33			

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
5 0	y⇄( )	24			
5 1	−	34			
5 2	f	15			
5 3	x←( )	67			
5 4	1	01			
5 5	↑	27			
5 6	×	36			
5 7	x←( )	67			
5 8	0	00			
5 9	+	33			
5 a	y→( )	40			
5 b	0	00			
5 c	CLEAR x	37			
5 d	x→( )	23			
6 0	1	01			
6 1	ROLL ↓	31			
6 2	y⇄( )	24			
6 3	9	11			
6 4	b	14			
6 5	IF x>y	53			
6 6	0	00			
6 7	2	02			
6 8	x←( )	67			
6 9	5	05			
6 a	↑	27			
6 b	×	36			
6 c	x←( )	67			
6 d	4	04			
7 0	↑	27			
7 1	×	36			
7 2	↓	25			
7 3	+	33			
7 4	x←( )	67			
7 5	3	03			
7 6	↑	27			
7 7	×	36			
7 8	↓	25			
7 9	+	33			
7 a	x←( )	67			
7 b	2	02			
7 c	↑	27			
7 d	×	36			
8 0	↓	25			
8 1	+	33			
8 2	x←( )	67			
8 3	8	10			
8 4	↑	27			
8 5	×	36			
8 6	↓	25			
8 7	+	33			
8 8	x←( )	67			
8 9	−	34			
8 a	e	12			
8 b	↑	27			
8 c	×	36			
8 d	↓	25			
9 0	+	33			
9 1	x←( )	67			
9 2	−	34			
9 3	f	15			
9 4	↑	27			
9 5	×	36			
9 6	↓	25			
9 7	+	33			
9 8	y→( )	40			
9 9	1	01			
9 a	CLEAR x	37			
9 b	↑	27	ENTER		
9 c	STOP	41	Program 2		
9 d	↓	25			
a 0	c	16			
a 1	IF x=y	50			
a 2	5	05			
a 3	3	03			
a 4	1	01			
a 5	+	33			
a 6	a	13			
a 7	↑	27			
a 8	d	17			
a 9	IF x=y	50			
a a	c	16			
a b	5	05			
a c	1	01			
a d	+	33			
b 0	y→( )	40			
b 1	a	13			
b 2	x←( )	67			
b 3	9	11			
b 4	ROLL ↓	31	ENTER		
b 5	STOP	41	X_{ijr}		
b 6	CONT	47			
b 7	↑	27			
b 8	×	36			
b 9	ACC +	60			
b a	y⇄( )	24			
b b	7	07			
b c	+	33			
b d	y⇄( )	24			
c 0	7	07			
c 1	↓	25			
c 2	GO TO ( )	44			
c 3	a	13			
c 4	6	06			
c 5	↓	25			
c 6	x←( )	67			
c 7	7	07			
c 8	y⇄( )	24			
c 9	1	01			
c a	+	33			
c b	y⇄( )	24			
c c	1	01			
c d	↑	27			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
d 0	×	36			
(−) 1	ROLL ↓	31			
2	y⇄( )	24			
3	6	06			
4	+	33			
5	y⇄( )	24			
6	6	06			
7	CLEAR x	37			
8	x→( )	23			
9	7	07			
a	x→( )	23			
b	a	13			
c	ROLL ↑	22			
d	RETURN	77			

Storage

Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
0 0	f	15			
(−) 1	↑	27			
2	×	36			
3	d	17			
4	÷	35			
5	c	16			
6	÷	35			
7	b	14			
8	÷	35			
9	e	12			
a	x⇄y	30			
b	−	34			
c	y→( )	40			
d	e	12			
1 0	x→( )	23			
(−) 1	f	15			
2	x←( )	67			
3	0	00			
4	↑	27			
5	c	16			
6	÷	35			
7	d	17			
8	÷	35			
9	f	15			
a	−	34			
b	y→( )	40			
c	5	05			
d	x←( )	67			
2 0	1	01			
(−) 1	↑	27			
2	b	14			
3	÷	35			
4	d	17			
5	÷	35			
6	f	15			
7	−	34			
8	y→( )	40			
9	4	04			
a	x←( )	67			
b	6	06			
c	↑	27			
d	d	17			

Step	Key	Code	Display x	Display y	Display z
3 0	÷	35			
(−) 1	x←( )	67			
2	5	05			
3	−	34			
4	x←( )	67			
5	1	01			
6	↑	27			
7	d	17			
8	÷	35			
9	b	14			
a	÷	35			
b	↓	25			
c	−	34			
d	y→( )	40			
4 0	3	03			
(−) 1	RCL	61			
2	x←( )	67			
3	5	05			
4	−	34			
5	x←( )	67			
6	4	04			
7	−	34			
8	x←( )	67			
9	3	03			
a	−	34			
b	d	17			
c	↑	27			
d	1	01			
5 0	−	34			
(−) 1	c	16			
2	×	36			
3	b	14			
4	×	36			
5	↓	25			
6	x⇄y	30			
7	y→( )	40			
8	d	17			
9	÷	35			
a	y→( )	40			
b	2	02			
c	x←( )	67			
d	5	05			

Step	Key	Code	Display x	Display y	Display z
6 0	×	36			
(−) 1	b	14			
2	↑	27			
3	1	01			
4	−	34			
5	y→( )	40			
6	b	14			
7	↓	25			
8	÷	35			
9	↑	27			
a	d	17	DISPLAY		
b	STOP	41	V_1	V_2	F_{row}
c	CONT	47			
d	CONT	47			
7 0	x←( )	67			
(−) 1	2	02			
2	↑	27			
3	x←( )	67			
4	4	04			
5	×	36			
6	c	16			
7	↑	27			
8	1	01			
9	−	34			
a	y→( )	40			
b	c	16			
c	↓	25			
d	÷	35			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
8 0	↑	27			
(−) 1	d	17	DISPLAY		
2	STOP	41	V_1	V_2	$F_{col.}$
3	CONT	47			
4	CONT	47			
5	x←( )	67			
6	2	02			
7	↑	27			
8	x←( )	67			
9	3	03			
a	×	36			
b	b	14			
c	↑	27			
d	c	16			
9 0	×	36			
(−) 1	↓	25			
2	÷	35			
3	↑	27			
4	d	17	DISPLAY		
5	STOP	41	V_1	V_2	F_{inter}
6	CONT	47			
7	CONT	47			
8	STOP	41			

Storage

9100B ELECTRONICS PROGRAM LISTING

71001 - TCHEBYSHEFF FILTER DESIGN
Calculates component values for Tchebysheff low pass filters with equal terminations.

71003 - BUTTERWORTH FILTER DESIGN
Calculates component values for Butterworth low pass filters between equal terminations.

71004 - MINIMUM LOSS PADS
Calculates resistive minimum loss pad and gives resistor values and loss in dB.

71005 - TCHEBYSHEFF EVALUATION
Used to determine filter order or the frequency response of a particular Tchebysheff filter.

71006 - ATTENUATOR PADS T OR Π
Calculates resistor values for either T or Π pads.

71007 - BAND PASS FILTER DESIGN
Calculates ideal component values and evaluates the frequency response by the image parameter method for a band pass filter.

71008 - STUB MATCHED TRANSMISSION LINE
Calculates the distance from a load to a point where a shorted stub is to be placed and the length of the stub to match a transmission line.

71009 - TRANSMISSION LINE
Calculates the impedance at any point on a transmission line either toward the generator or toward the load, the voltage reflection (magnitude and phase) and the VSWR on the line.

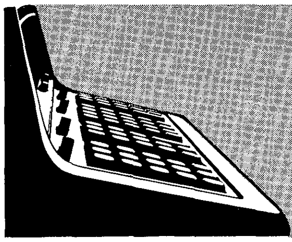
71010 - WYE →DELTA AND/OR DELTA →WYE CONVERSION
Transforms impedances wired in delta configuration to the equivalent wye configuration and vice-versa. Loop and nodal analyses are used to perform the transformations.

9100B ONLY

71501 - TCHEBYSHEFF FILTER DESIGN - FINITE TERMINATIONS
Calculates component values for Tchebysheff low pass filters with finite terminations (equal or unequal).

71502 - S PARAMETER TO Y PARAMETER CONVERSION
Converts S parameters for linear (active or passive) circuits to Y parameters.

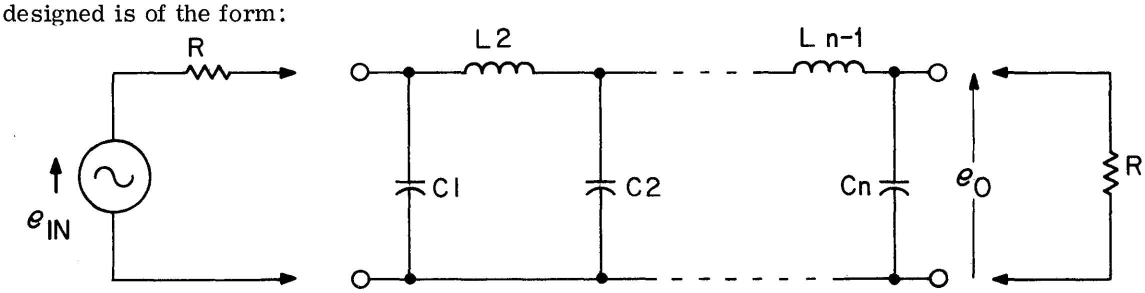
71503 - FREQUENCY RESPONSE FROM POLES AND ZEROES WITH PLOT
Given the zeroes and poles of a complex function f(s), the magnitude and phase response is computed over a specified frequency range. The program can consider any combination of six poles and zeroes of the form $r_i = \alpha + jw$.



PART NO.
09100-71001

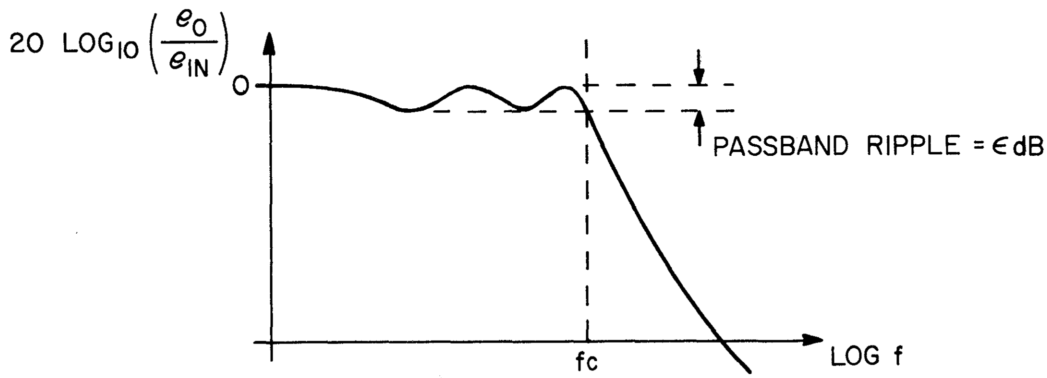
TCHEBYSHEFF FILTER DESIGN

This program computes component values for Tchebysheff low pass filters between equal terminations. The network designed is of the form:



The first element of the filter is a shunt C. The order (no. of poles) must be odd. (Even order requires unequal terminations for realizability.)

Frequency response of filter is:



User specifies: Filter Order n (must be odd)
Pass Band Ripple ϵ_{dB} (in dB)
Termination Resistance R (in ohms)
Corner ($-\epsilon_{dB}$) Frequency f_c (in Hz)

The equations used are:

$$G_1 = \frac{2a_1}{\gamma}$$

$$G_K = \frac{4a_{K-1}\, a_K}{b_{K-1}\, G_{K-1}} \qquad K = 2, 3, 4, \cdots n$$

where $$\gamma = \sinh\left[\frac{\ln\left(\coth \frac{\epsilon_{dB}}{40 \log_{10} e}\right)}{2n}\right]$$

$$a_K = \sin\left[\frac{(2K-1)\pi}{2n}\right] \quad K = 1, 2, 3, \cdots n \quad \text{and} \quad b_K = \gamma^2 + \sin^2\left(\frac{K\pi}{n}\right) \quad K = 1, 2, 3, \cdots n$$

$$C_K = \frac{G_K}{2\pi f_c R} \quad K = 1, 3, 5, \cdots n \quad \text{and} \quad L_K = \frac{RG_K}{2\pi f_c} \quad K = 2, 4, 6, \cdots (n-1)$$

Reference: A Handbook on Electrical Filters

White Electromagnetics, Inc. 1963

USER INSTRUCTIONS

SET [ ] [RADIANS] [FLOATING] [ ]

ENTER PROGRAM (Starting address is 0-0)

This program is self-destructive ie., each solution requires reloading the calculator.

PRESS: END

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: n → Y, ϵ_{dB} → X

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: R → Y, f_c → X

→ PRESS: CONTINUE

DISPLAY

(K odd)

0	——	Z
C_K	——	Y
K	——	X

(C_K is in farads)

(K even)

0	——	Z
L_K	——	Y
K	——	X

(L_K is in henries)

Displays component values until K = N. Next CONTINUE clears DISPLAY. The program steps must be reentered to enter another problem.

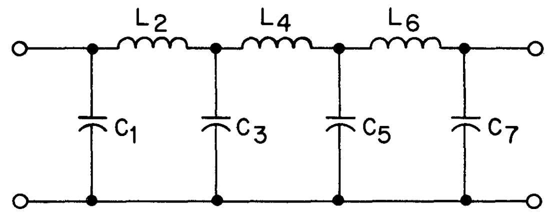
EXAMPLE

7th Order (n)

0.1 dB Ripple (ϵ_{dB})

50 Ω Impedance Level (R)

3.2 MHz Corner Frequency (f_c)



C_1 = 1175 pF

C_3 = 2086 pF

C_5 = 2086 pF

C_7 = 1175 pF

L_2 = 3.538 μH

L_4 = 3.913 μH

L_6 = 3.538 μH

FROM c-5 → step 0-3

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
0	0	GO TO ( )( )	44									
	1	a	13									
	2	0	00									
	3	CLEAR	20									
	4	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
	5	STOP	41	fc	R	0	ENTER R, f_c					
	6	y→( )	40	STORE R								
	7	f	15									
	8	x⇄y	30									
	9	2	02									
	a	×	36									
	b	π	56	CALCULATE $\omega_C = 2\pi fc$								
	c	×	36									
	d	y→( )	40									
1	0	e	12									
	1	↑	27									
	2	2	02									
	3	÷	35									
	4	d	17									
	5	÷	35									
	6	↓	25									
	7	sin x	70	CALCULATE G_1 = FIRST NORMALIZED COMPONENT VALUE								
	8	↑	27									
	9	2	02									
	a	×	36									
	b	a	13									
	c	÷	35									
	d	y→( )	40									
2	0	b	14									
	1	↑	27									
	2	×	36	REPLACE γ WITH γ^2								
	3	y→( )	40									
	4	a	13									
	5	CLEAR x	37									
	6	ROLL ↑	22									
	7	↑	27									
	8	e	12	CALCULATE C_1								
	9	÷	35									
	a	f	15									
	b	÷	35									
	c	1	01	DISPLAY 1 TO INDICATE FIRST COMPONENT								
	d	STOP	41	1	C_1	0	DISPLAY					

FROM 9-5 → (3-4)

Step		Key	Code	Display x	y	z	Storage f	e	d	c	b	a
3	0	2	02	INITIALIZE K TO 2								
	1	x→( )	23									
	2	c	16									
	3	SET FLAG	54	FLAG IS USED TO ALTERNATE DENORMALIZATION OF L_K AND C_K								
	4	4	04	CALCULATE $G_K = K^{th}$ NORMALIZED COMPONENT VALUE (steps 3-4 through 5-d)								
	5	↑	27									
	6	c	16									
	7	↑	27									
	8	2	02									
	9	×	36									
	a	3	03									
	b	—	34									
	c	π	56									
	d	×	36									
4	0	2	02									
	1	÷	35									
	2	d	17									
	3	÷	35									
	4	↓	25									
	5	sin x	70									
	6	×	36									
	7	c	16									
	8	↑	27									
	9	2	02									
	a	×	36									
	b	1	01									
	c	—	34									
	d	π	56									
5	0	×	36									
	1	2	02									
	2	÷	35									
	3	d	17									
	4	÷	35									
	5	↓	25									
	6	sin x	70									
	7	×	36									
	8	c	16									
	9	↑	27									
	a	1	01									
	b	—	34									
	c	π	56									
	d	×	36									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	d	17									
	1	÷	35									
	2	↓	25									
	3	sin x	70									
	4	↑	27									
	5	×	36									
	6	a	13									
	7	+	33									
	8	↓	25									
	9	÷	35									
	a	b	14									
	b	÷	35									
	c	y→( )	40									
	d	b	14									
7	0	e	12	CALCULATE G_K/ω_c								
	1	÷	35									
	2	f	15	RECALL R								
	3	IF FLAG	43	ALTERNATING BRANCH								
	4	8	10									
	5	2	02									
	6	÷	35	COMPLETES C_K CALCULATION								
	7	0	00	POSITION DISPLAY AND RECALL K								
	8	x⇄y	30									
	9	↑	27									
	a	c	16									
	b	STOP	41	K	C_K	0	DISPLAY					
	c	SET FLAG	54									
	d	GO TO ( ) ( )	44	BRANCH TO ISOLATE L_K CALCULATION								
8	0	8	10									
	1	8	10									
	2	×	36	COMPLETES L_K CALCULATION								
	3	0	00	POSITION DISPLAY AND RECALL K								
	4	x⇄y	30									
	5	↑	27									
	6	c	16									
	7	STOP	41	K	L_K	0	DISPLAY					
	8	↑	27	INCREMENT K BY 1								
	9	1	01									
	a	+	33									
	b	y→( )	40									
	c	c	16									
	d	d	17	RECALL n								

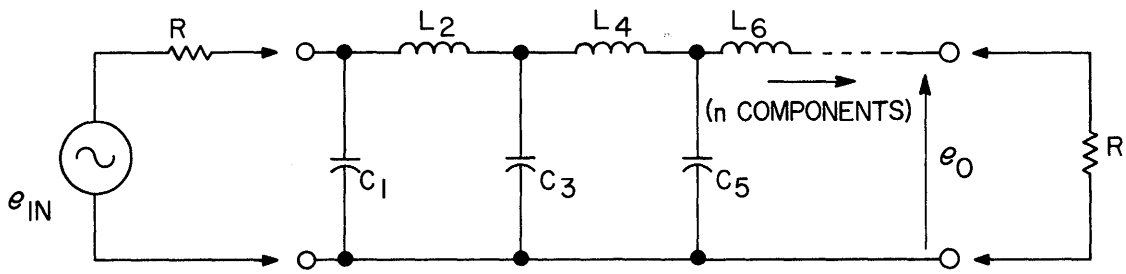
Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	IF $x<y$	52									
	1	9	11	BRANCH TO PROGRAM END IF COMPLETED (K=n+1)								
	2	6	06									
	3	GO TO ( )( )	44									
	4	3	03	BRANCH BACK TO START OF G_K LOOP								
	5	4	04									
	6	CLEAR	20	CLEAR DISPLAY								
	7	STOP	41	PROGRAM HALTS HERE								
	8	STOP	41									
	9	STOP	41									
	a	STOP	41	UNUSED STATEMENTS								
	b	STOP	41									
	c	STOP	41									
	d	STOP	41									
a	0	CLEAR	20	THIS IS THE STARTING ADDRESS								
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	STOP	41	ϵ_{dB}	n	0	ENTER n AND ϵ_{dB}					
	3	$y \to (\ )$	40	STORE n								
	4	d	17									
	5	$x \rightleftarrows y$	30									
	6	1	01									
	7	e^x	74									
	8	log x	75									
	9	↑	27									
	a	4	04									
	b	0	00									
	c	×	36									
	d	↓	25									
b	0	÷	35									
	1	↓	25	CALCULATE γ								
	2	hyper ▾	67									
	3	tan x	71									
	4	↑	27									
	5	1	01									
	6	$x \rightleftarrows y$	30									
	7	÷	35									
	8	↓	25									
	9	ln x	65									
	a	↑	27									
	b	2	02									
	c	÷	35									
	d	d	17									

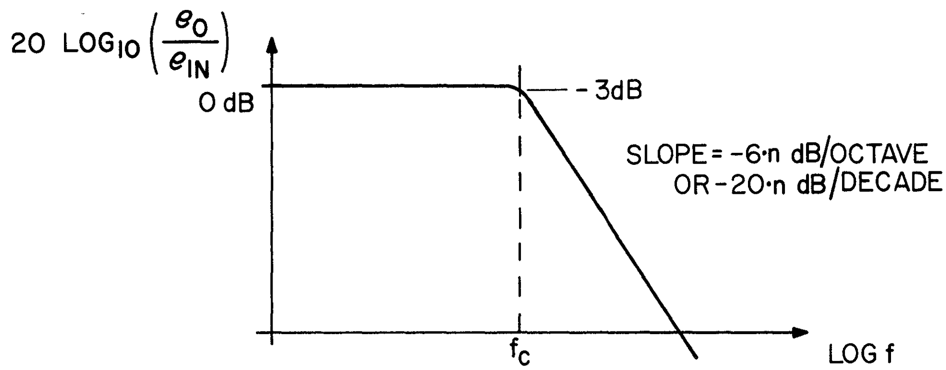
Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
c	0	÷	35									
	1	↓	25									
	2	hyper ▾	67									
	3	sin x	70									
	4	x→( )	23	STORES Y IN a, DESTROYING PROGRAM STEPS ALREADY EXECUTED								
	5	a	13									
	6	GO TO ( )( )	44									
	7	0	00	BRANCH TO CONTINUE CALCULATION								
	8	3	03									
	9	END	46									
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

BUTTERWORTH FILTER DESIGN

This program calculates component values for Butterworth low-pass filters between equal terminations. The user supplies filter order n, termination resistance R, and corner (3 dB) frequency f_c. The filter designed is of this form:





Reference: Simplified Modern Filter Design
by P. R. Geffe

J. F. Rider 1963

Handbook on Electrical Filters

White Electromagnetics Inc. 1963

USER INSTRUCTIONS	EXAMPLES

SET: [ ] [RADIANS] [FLOATING] [ ]

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

ENTER DATA: n → Z, R(in ohms) → Y, f_c(in hertz) → X

(K is initialized to 1)

→ PRESS: CONTINUE

DISPLAY

0	——	Z
C_K	——	Y
K	——	X

(C_K is in farads)

K + 2 > n? NO → K is incremented by 2 (return to PRESS: CONTINUE)

YES ↓

(K is initialized to 2)

→ PRESS: CONTINUE

DISPLAY

0	——	Z
L_K	——	Y
K	——	X

(L_K is in henries)

K + 2 > n? NO → K is incremented by 2 (return to PRESS: CONTINUE)

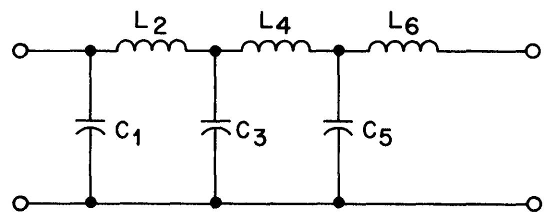
YES → (return to first PRESS: CONTINUE)

EXAMPLES

n = 6

R = 50 Ω

f_c = 10 mHz



C_1 = 164.8 pF

C_3 = 614.9 pF

C_5 = 450.2 pF

L_2 = 1.125 μH

L_4 = 1.537 μH

L_6 = .412 μH

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	f_c	R	n	ENTER n, R, f_c					
	2	x→( )	23	STORE INPUT (steps 0-2 to 0-8)								
	3	c	16									
	4	y→( )	40									
	5	b	14									
	6	ROLL ↑	22									
	7	x→( )	23									
	8	a	13									
	9	CLEAR	20									
	a	1	01	INITIALIZE COUNTER FOR C'S (steps 0-a to 0-c)								
	b	x→( )	23									
	c	d	17									
	d	↑	27	CALCULATE $\frac{\sin}{\pi f_c R}\left(\frac{(2K-1)\pi}{2n}\right)$ (steps 0-d to 2-4)								
1	0	2	02									
	1	×	36									
	2	1	01									
	3	−	34									
	4	π	56									
	5	×	36									
	6	2	02									
	7	÷	35									
	8	a	13									
	9	÷	35									
	a	↓	25									
	b	sin x	70									
	c	x⇄y	30									
	d	π	56									
2	0	÷	35									
	1	c	16									
	2	÷	35									
	3	b	14									
	4	÷	35									
	5	d	17	RECALL K								
	6	STOP	41	K	C_K	0	DISPLAY					
	7	↑	27	INCREMENT K (steps 2-7 to 2-9)								
	8	2	02									
	9	+	33									
	a	a	13	RECALL n								
	b	IF x<y	52	BRANCH IF ALL C'S ARE CALCULATED (steps 2-b to 2-d)								
	c	3	03									
	d	6	06									

Flow annotations: FROM 5-a → step 0-0; FROM 3-5 → step 0-c.

Step		Key	Code	Display x	y	z	Comments
3	0	y→( )	40				STORE K
	1	d	17				
	2	CLEAR	20				
	3	GO TO ( )( )	44				
	4	0	00				
	5	c	16				
	6	2	02				INITIALIZE COUNTER FOR L'S (FROM 2-d)
	7	x→( )	23				
	8	d	17				(FROM 6-2)
	9	↑	27				CALCULATE $\frac{R \sin}{\pi f_c}\left(\frac{(2K-1)\pi}{2n}\right)$
	a	2	02				
	b	×	36				
	c	1	01				
	d	−	34				
4	0	π	56				
	1	×	36				
	2	2	02				
	3	÷	35				
	4	a	13				
	5	÷	35				
	6	CLEAR x	37				
	7	ROLL ↓	31				
	8	sin x	70				
	9	x⇄y	30				
	a	π	56				
	b	÷	35				
	c	c	16				
	d	÷	35				
5	0	b	14				
	1	×	36				
	2	d	17	RECALL K			
	3	STOP	41	K	L_K	0	DISPLAY
	4	↑	27				INCREMENT K
	5	2	02				
	6	+	33				
	7	a	13				
	8	IF x<y	52				BRANCH IF ALL L'S ARE CALCULATED
	9	0	00				
	a	0	00				
	b	y→( )	40				STORE K
	c	d	17				
	d	CLEAR	20				

Storage: f, e, d, c, b, a (empty)

HEWLETT · PACKARD

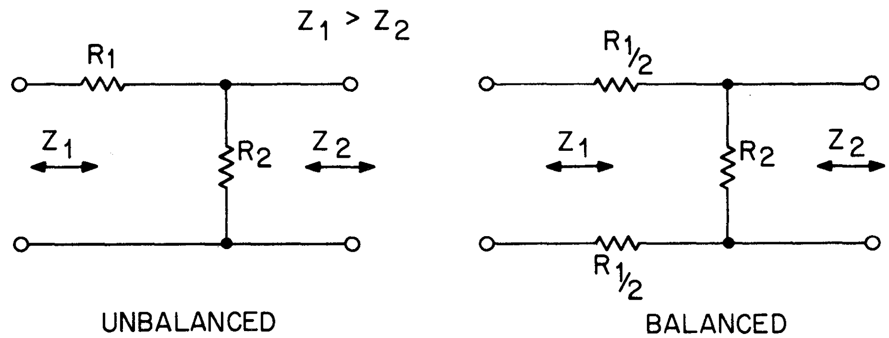
 PART NO. 09100-71003

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	GO TO ( ) ( )	44									
	1	3	03									
	2	8	10									
	3	END	46									
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

MINIMUM LOSS PADS

Given 2 impedances to be matched, program calculates resistive minimum loss pad and displays resistor values and loss in dB.



The equations used are:

$$R_1 = Z_1\sqrt{1 - \frac{Z_2}{Z_1}}$$

$$R_2 = \frac{Z_2}{\sqrt{1 - \frac{Z_2}{Z_1}}}$$

$$A(dB) = \frac{20}{\ln 10}\ \text{arccosh}\sqrt{\frac{Z_1}{Z_2}}$$

Reference: Reference Data for Radio Engineers, ITT
Fourth Edition 1963

USER INSTRUCTIONS	EXAMPLES

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

ENTER DATA: $Z_2 \rightarrow Y$, $Z_1 \rightarrow X$

$Z_1 > Z_2$

PRESS: CONTINUE

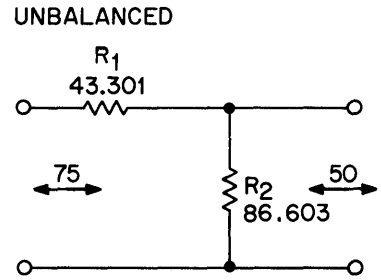
DISPLAY

A(dB)	Z
R_2	Y
R_1	X

EXAMPLES

Z_1 = 75 Ω

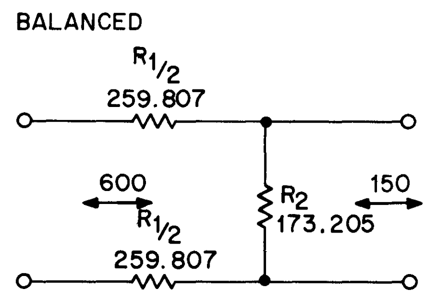
Z_2 = 50 Ω



Loss = 5.719 dB

Z_1 = 600

Z_2 = 150



Loss = 11.439 dB

R_1 = 519.615

R_2 = 173.205

PAGE 1 PART NO. 09100-71004

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	STOP	41	Z_1	Z_2	0	ENTER Z_2, Z_1					
2	ACC +	60									
3	↑	27									
4	↓	25									
5	÷	35									
6	1	01									
7	$x \rightleftarrows y$	30	CALCULATE R_2								
8	−	34									
9	$x \rightleftarrows y$	30									
a	$\sqrt{x}$	76									
b	ROLL ↑	22									
c	$x \rightleftarrows y$	30									
d	÷	35									
1 0	↑	27									
1	f	15									
2	×	36	CALCULATE R_1								
3	y→( )	40									
4	a	13									
5	$x \rightleftarrows y$	30									
6	e	12									
7	÷	35									
8	↓	25									
9	$\sqrt{x}$	76									
a	arc ▾	72									
b	hyper ▾	67									
c	cos x	73	CALCULATE A_{dB}								
d	↑	27									
2 0	2	02									
1	0	00									
2	×	36									
3	1	01									
4	0	00									
5	ln x	65									
6	÷	35									
7	a	13									
8	ROLL ↑	22	POSITION DISPLAY								
9	$x \rightleftarrows y$	30									
a	END	46	R_1	R_2	A	DISPLAY					
b											
c											
d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

PART NO.
09100-71005

TCHEBYSHEFF EVALUATION

This program evaluates the response of a Tchebysheff filter. The programmed input-output routine is designed to facilitate the use of the program in determining the optimum filter for a given application.

User specifies:

- Pass Band Ripple - ϵ_{dB} (dB)
- Corner ($-\epsilon_{dB}$) Frequency - f_c } in any consistent units
- Frequency at which response is to be found - f_i } in any consistent units
- Filter Order - n

The equations are:

$$G(\Omega)_{dB} = \log_{10} \frac{1}{1 + \epsilon^2 T_n^2(\Omega)}$$

where $\Omega = \frac{f_i}{f_c}$

$G(\Omega)_{dB}$ = the attenuation in dB below the minimum attenuation

$$\epsilon^2 = 10^{\cdot \epsilon_{dB}} - 1$$

T_n is the Tchebysheff polynomial

$$T_n(\Omega) = \cos(n \cos^{-1}(\Omega)) \qquad \Omega \leq 1$$

$$T_n(\Omega) = \cosh(n \cosh^{-1}(\Omega)) \qquad \Omega \geq 1$$

USER INSTRUCTIONS

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: $f_c \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: $f_i \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
3	——	X

ENTER DATA: $n \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
4	——	X

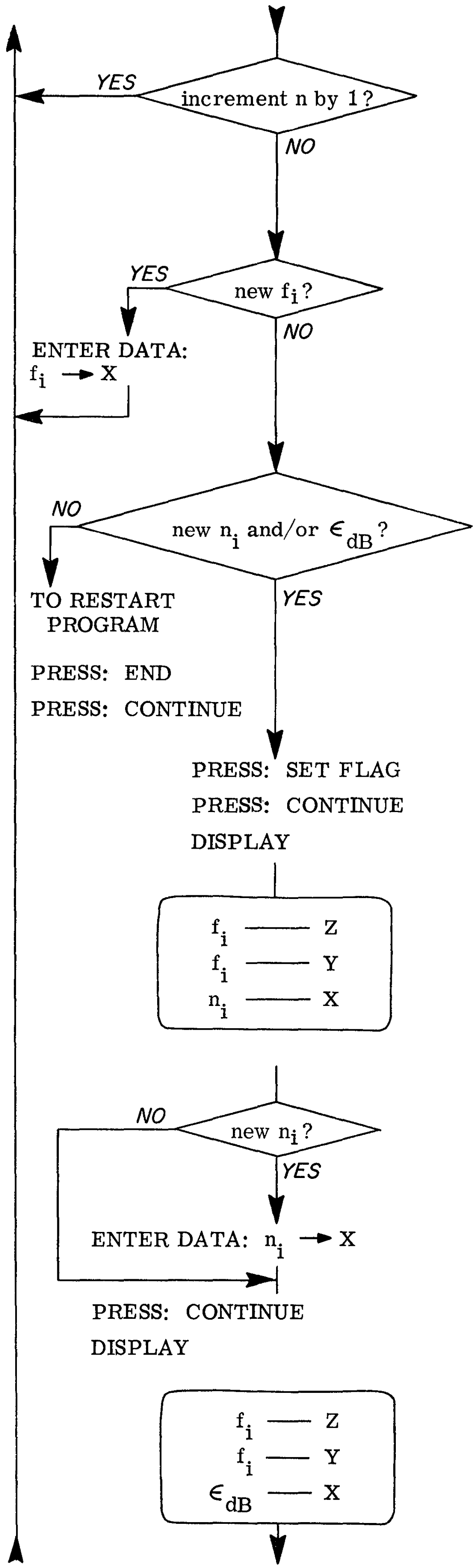
ENTER DATA: $\epsilon_{dB} \rightarrow X$

PRESS: CONTINUE

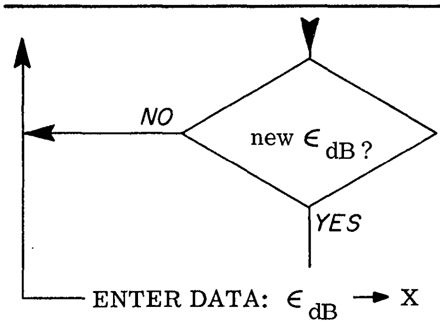
DISPLAY

f_i	——	Z
n_i	——	Y
A_{dB}	——	X

USER INSTRUCTIONS (Cont'd)



USER INSTRUCTIONS (Cont'd)



EXAMPLES

The following general filter requirements are known:

1. Filter cutoff frequency is 10 MHz
2. Minimum ripple is desired ($0 < \max \leq .1$ dB)
3. At 14 MHz amplitude attenuation of 15 dB is required
4. At 20 MHz 35 dB attenuation is required
5. Minimize components
6. Prefer matched terminations i.e., n odd

Initial design parameters are:

f_c = 10 MHz
f_i = 20 MHz
n = 2
ϵ_{dB} = .025 dB

Results are

A_{dB} = 1.1 dB

thus increase n (increment n)

n = 3
A_{dB} = 6.9 dB

increase n (increment n)

n = 4
A_{dB} = 17.4 dB

increase n

n = 5
A_{dB} = 28.8 dB

increase n

n = 6
A_{dB} = 40.2 dB

EXAMPLES (Cont'd)

Because of matching preference, increase ϵ_{dB} and decrement n to 5.

n = 5
ϵ_{dB} = .05
A_{dB} = 31.8

further increase ϵ_{dB}

ϵ_{dB} = .08 dB
A_{dB} = 33.9 dB

further increase ϵ_{dB}

ϵ_{dB} = .1
A_{dB} = 34.8

try increasing n to 7 since want odd and decreasing ϵ_{dB}

n_i = 7
ϵ_{dB} = .01
A_{dB} = 47.7 dB

ϵ_{dB} = .01 is about the minimum ripple achievable so we check attenuation at 14 MHz.

f_i = 14 MHz
A_{dB} = 20.4 dB

Thus the requirements are met. The element values may be found from program #09100-71001 Tchebysheff for Equal Terminations.

Step	Key	Code	Display x	Display y	Display z	Storage / Remarks
0 0	CLEAR	20				
1	3	03				
2	$x \to (\ )$	23				
3	f	15	STORE DISPLAY CONSTANTS			
4	4	04				
5	$x \to (\ )$	23				
6	e	12				
7	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY			
8	STOP	41	f_c	0	0	ENTER f_c
9	$x \to (\ )$	23	STORE f_c			
a	d	17				
b	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY			
c	STOP	41	f_i	0	0	ENTER f_i
d	$x \to (\ )$	23	STORE f_i			
1 0	c	16				
1	↑	27				
2	d	17				
3	÷	35	CALCULATE $\frac{f_i}{f_c}$			
4	$y \to (\ )$	40				
5	b	14				
6 (FROM 6-8)	↓	25				
7	f	15	RECALL AND DISPLAY 3 TO INDICATE THIRD ENTRY			
8	STOP	41	n / n	0 / f_i	0 / f_i	ENTER n / DISPLAY
9	$x \to (\ )$	23	STORE n			
a	f	15				
b	e	12	RECALL AND DISPLAY 4 TO INDICATE FOURTH ENTRY			
c	STOP	41	ϵ_{dB} / ϵ_{dB}	0 / f_i	0 / f_i	ENTER ϵ_{dB} / DISPLAY
d	$x \to (\ )$	23	STORE ϵ_{dB}			
2 0	e	12				
1	1	01				
2	0	00				
3	↑	27				
4	ln x	65				
5	$x \rightleftarrows y$	30				
6	÷	35				
7	e	12				
8	×	36	CALCULATE ϵ^2			
9	↓	25				
a	e^x	74				
b	↑	27				
c	1	01				
d	−	34				

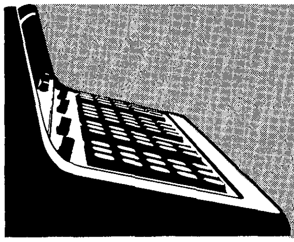
Step	Key	Code	Display x	Display y	Display z	f	e	d	c	b	a
3 0	y→( )	40									
1	a	13									
2	b	14									
3	↑	27									
4	1	01									
5	IF x > y	53									
6	4	04	BRANCH DEPENDING ON Ω								
7	7	07									
8	b	14									
9	arc ▾	72									
a	hyper ▾	67									
b	cos x	73									
c	↑	27									
d	f	15									
4 0	×	36	CALCULATE $T_n(\Omega)$ $\Omega \geq 1$								
1	↓	25									
2	hyper ▾	67									
3	cos x	73									
4	GO TO ( )( )	44									
5	5	05									
6	1	01									
7	↓	25									
8	arc ▾	72									
9	cos x	73									
a	↑	27									
b	f	15	CALCULATE $T_n(\Omega)$ $\Omega < 1$								
c	×	36									
d	↓	25									
5 0	cos x	73									
1	↑	27									
2	×	36									
3	a	13									
4	×	36									
5	1	01									
6	+	33									
7	↓	25									
8	log x	75	CALCULATE A_{dB}								
9	↑	27									
a	1	01									
b	0	00									
c	×	36									
d	y→( )	40									

FROM 7-9 & 8-4 (→ step 3-2)

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 0	9	11									
1	c	16									
2	↑	27									
3	f	15									
4	ROLL ↑	22									
5	STOP	41	A_{dB}	n_i	f_i	DISPLAY					
6	IF FLAG	43									
7	1	01	BRANCH FOR NEW n_i AND ϵ_{dB}								
8	6	06									
9	y⇄( )	24									
a	9	11									
b	IF x = y	50									
c	7	07	BRANCH TO INCREMENT n								
d	a	13									
7 0	↑	27									
1	d	17									
2	y→( )	40									
3	c	16									
4	÷	35									
5	y→( )	40	LOAD IN NEW f_i								
6	b	14									
7	GO TO ( )( )	44									
8	3	03									
9	2	02									
a	f	15									
b	↑	27									
c	1	01									
d	+	33	INCREMENT n								
8 0	y→( )	40									
1	f	15									
2	GO TO ( )( )	44									
3	3	03									
4	2	02									
5	END	46									
6											
7											
8											
9											
a											
b											
c											
d											

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

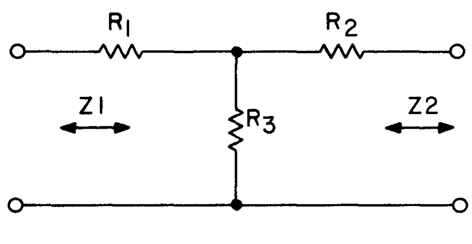


PART NO.
09100-71006

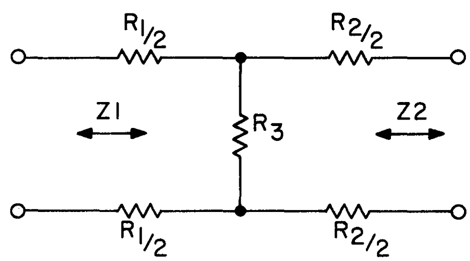
ATTENUATOR PADS T OR Π

Given input and output impedances and attenuation desired, program calculates resistor values for either T or Π pads.

'T' OR 'H' PAD



UNBALANCED



BALANCED

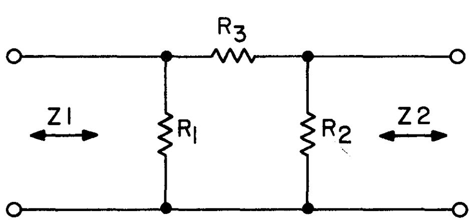
$$\theta = \frac{A(dB)}{20} \ln 10$$

$$R_3 = \frac{\sqrt{Z_1 Z_2}}{\sinh \theta}$$

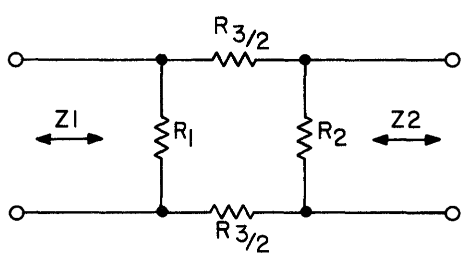
$$R_2 = \frac{Z_2}{\tanh \theta} - R_3$$

$$R_1 = \frac{Z_1}{\tanh \theta} - R_3$$

'Π' OR 'O' PAD



UNBALANCED



BALANCED

$$R_3 = \sqrt{Z_1 Z_2} \sinh \theta$$

$$R_2 = \frac{R_3 Z_2 \tanh \theta}{R_3 - Z_2 \tanh \theta}$$

$$R_1 = \frac{R_3 Z_1 \tanh \theta}{R_3 - Z_1 \tanh \theta}$$

Reference: Reference Data for Radio Engineer, ITT

4th Edition 1963

USER INSTRUCTIONS

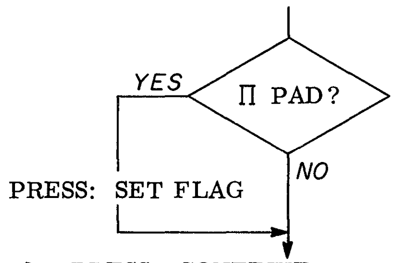
SET: [| FIXED]

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA: $Z_1 \rightarrow Z$, $Z_2 \rightarrow Y$, $A_{dB} \rightarrow X$



PRESS: CONTINUE

DISPLAY

R_3	——	Z
R_2	——	Y
R_1	——	X

PRESS: CONTINUE

DISPLAY

Z_1	——	Z
Z_2	——	Y
A_{dB}	——	X

ENTER NEW VALUES OF A_{dB}, Z_1 and/or Z_2 AS BEFORE

PRESS: CONTINUE

DISPLAY NEW VALUES OF R_1, R_2 and R_3

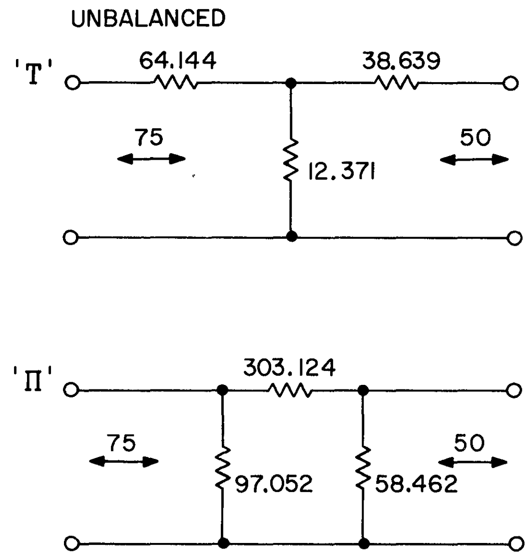
NOTE: If SET FLAG is pressed, program continues Π calculation. To change from Π program back to T program, press IF FLAG after entering new data, then press CONTINUE. Thus the selection of Π or T pads needs to be made only once at entry point.

EXAMPLES

Input impedance 75 Ω

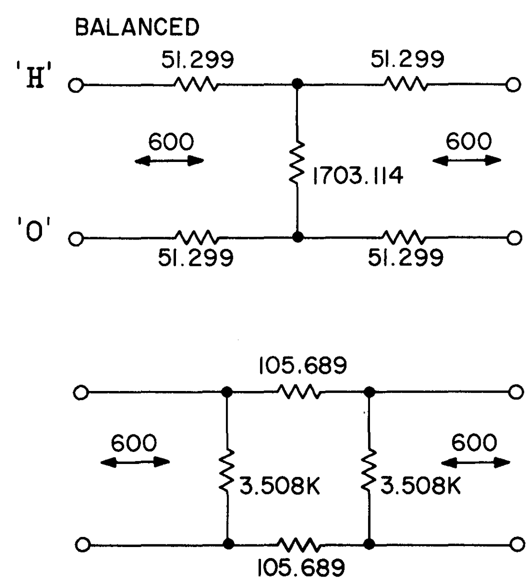
Output impedance 50 Ω

Attenuation 20 dB



Input and output impedance 600 Ω

Attenuation 3 dB



R_1 = 102.598	R_1 = 3508.826
R_2 = 102.598	R_2 = 3508.826
R_3 = 1703.114	R_3 = 211.378

All units are in ohms

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	A(dB)	Z_2	Z_1	ENTER Z_1, Z_2, A(dB)					
	2	ROLL ↓	31	STORE DATA								
	3	y→( )	40									
	4	a	13									
	5	x→( )	23									
	6	b	14									
	7	↓	25	CALCULATE θ								
	8	2	02									
	9	0	00									
	a	÷	35									
	b	1	01									
	c	0	00									
	d	ln x	65									
1	0	×	36									
	1	a	13									
	2	ROLL ↑	22	STORE A								
	3	x→( )	23									
	4	c	16									
	5	b	14	CALCULATE $\sqrt{Z_1 Z_2}$								
	6	×	36									
	7	↓	25									
	8	√x	76									
	9	x⇄y	30	CALCULATE sinh θ								
	a	hyper ▾	67									
	b	sin x	70									
	c	IF FLAG	43	BRANCH IF Π PAD IS DESIRED								
	d	4	04									
2	0	1	01									
	1	÷	35	CALCULATE R_3								
	2	a	13	CALCULATE R_1								
	3	ROLL ↑	22									
	4	hyper ▾	67									
	5	tan x	71									
	6	÷	35									
	7	ROLL ↓	31									
	8	x⇄y	30									
	9	−	34									
	a	y→( )	40									
	b	d	17									
	c	ROLL ↓	31									
	d	b	14									

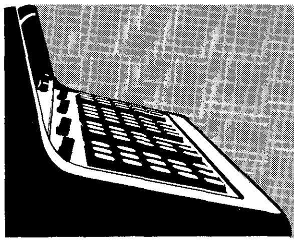
Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	x⇄y	30									
1	÷	35	CALCULATE R_2								
2	↓	25									
3	x⇄y	30									
4	—	34									
5	d	17									
6	STOP	41	R_1	R_2	R_3	DISPLAY					
7 (FROM 6-b)	a	13									
8	↑	27									
9	b	14	RECALL AND POSITION INPUT								
a	↑	27									
b	c	16									
c	GO TO ( )( )	44									
d	0	00	RETURN TO INITIAL ENTRY								
4 0	1	01									
1 (FROM 1-c)	×	36	BRANCH TO THIS POSITION FOR Π PAD, CALCULATE R_3								
2	a	13									
3	ROLL ↑	22									
4	hyper ▾	67									
5	tan x	71									
6	×	36									
7	ROLL ↑	22									
8	ACC +	60									
9	↓	25									
a	f	15	CALCULATE R_1								
b	×	36									
c	ROLL ↑	22									
d	—	34									
5 0	↓	25									
1	÷	35									
2	y→( )	40									
3	d	17									
4	b	14									
5	↑	27									
6	e	12									
7	×	36									
8	↑	27									
9	↓	25									
a	f	15	CALCULATE R_2								
b	×	36									
c	ROLL ↑	22									
d	—	34									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6 0	↓	25									
1	÷	35									
2	d	17									
3	↑	27									
4	f	15									
5	ROLL ↓	31									
6	STOP	41	R_1	R_2	R_3	DISPLAY					
7	CLEAR	20									
8	SET FLAG	54									
9	GO TO ( )( )	44									
a	3	03									
b	7	07									
c	END	46									
d											

Steps 62–65: POSITION DISPLAY

Steps 68–6b: SET FLAG AND BRANCH FOR ANOTHER Π PAD

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-71007

BAND PASS FILTER DESIGN

Given the image impedance level and the desired bandpass of a filter the program calculates the ideal component values. If values of closest commercially available components are substituted for these ideal values, the program then calculates the frequency response of the proposed filter.

F_L = low frequency
F_H = high frequency } Frequency range over which filter is to be tested.

A_{dB} = Attenuation in dB

Dimensions: Increment, f_1, f_2, F_L, F_H, given in cycles/sec.

Attenuation given in decibels (dB)

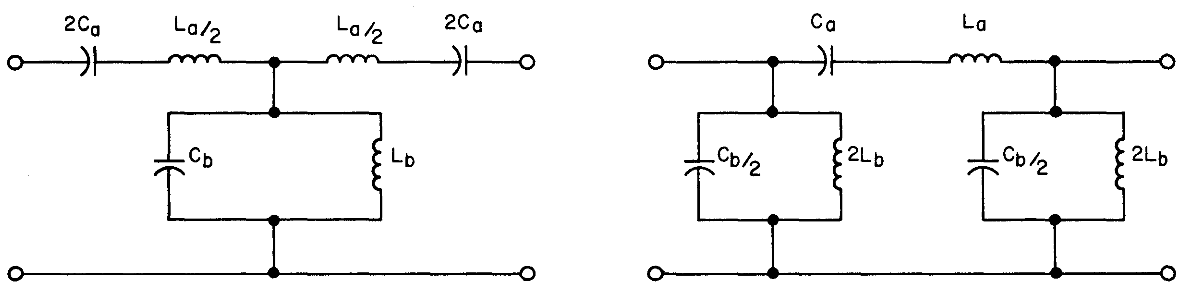
L_b, L_a given in henries

C_a, C_b given in farads

The equations used are:

R = Impedance; f_1 = Low cutoff; f_2 = High Cutoff

T Π



$$C_a = \frac{f_2 - f_1}{4\pi f_1 f_2 R} \qquad C_b = \frac{1}{\pi (f_2 - f_1) R}$$

$$L_a = \frac{R}{\pi (f_2 - f_1)} \qquad L_b = \frac{R (f_2 - f_1)}{4 \pi f_1 f_2}$$

$$\frac{X_a}{4 X_b} = \frac{(\omega^2 C_a L_a - 1)(1 - \omega^2 C_b L_b)}{4 \omega^2 C_a L_b} \quad \text{when } \omega = 2\pi f$$

Attenuation in dB

If

$$0 < \frac{X_a}{4X_b} \qquad (40 \text{ Log}_{10} e) \left(\sinh^{-1} \sqrt{\frac{X_a}{4X_b}} \right)$$

	(Attenuation in dB)
$-1 < \frac{X_a}{4X_b} < 0$	0
$\frac{X_a}{4X_b} < -1$	$(40 \text{ Log}_{10} e)\left(\cosh^{-1}\sqrt{-\frac{X_a}{4X_b}}\right)$

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
0	——	X

ENTER DATA: $R \rightarrow Z$, $f_2 \rightarrow Y$, $f_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

C_a	——	Z
L_a	——	Y
1	——	X

Approx. C_a, L_a with realistic component values?

YES: Enter approximations: $C_{a_o} \rightarrow Z$, $L_{a_o} \rightarrow Y$, $0 \rightarrow X$

NO

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

DISPLAY

C_b	——	Z
L_b	——	Y
2	——	X

Approx. C_b, L_b with realistic component values?

YES: Enter approximations: $C_{b_o} \rightarrow Z$, $L_{b_o} \rightarrow Y$, $0 \rightarrow X$

NO

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
3	——	X

ENTER DATA: $F_L \rightarrow Z$, $F_U \rightarrow Y$,

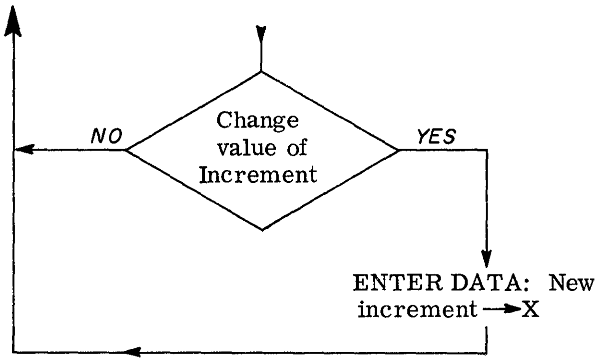
Increment $\rightarrow X$

PRESS: CONTINUE

DISPLAY

A_{db}	——	Z
f	——	Y
Increment	——	X

USER INSTRUCTIONS (con't)



Note (1): When the frequency being used is incremented enough to exceed the value of F_H entered, the calculator will return to the initial display.

Note (2): Substitution of "Pause" instead of "Stop" in step 9 - 0 will provide continuous incrementing and attenuation evaluation without use of "continue" key.

EXAMPLE

SET: digit dial to 8

SET: [| FIXED POINT]

(A) R = 50

f_2 = 3000

f_1 = 300

Results: L_a = 5.89 mh

C_a = 4.77 μf

L_b = 11.94 mh

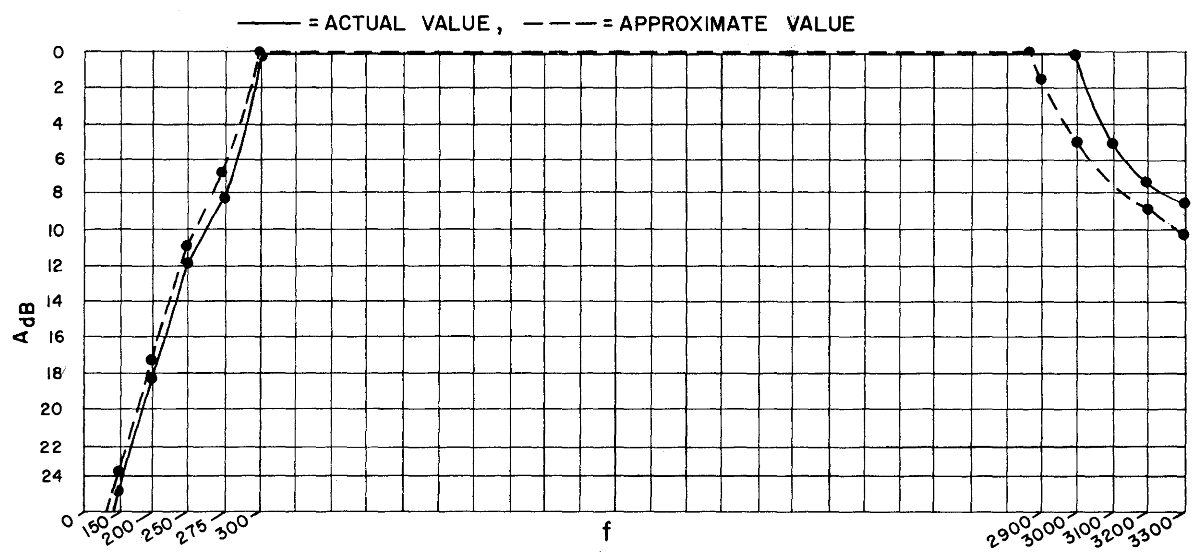
C_b = 2.36 μf

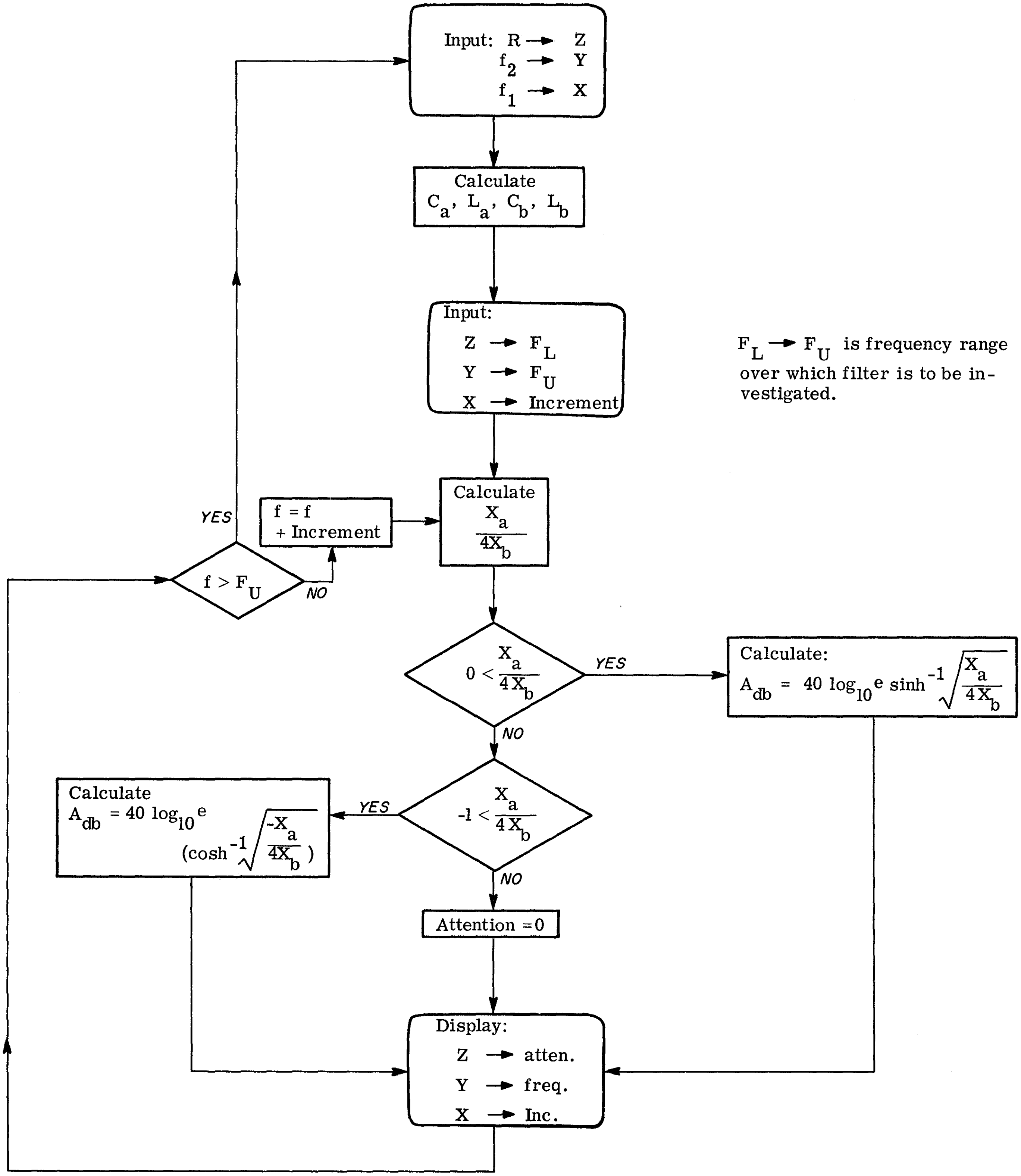
ENTER DATA: F_U = 3500, F_L = 150
Increment

(B) Reset problem - use same values as part (A)

After displaying L_a and C_a set in approximate values of; C_a = 5 f, L_a = 6mh (realistic component values)

After displaying L_b and C_b set in approximate values of; L_b = 12 mh, C_b = 2.5 f (realistic component values)




Input: R → Z
f_2 → Y
f_1 → X
Calculate C_a, L_a, C_b, L_b
Input:
Z → F_L
Y → F_U
X → Increment
$F_L \rightarrow F_U$ is frequency range over which filter is to be investigated.
Calculate $\frac{X_a}{4X_b}$
f = f + Increment
YES
$f > F_U$
NO
$0 < \frac{X_a}{4X_b}$
YES
Calculate: $A_{db} = 40 \log_{10} e \sinh^{-1} \sqrt{\frac{X_a}{4X_b}}$
NO
$-1 < \frac{X_a}{4X_b}$
YES
Calculate $A_{db} = 40 \log_{10} e \, (\cosh^{-1} \sqrt{\frac{-X_a}{4X_b}})$
NO
Attention =0
Display:
Z → atten.
Y → freq.
X → Inc.

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	f_1	f_2	R	ENTER R, f_2, f_1					
	2	ACC +	60									
	3	−	34									
	4	y→( )	40	STORE f_1, f_2, AND $f_2 - f_1$								
	5	d	17									
	6	ROLL ↑	22									
	7	x→( )	23	STORE R								
	8	b	14									
	9	e	12									
	a	×	36									
	b	4	04									
	c	×	36									
	d	π	56									
1	0	×	36									
	1	y→( )	40									
	2	c	16									
	3	ROLL ↓	31	CALCULATE C_a AND L_a								
	4	÷	35									
	5	b	14									
	6	÷	35									
	7	↑	27									
	8	d	17									
	9	÷	35									
	a	π	56									
	b	÷	35									
	c	1	01									
	d	STOP	41	1	L_a	C_a	DISPLAY					
2	0	↓	25									
	1	×	36									
	2	1	01	CALCULATE AND STORE C_aL_a								
	3	y⇄( )	24									
	4	c	16									
	5	ROLL ↑	22									
	6	x→( )	23									
	7	a	13									
	8	d	17									
	9	÷	35									
	a	π	56									
	b	÷	35	CALCULATE C_b AND L_b								
	c	b	14									
	d	÷	35									

Step	Key	Code	Display x	Display y	Display z	Storage
3 0	ROLL ↑	22				
1	÷	35				
2	d	17				
3	×	36				
4	2	02				
5	STOP	41	2	L_b	C_b	DISPLAY
6	ROLL ↓	31	CALCULATE AND STORE C_bL_b			
7	×	36				
8	y⇄( )	24				
9	a	13				
a	×	36	CALCULATE AND STORE $4C_aL_b$			
b	4	04				
c	×	36				
d	y→( )	40				
4 0	b	14				
1	CLEAR	20				
2	3	03				
3	STOP	41	INCREMENT	F_u	F_L	ENTER F_L, F_u, INCREMENT
4	ACC +	60				
5	ROLL ↓	31				
6 (FROM 9-7)	2	02				
7	×	36				
8	π	56				
9	×	36				
a	↓	25				
b	↑	27				
c	×	36				
d	y→( )	40				
5 0	d	17	CALCULATE $\frac{X_a}{4X_b}$			
1	c	16				
2	×	36				
3	1	01				
4	−	34				
5	b	14				
6	÷	35				
7	d	17				
8	÷	35				
9	↑	27				
a	a	13				
b	×	36				
c	1	01				
d	x⇄y	30				

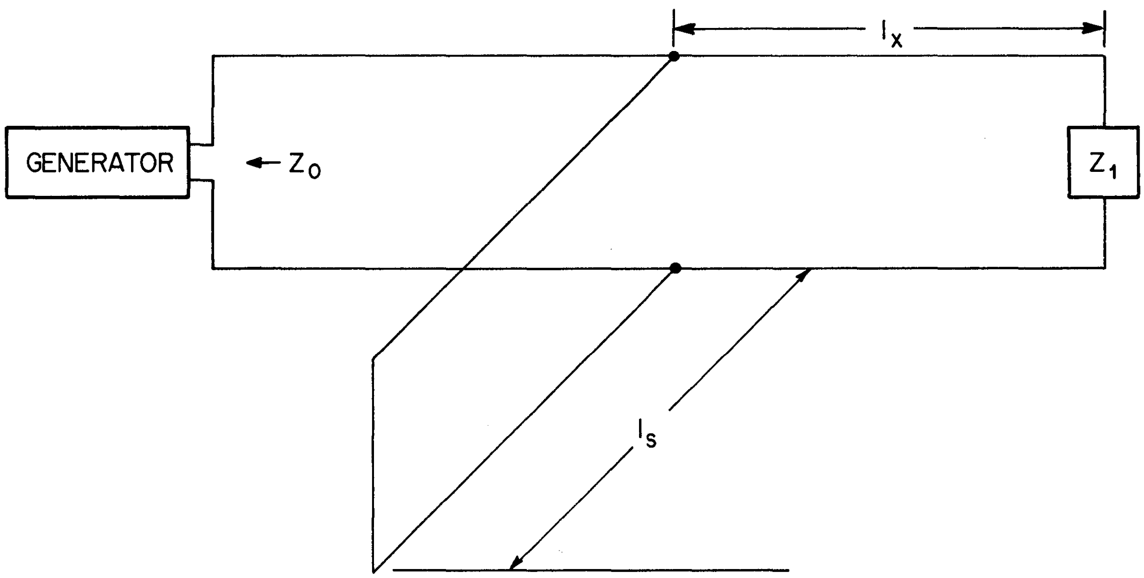
Step	Key	Code	Remarks
6 0	−	34	
1	ROLL ↓	31	
2	×	36	
3	CLEAR x	37	
4	IF $x<y$	52	
5	7	07	
6	7	07	
7	1	01	
8	CHG SIGN	32	IF $-1< \frac{X_a}{4X_b} <0$, ENTER 0 FOR A_{dB} AT 8-4
9	IF $x<y$	52	
a	8	10	
b	4	04	
c	ROLL ↓	31	
d	CHG SIGN	32	
7 0	$\sqrt{x}$	76	CALCULATE $\cosh^{-1}\sqrt{\frac{-X_a}{4X_b}}$
1	arc ▾	72	
2	hyper ▾	67	
3	cos x	73	
4	GO TO ( )( )	44	
5	7	07	
6	c	16	
7	$x \rightleftarrows y$	30	CALCULATE $\sinh^{-1}\sqrt{\frac{X_a}{4X_b}}$
8	$\sqrt{x}$	76	
9	arc ▾	72	
a	hyper ▾	67	
b	sin x	70	
c	↑	27	
d	1	01	
8 0	e^x	74	CALCULATE A_{dB} FOR $\frac{X_a}{4X_b} > 0$ OR $\frac{X_a}{4X_b} < -1$
1	log x	75	
2	×	36	
3	4	04	
4	0	00	
5	×	36	
6	d	17	CALCULATE f FOR EACH ATTENUATION AND RECALL INCREMENT
7	$\sqrt{x}$	76	
8	↑	27	
9	2	02	
a	÷	35	
b	π	56	
c	÷	35	
d	f	15	

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	STOP	41	INCREMENT	f	ATTENUATION	DISPLAY					
	1	$x \rightarrow ()$	23									
	2	f	15									
	3	+	33	CALCULATE $F_L + \sum_1^n$ INCREMENT								
	4	e	12									
	5	IF $x > y$	53									
	6	4	04	IF $F_u > F_L + \sum_1^n$ INCREMENT, INCREMENT f AND								
	7	6	06	CALCULATE NEW ATTENUATION								
	8	END	46									
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

PART NO.
09100-71008

STUB MATCHED TRANSMISSION LINE

This program solves for l_x, the distance from the load to the point where $Y_{in}/Y_o = 1 + jb_{in}/Y_o$ in wavelengths and meters and for l_s, the length of a shorted stub to match the line in wavelengths and meters.



The equations used are:

$$\frac{Y_{in}}{Y_o} = \frac{\frac{g_1}{Y_o} + j\left(\frac{b_1}{Y_o} + \tan \beta 1\right)}{1 - \frac{b_1}{Y_o} \tan \quad 1 + j\frac{g_1}{Y_o} \tan \beta 1}$$

The iteration technique solves for:

$$\frac{Y_{in}}{Y_o} = 1 + j\,\frac{b_{in}}{Y_o}$$

The length of the shorted stub in:

$$l_s = \left(.25 - \frac{\tan^{-1} \frac{b_{in}}{Y_o}}{360} \right)$$

Reference: Microwave Theory and Measurements
Hewlett-Packard Microwave Division
Prentice-Hall, Inc., 1962

USER INSTRUCTIONS

SET: DEGREES

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

1	———	Z
0	———	Y
0	———	X

ENTER DATA: $Z_o \rightarrow Z$, $X_L \rightarrow Y$, $R_L \rightarrow X$

PRESS: CONTINUE

Note: Program pauses at each iteration, stops when completed.

DISPLAY

l_x	———	Z	
b_{in}	———	Y	(In Wavelengths)
g_{in}	———	X	

PRESS: CONTINUE

DISPLAY

2	———	Z
l_s	———	Y
l_x	———	X

(X contains distance from load in wavelenths)
(Y contains lengths of shorted stub in wavelengths)

ENTER DATA: Frequency (f) $\rightarrow$ Y, Propagation Constant (k) $\rightarrow$ X (k = 1 for air)

PRESS: CONTINUE

DISPLAY

3	———	Z	
l_s	———	Y	(In meters)
l_x	———	X	

USER INSTRUCTIONS (con't)

(X contains the distance from load in meters, Y contains the length of shorted stub in meters)

PRESS: CONTINUE to enter new problem

EXAMPLE

$Z_o = 50\ \Omega$

$X_L = 25\ \Omega$

$R_L = 50\ \Omega$

k = 1.000

$f = 1 \times 10^6$

$g_{in} = 1.000$

$b_{in} = .500$

METERS

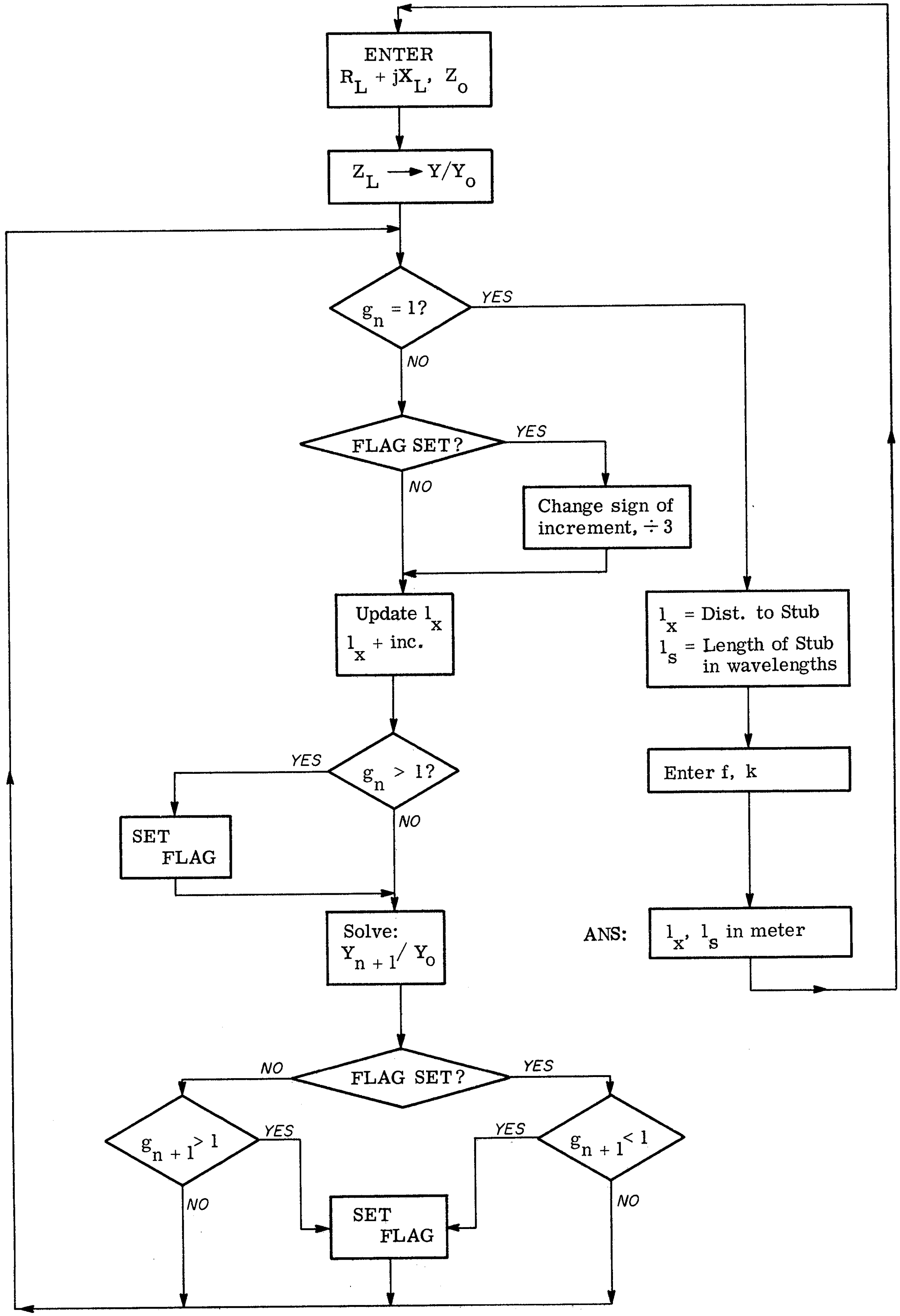
$l_x = 74.944$

$l_s = 52.823$

WAVELENGTHS

$l_x = .250$

$l_s = .176$



ENTER
$R_L + jX_L$, Z_o
$Z_L \rightarrow Y/Y_o$
$g_n = 1$?
YES
NO
FLAG SET?
YES
NO
Change sign of increment, ÷ 3
Update l_x
l_x + inc.
l_x = Dist. to Stub
l_s = Length of Stub in wavelengths
Enter f, k
$g_n > 1$?
YES
NO
SET FLAG
Solve:
Y_{n+1}/Y_o
ANS:
l_x, l_s in meter
FLAG SET?
NO
YES
$g_{n+1} > 1$
YES
NO
$g_{n+1} < 1$
YES
NO
SET FLAG

FROM 6-2 → step 0-0

FROM 7-4, 7-8, 7-c → step 1-7

Step 2-6 → (branch)

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	$x \rightarrow ()$	23									
2	c	16									
3	·	21									
4	2	02									
5	$x \rightarrow ()$	23									
6	d	17									
7	1	01									
8	ROLL ↓	31									
9	STOP	41	R_L	X_L	Z_0 (1)	ENTER Z_0, X_L, R_L					
a	TO POLAR	62									
b	ln x	65									
c	ACC −	63									
d	RCL	61									
1 0	ACC −	63	CONVERT Z TO Y								
1	e^x	74									
2	ROLL ↑	22									
3	×	36									
4	↓	25									
5	TO RECT	66									
6	ACC +	60									
7	RCL	61									
8	↑	27									
9	$\|y\|$	55									
a	ENTER EXP	26									
b	8	10									
c	×	36	ROUND gn TO 8 SIGNIFICANT DIGITS								
d	$x \rightleftarrows y$	30									
2 0	int x	64									
1	$x \rightleftarrows y$	30									
2	÷	35									
3	1	01									
4	IF $x = y$	50									
5	7	07	BRANCH IF ITERATION COMPLETED								
6	d	17									
7	d	17									
8	↑	27									
9	IF FLAG	43									
a	SET FLAG	54									
b	3	03	CHANGE INCREMENT (÷3, CHANGE SIGN IF CROSSOVER OCCURED)								
c	IF FLAG	43									
d	CHG SIGN	32									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	÷	35									
1	y→( )	40									
2	d	17									
3	c	16									
4	+	33	UPDATE λ_x								
5	y→( )	40									
6	c	16									
7	1	01									
8	ROLL ↑	22									
9	IF x > y	53									
a	SET FLAG	54									
b	SET FLAG	54									
c	d	17									
d	↑	27									
4 0	3	03									
1	6	06									
2	0	00									
3	×	36									
4	↓	25									
5	tan x	71									
6	↑	27									
7	e	12									
8	y→( )	40									
9	e	12									
a	↑	27									
b	ROLL ↑	22									
c	+	33									
d	f	15									
5 0	TO POLAR	62									
1	ln x	65									
2	y⇄( )	24									
3	e	12									
4	x⇄y	30									
5	y⇄( )	24									
6	f	15	SOLVE $\frac{Y}{Y_0} = \frac{\frac{g_1}{Y_0} + j\frac{b_1}{Y_0} + \text{TAN } \lambda_x}{(1 - \frac{b_1}{Y_0}\text{TAN } \lambda_x) + j\frac{g_1}{Y_0}\text{TAN } \lambda_x}$								
7	×	36									
8	ROLL ↑	22									
9	CHG SIGN	32									
a	×	36									
b	1	01									
c	+	33									
d	c	16									

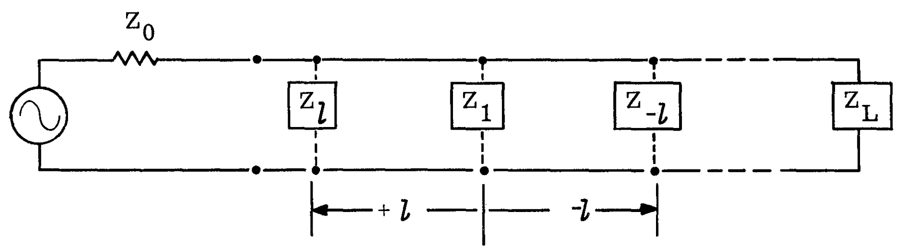
Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	ROLL ↓	31									
	1	TO POLAR	62									
	2	ln x	65									
	3	ACC −	63									
	4	RCL	61									
	5	ACC −	63									
	6	e^x	74									
	7	TO RECT	66									
	8	ACC +	60									
	9	PAUSE	57	g_{n+1}	b_{n+1}	l_{n+1}	DISPLAY IN WAVELENGTHS					
	a	x⇄y	30									
	b	1	01									
	c	|y|	55									
	d	IF FLAG	43									
7	0	7	07									
	1	6	06									
	2	IF x>y	53									
	3	1	01									
	4	7	07									
	5	SET FLAG	54									
	6	IF x<y	52									
	7	1	01									
	8	7	07									
	9	SET FLAG	54									
	a	GO TO ( )( )	44									
	b	1	01									
	c	7	07									
	d	C	16									
8	0	ROLL ↓	31									
	1	RCL	61									
	2	ACC −	63									
	3	STOP	41	$g_{in}=1$	b_{in}	l_x	ANSWER: Y_{in} AT l_x WAVELENGTHS FROM LOAD					
	4	↓	25									
	5	arc ▾	72									
	6	tan x	71									
	7	↑	27									
	8	3	03									
	9	6	06	CONVERT FROM WAVELENGTHS TO METERS								
	a	0	00	FIND LENGTH OF STUB		$l_s = (.25 - \frac{\tan^{-1}\frac{b_{in}}{Y_0}}{360})$						
	b	÷	35									
	c	.	21									
	d	2	02									

FROM 2-6

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	5	05									
	1	x⇄y	30									
	2	−	34									
	3	2	02									
	4	ROLL ↓	31									
	5	x⇄y	30									
	6	ACC +	60									
	7	STOP	41	l_x	l_s	2	ANSWER: DISTANCE TO STUB IN X, LENGTH OF SHORTED MATCHING STUB IN Y					
	8	÷	35	k	f	0						
	9	2	02				ENTER: FREQUENCY(f) IN Y PROP. CONSTANT(k) IN X					
	a	9	11									
	b	9	11									
	c	7	07									
	d	7	07									
a	0	6	06									
	1	ENTER EXP	26									
	2	8	10									
	3	x⇄y	30	CONVERT FROM WAVELENGTHS TO METERS								
	4	÷	35									
	5	e	12									
	6	x⇄y	30									
	7	×	36									
	8	↑	27									
	9	f	15									
	a	×	36									
	b	3	03									
	c	ROLL ↓	31									
	d	STOP	41	l_x (meters)	l_s (meters)	3	ANSWER: DISTANCE TO STUB IN X, LENGTH OF STUB IN Y					
b	0	GO TO ( )( )	44									
	1	0	00									
	2	0	00									
	3	END	46									
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

TRANSMISSION LINE

Given the characteristic line impedance, generator frequency, measured impedance at some point on the line and propagation constant, the program solves for the impedance at any other point on the line either toward the generator or toward the load, the voltage reflection coefficient (magnitude and phase) at the point where impedance is determined, and the VSWR on the line.



Given: Characteristic impedance (Z_0) of the line
Frequency (f)
Propagation constant (k) (k = 1 for air)
Measured impedance (Z_1) at some point on the line

The Lossless Equations used are:

$$Z_l = Z_0 \frac{Z_1 + jZ_0 \text{ TAN}\beta l}{Z_0 + jZ_1 \text{ TAN}\beta l}$$

where: $\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{ck}$

c = velocity of light

$Z_1 = R_1 + jX_1$

NOTE: $+l$ implies toward the generator
$-l$ implies toward the load

For the case: $\text{TAN}\,\beta l = \infty$, i.e., $\beta l = \pi \left(\frac{2N-1}{2}\right)$ where $N = 0, \pm 1, \pm 2 \cdots$;

Then $Z_l = \frac{Z_0^2}{Z_1}$

The reflection coefficient is:

$$\rho_v = \frac{Z_l - Z_0}{Z_l + Z_0} = |\rho_v| e^{j\theta_{\rho_v}}$$

$$\text{VSWR} = \frac{1 + |\rho_v|}{1 - |\rho_v|}$$

Reference: Microwave Theory and Measurements, Hewlett-Packard Microwave Division
Prentice-Hall, Inc. 1962

USER INSTRUCTIONS

SET: DEGREES

ENTER PROGRAM: (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: $Z_0 \rightarrow Z$, $f \rightarrow Y$, $k \rightarrow X$

PRESS: CONTINUE

DISPLAY

1	———	Z
0	———	Y
0	———	X

ENTER DATA: $X_1 \rightarrow Y$, $R_1 \rightarrow X$ ($Z_1 = R_1 + jX_1$)

PRESS: CONTINUE

DISPLAY

2	———	Z
0	———	Y
0	———	X

(1)

ENTER DATA: $l \rightarrow X$ ($+l \rightarrow$ Generator
$-l \rightarrow$ Load)

l in meters?

YES: PRESS: SET FLAG
PRESS: CONTINUE
(2)

NO: PRESS: CONTINUE
(2)

USER INSTRUCTIONS (Con't)

(2)

DISPLAY

3	———	Z
X_l	———	Y
R_l	———	X

$$Z_l = R_l + jX_l$$

PRESS: CONTINUE

DISPLAY

4	———	Z
θ_{ρ_ν}	———	Y
$\lvert\rho_\nu\rvert$	———	X

(1)

PRESS: CONTINUE

FINAL DISPLAY

5	———	Z
0	———	Y
VSWR	———	X

PRESS: CONTINUE to enter new X_1, R_1, or l.

EXAMPLE

Z_o = 50 ohms
f = 1×10^6 hertz
k = 1
X_1 = 50
R_1 = 50 } where $Z_1 = R_1 + jX_1$
l = 1.75

	In Wavelengths	In Meters
X_l =	-25.000	51.757
R_l =	25.000	53.876
θ_{ρ_ν} =	-116.565	59.232
$\lvert\rho_\nu\rvert$ =	.447	.447
VSWR=	2.618	2.618

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0-0	CLEAR	20									
0-1	x→( )	23									
0-2	b	14									
0-3	x→( )	23									
0-4	a	13									
0-5	STOP	41	k	f	Z_0	ENTER Z_0, f, k					
0-6	÷	35									
0-7	2	02									
0-8	9	11									
0-9	9	11									
0-a	7	07									
0-b	7	07									
0-c	6	06									
0-d	ENTER EXP	26									
1-0	8	10									
1-1	÷	35									
1-2	y→( )	40									
1-3	c	16									
1-4	ROLL ↑	22									
1-5	x→( )	23									
1-6	d	17									
1-7 (FROM 8-9)	1	01									
1-8	↑	27									
1-9	a	13									
1-a	↑	27									
1-b	b	14									
1-c	STOP	41	R_1	X_1	1	ENTER X_1, R_1					
1-d	x→( )	23									
2-0	b	14									
2-1	y→( )	40									
2-2	a	13									
2-3	2	02									
2-4	ROLL ↓	31									
2-5	RCL	61									
2-6	STOP	41	$\pm l$	0	2	ENTER $\pm l$					
2-7	↑	27									
2-8	3	03									
2-9	6	06									
2-a	0	00	CALCULATE TAN βl (steps 2-7 to 2-d)								
2-b	×	36									
2-c	c	16									
2-d	IF FLAG	43									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	×	36									
	1	CLEAR x	37									
	2	↓	25									
	3	tan x	71									
	4	↑	27									
	5	↑	27									
	6	|y|	55									
	7	ENTER EXP	26									
	8	9	11									
	9	IF x < y	52	TEST FOR $TAN\beta l > 10^9$, IF SO, SOLVE $Z_l = \frac{Z_0^2}{Z_1}$								
	a	8	10									
	b	a	13									
	c	↓	25									
	d	d	17									
4	0	×	36									
	1	a	13									
	2	+	33									
	3	b	14									
	4	TO POLAR	62									
	5	ln x	65									
	6	ACC +	60									
	7	b	14									
	8	ROLL ↑	22									
	9	×	36									
	a	↑	27									
	b	a	13									
	c	CHG SIGN	32									
	d	×	36									
5	0	d	17									
	1	+	33									
	2	↓	25	CALCULATE $Z_l = Z_0\left[\frac{Z_1 + jZ_0 TAN\beta l}{Z_0 + jZ_1 TAN\beta l}\right]$								
	3	TO POLAR	62									
	4	ln x	65									
	5	ACC −	63									
	6	RCL	61									
	7	ACC −	63									
	8	e^x	74									
	9	↑	27									
	a	d	17									
	b	×	36									
	c	3	03									
	d	ROLL ↓	31									

← FROM 9-9 (to step 5-c)

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	TO RECT	66									
	1	STOP	41	R_l	X_l	3	DISPLAY					
	2	↑	27									
	3	d	17									
	4	−	34									
	5	ROLL ↓	31									
	6	TO POLAR	62									
	7	ln x	65									
	8	ACC +	60									
	9	e^x	74									
	a	TO RECT	66									
	b	ROLL ↑	22	CALCULATE REFLECTION COEFFICIENT ρ_v								
	c	+	33									
	d	+	33									
7	0	4	04									
	1	ROLL ↓	31									
	2	TO POLAR	62									
	3	ln x	65									
	4	ACC −	63									
	5	RCL	61									
	6	e^x	74									
	7	STOP	41	$\lvert\rho_v\rvert$	θ_{ρ_v}	4	DISPLAY					
	8	↑	27									
	9	↑	27									
	a	1	01									
	b	+	33									
	c	ROLL ↑	22									
	d	−	34	CALCULATE VSWR								
8	0	5	05									
	1	ROLL ↓	31									
	2	÷	35									
	3	CLEAR x	37									
	4	$x \rightleftarrows y$	30									
	5	STOP	41	VSWR	0	5	DISPLAY					
	6	CLEAR	20									
	7	GO TO ( )( )	44									
	8	1	01									
	9	7	07									
FROM 3-b →	a	d	17									
	b	↑	27									
	c	×	36									
▲	d	a	13									

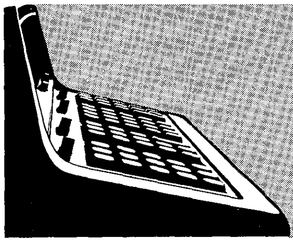
▲ Denotes Revision

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	↑	27									
	1	b	14									
	2	TO POLAR	62									
	3	x⇄y	30									
	4	CHG SIGN	32									
	5	ROLL ↓	31									
	6	÷	35									
	7	GO TO ( )( )	44									
	8	5	05									
	9	c	16									
	a	END	46									
	b											
	c											
	d											

Steps 90–99: CALCULATE SPECIAL CASE FOR TAN $\beta l > 10^9$

$$Z_l = \frac{Z_0^2}{Z_1}$$

▲ Denotes Revision

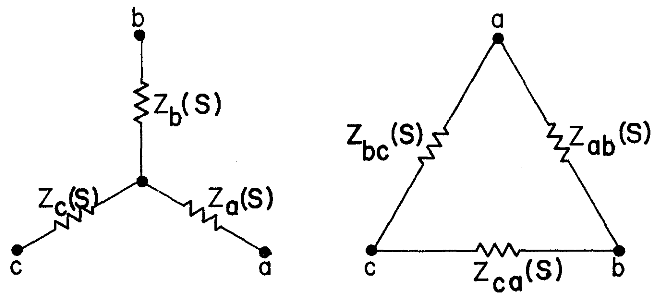


WYE→DELTA and/or DELTA→WYE CONVERSION

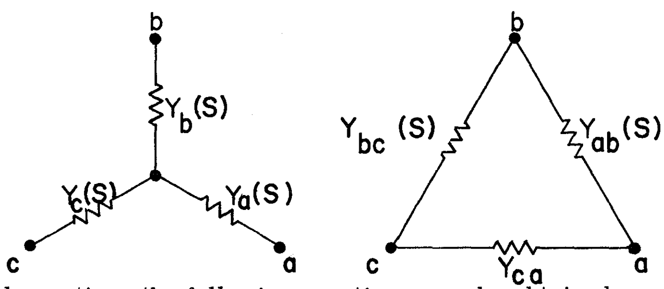
PART NO.
09100-71010

The following notation is used:

Delta→Wye Conversion:



Wye→Delta Conversion:



By loop and nodal equations the following equations may be obtained:

Delta→Wye:

$$Z_a(S) = \frac{Z_{ab}(S)\,Z_{ca}(S)}{Z_{ab}(S) + Z_{bc}(S) + Z_{ca}(S)}$$

$$Z_b(S) = \frac{Z_{bc}(S)\,Z_{ab}(S)}{Z_{ab}(S) + Z_{bc}(S) + Z_{ca}(S)}$$

$$Z_c(S) = \frac{Z_{bc}(S)\,Z_{ca}(S)}{Z_{ab}(S) + Z_{bc}(S) + Z_{ca}(S)}$$

Wye→Delta:

$$Y_{ab}(S) = \frac{Y_a(S)\;Y_b(S)}{Y_a(S) + Y_b(S) + Y_c(S)}$$

$$Y_{bc}(S) = \frac{Y_b(S)\;Y_c(S)}{Y_a(S) + Y_b(S) + Y_c(S)}$$

$$Y_{ca}(S) = \frac{Y_c(S)\;Y_a(S)}{Y_a(S) + Y_b(S) + Y_c(S)}$$

Because of the similarity of the two sets of formulas, the same basic relationship is used in the program for either wye to delta or delta to wye conversion. Thus, circuit values are given as impedances when converting from Delta to Wye, and as admittances when converting from Wye to Delta.

Reference: Introductory Circuit Analysis - S. Ivar Pearson and George J. Maler
Publisher - John Wiley & Sons, Inc. 1965

USER INSTRUCTIONS

Notes: To convert from Delta to Wye, the inputs must be in ohms (Ω) and in rectangular form, i. e. Z = R + jX.

To convert from Wye to Delta, the inputs must be in mhos (℧) and in rectangular form, i. e. Y = G + jB.

Outputs are in the same dimensions as the inputs and are in rectangular form.

If one wishes to transform impedances from Wye to Delta or admittances from Delta to Wye then the data must be inverted before input by displaying each input in Polar form, (use TO POLAR key), taking the inverse of the magnitude, changing the sign of the angle and after placing these quantities in the X and Y registers respectively pressing the TO RECT key.

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

Value	Register
0	Z
0	Y
1	X

ENTER DATA: X_{ab} or B_b → Y, R_{ab} or G_b → X

PRESS: CONTINUE

DISPLAY

Value	Register
0	Z
0	Y
2	X

ENTER DATA: X_{bc} or B_c → Y, R_{bc} or G_c → X

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

DISPLAY

Value	Register
0	Z
0	Y
3	X

ENTER DATA: X_{ca} or B_a → Y, R_{ca} or G_a → X

PRESS: CONTINUE

DISPLAY

Value	Register
1	Z
B_{ab} or X_a	Y
G_{ab} or R_a	X

PRESS: CONTINUE

Value	Register
2	Z
B_{bc} or X_b	Y
G_{bc} or R_b	X

PRESS: CONTINUE

Value	Register
3	Z
B_{ca} or X_c	Y
G_{ca} or R_c	X

Note: To reset problem, re-enter program card and proceed with user instructions.

EXAMPLES

(A) Wye → Delta

Input: $Y_b = 2 + j0$ ℧ Solution: $Y_{ab} = .3333$ ℧

$Y_c = 3 + j0$ ℧ $Y_{bc} = 1.0000$ ℧

$Y_a = 1 + j0$ ℧ $Y_{ca} = .5000$ ℧

(B) Delta → Wye

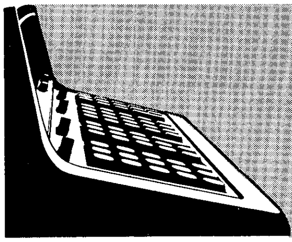
Input: $Z_{ab} = 0 + j5\Omega$ Solution: $Z_a = 2.5 + j2.5\Omega$

$Z_{bc} = 0 + j5\Omega$ $Z_b = -1.25 + j1.25\Omega$

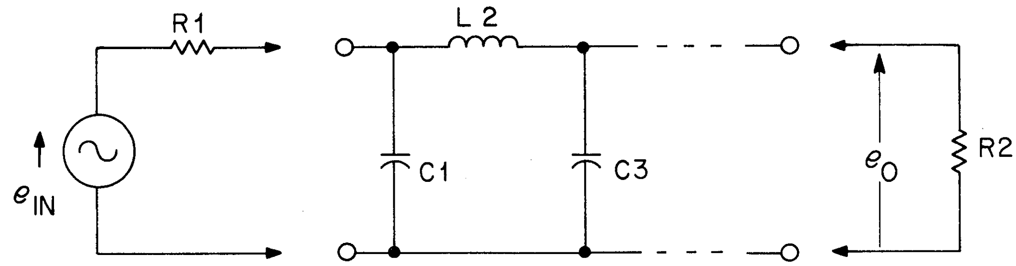
$Z_{ca} = 10 + j0\Omega$ $Z_c = 2.5 + j2.5\Omega$

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a)
0 0	CLEAR	20				
1	x→( )	23				
2	7	07				
3	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY			
4	STOP	41	R_{ab} or G_b	X_{ab} or B_b	0	ENTER X_{ab}, R_{ab} or G_b, B_b
5	ACC +	60				
6	TO POLAR	62	STORE Z_{ab} OR Y_b IN POLAR FORM (steps 6–a)			
7	x→( )	23				
8	d	17				
9	y→( )	40				
a	c	16				
b	↓	25				
c	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY			
d	STOP	41	R_{bc} or G_c	X_{bc} or B_c	0	ENTER X_{bc}, R_{bc} or G_c, B_c
1 0	ACC +	60	CALCULATE PARTIAL DENOMINATOR			
1	TO POLAR	62	STORE Z_{bc} OR Y_c IN POLAR FORM (steps 1–5)			
2	x→( )	23				
3	8	10				
4	y→( )	40				
5	9	11				
6	↓	25				
7	3	03	DISPLAY 3 TO INDICATE THIRD ENTRY			
8	STOP	41	R_{ca} or G_a	X_{ca} or B_a	0	ENTER X_{ca}, R_{ca} or G_a, B_a
9	ACC +	60	FINISH CALCULATING DENOMINATOR			
a	TO POLAR	62	STORE Z_{ca} OR Y_a IN POLAR FORM (steps 1a–2 0)			
b	x→( )	23				
c	b	14				
d	y→( )	40				
2 0	a	13				
1	RCL	61	RECALL DENOMINATOR IN RECTANGULAR FORM AND CLEAR ACCUMULATORS (steps 1–2)			
2	ACC −	63				
3	TO POLAR	62	STORE DENOMINATOR IN POLAR FORM (steps 3–4)			
4	ACC +	60				
5 (FROM 5-7)	c	16	CALCULATE ANGLE OF EQUIVALENT ELEMENT (steps 5–a)			
6	↑	27				
7	a	13				
8	+	33				
9	e	12				
a	−	34				
b	y⇄( )	24	INCREMENT AND STORE COUNT FOR DISPLAY (steps b–d)			
c	7	07				
d	1	01				

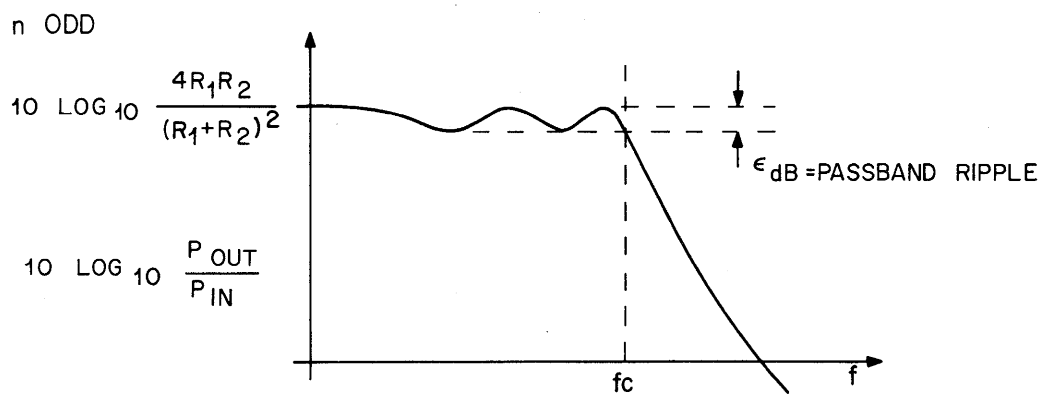
Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	Remarks
3	0	+	33										
	1	y⇄( )	24										
	2	7	07										
	3	d	17										CALCULATE MAGNITUDE OF EQUIVALENT ANGLE
	4	↑	27										
	5	b	14										
	6	×	36										
	7	f	15										
	8	÷	35										
	9	ROLL ↑	22										RECALL COUNT, POSITION, AND DISPLAY ELEMENT IN RECTANGULAR FORM
	a	y⇄( )	24										
	b	7	07										
	c	y→( )	40										
	d	7	07										
4	0	ROLL ↑	22										
	1	TO RECT	66										
	2	STOP	41	X_a	Y_a	0							DISPLAY
	3	y⇄( )	24										ROTATE REGISTER CONTENTS IN PREPARATION FOR NEXT EQUIVALENT ELEMENT
	4	8	10										
	5	y⇄( )	24										
	6	d	17										
	7	y⇄( )	24										
	8	b	14										
	9	y→( )	40										
	a	8	10										
	b	y⇄( )	24										
	c	9	11										
	d	y⇄( )	24										
5	0	c	16										
	1	y⇄( )	24										
	2	a	13										
	3	y→( )	40										
	4	9	11										
	5	GO TO ( )( )	44										BRANCH TO CALCULATE NEXT ELEMENT
	6	2	02										
	7	5	05										
	8	END	46										
	9												
	a												
	b												
	c												
	d												

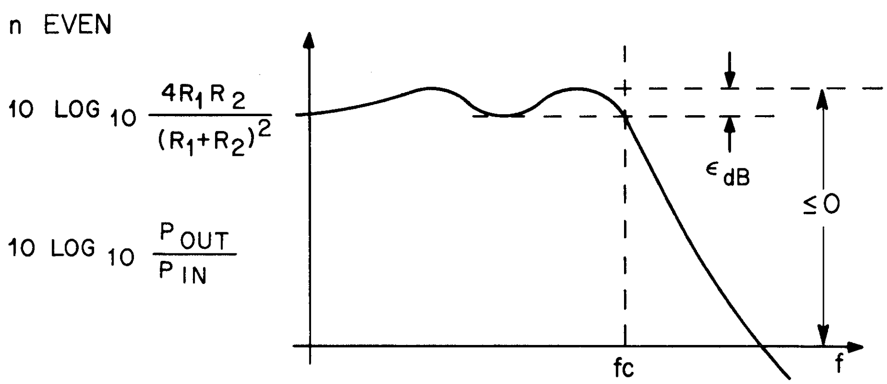


This program computes component values for Tchebysheff lowpass filters between equal or unequal finite terminations. The network designed is of the form:



The first element of the filter is a shunt C. The order (no. of poles) may be even or odd but if the order is even the mismatch loss between the terminating impedances must be equal to or greater than the ripple.





User specifies:

- Pass Band Ripple - ϵ_{dB} (dB)
- Corner (- ϵ_{dB}) Frequency - f_c (Hz)
- Filter Order - n
- Load Resistance - R_2 (Ω)
- Generator Resistance - R_1 (Ω)

The equations used are:

$$C_1 = \frac{4 \sin \frac{\pi}{2n}}{(\xi - \eta)\omega_c R_1}$$

$$L_{K+1} = \frac{16 \left(\sin \frac{\pi(2K-1)}{2n}\right)\left(\sin \frac{\pi(2K+1)}{2n}\right)}{\left(\xi^2 + \eta^2 + 4\sin^2 \frac{\pi K}{n} - 2\xi\eta \cos \frac{\pi K}{n}\right) C_K \omega_c^2}$$

$$C_{K+1} = \frac{16 \left(\sin \frac{\pi(2K-1)}{2n}\right)\left(\sin \frac{\pi(2K+1)}{2n}\right)}{\left(\xi^2 + \eta^2 + 4\sin^2 \frac{\pi K}{n} - 2\xi\eta \cos \frac{\pi K}{n}\right) L_K \omega_c^2}$$

$$K = 1, 2, \ldots, n$$

where $$\xi = 2 \sinh \left\{ \frac{1}{n} \sinh^{-1} \frac{1}{\epsilon} \right\}$$

$$\eta = 2 \sinh \left\{ \frac{1}{n} \ln \left(\frac{\sqrt{\epsilon^2 + 1 - A} + \sqrt{1 - A}}{\epsilon} \right) \right\}$$

$$\eta = \sqrt{10^{.1\epsilon \mathrm{dB}} - 1}$$

$$A = \frac{4R_1R_2}{(R_1 + R_2)^2} \text{ (n odd)}$$

or

$$A = \frac{4R_1R_2}{(R_1 + R_2)^2} \times 10^{.1\epsilon} \text{ dB (n even)}$$

NOTE: $A \leq 1$

In general, Tchebysheff filter designs with unequal terminations the solution will be non-unique. Filters designed by other methods may differ considerably from those formed by use of this program. This does not imply that either method is incorrect.

Reference: Network Analysis and Synthesis
by Louis Weinberg

McGraw-Hill 1962

USER INSTRUCTIONS

SET: [ ] [RADIANS] [FLOATING] [ ]

PRESS: END

→ ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $\epsilon_{dB} \rightarrow Z$, $f_c \rightarrow Y$, $n \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: $R_2 \rightarrow Y$, $R_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

n	Z
1	Y
C_1	X

PRESS: CONTINUE

→ DISPLAY

n	Z
K	Y
L_K	X

K EVEN

USER INSTRUCTIONS (Con't)

n	Z
K	Y
C_K	X

K ODD

After K = n the display clears. To solve another problem :

PRESS: END

For filters with L input, replace STEP (9 - 1) with a (X).

EXAMPLES

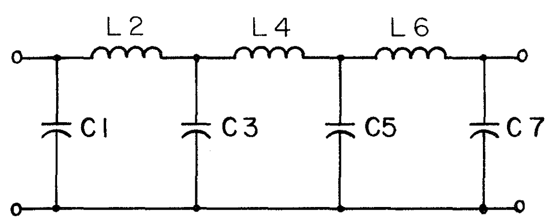
ϵ_{dB} = .1 dB ripple

f_c = 620,000 Hz

n = 6 (Filter Order)

R_2 = 600 (Load Impedance)

R_1 = 135 (Generator Impedance)



C_1 = .0117 μF

L_2 = 13.71 μH

C_3 = .0143 μF

L_4 = 12.94 μH

C_5 = .0113 μF

L_6 = 5.030 μH

NOTE: If execution stops at location +(3)(a) the attempted maximum power gain exceeds 1.

Gain at D. C.	Z
Attempted Max Power Gain	Y
– – – –	X

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
(+) 01	1	01	ENTER		
02	STOP	41	n	f_c	ϵ_{db}
03	x→( )	23			
04	b	14			
05	y→( )	40			
06	c	16			
07	↓	25			
08	1	01			
09	0	00			
0a	ln x	65			
0b	×	36			
0c	•	21			
0d	1	01			
10	×	36			
(+) 11	↓	25			
12	e^x	74			
13	x→( )	23			
14	d	17			
15	CLEAR	20			
16	2	02	ENTER		
17	STOP	41	R_1	R_2	0
18	ACC +	60			
19	+	33			
1a	↓	25			
1b	↑	27			
1c	×	36			
1d	↑	27			
20	RCL	61			
(+) 21	×	36			
22	↓	25			
23	x⇄y	30			
24	÷	35			
25	4	04			
26	×	36			
27	b	14			
28	↑	27			
29	2	02			
2a	÷	35			
2b	↓	25			
2c	↑	27			
2d	int x	64			

Step	Key	Code	Display x	Display y	Display z
30	IF x<y	52			
(+) 31	3	03			
32	b	14			
33	↓	25			
34	d	17			
35	×	36			
36	1	01			
37	IF x>y	53			
38	3	03			
39	c	16			
3a	STOP	41			
3b	↓	25			
3c	y→( )	40			
3d	e	12			
40	d	17			
(+) 41	↑	27			
42	1	01			
43	−	34			
44	↓	25			
45	√x	76			
46	x→( )	23			
47	d	17			
48	↑	27			
49	×	36			
4a	1	01			
4b	↑	27			
4c	e	12			
4d	−	34			
50	↓	25			
(+) 51	↑	27			
52	√x	76			
53	ROLL ↓	31			
54	+	33			
55	↓	25			
56	√x	76			
57	+	33			
58	d	17			
59	÷	35			
5a	↓	25			
5b	ln x	65			
5c	GO TO ( )	44			
5d	▲SUB▼	77			

Step	Key	Code	Display x	Display y	Display z
60	a	13			
(+) 61	2	02			
62	y→( )	40			
63	e	12			
64	1	01			
65	↑	27			
66	d	17			
67	÷	35			
68	↓	25			
69	arc ▼	72			
6a	hyper ▼	67			
6b	sin x	70			
6c	GO TO ( )	44			
6d	▲SUB▼	77			
70	a	13			
(+) 71	2	02			
72	y→( )	40			
73	d	17			
74	c	16			
75	↑	27			
76	2	02			
77	×	36			
78	π	56			
79	×	36			
7a	y→( )	40			
7b	c	16			
7c	↑	27			
7d	2	02			

Storage

Register	Contents
f	+ R_2
e	R_1
d	$10^{.1\epsilon_{db}}$
c	f_c
b	n
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
8 0 (+)	÷	35			
1	b	14			
2	÷	35			
3	↓	25			
4	sin x	70			
5	↑	27			
6	d	17			
7	↑	27			
8	e	12			
9	−	34			
a	↓	25			
b	÷	35			
c	c	16			
d	÷	35			
9 0 (+)	f	15			
1	÷	35			
2	4	04			
3	×	36			
4	b	14			
5	↑	27			
6	1	01			
7	ROLL ↑	22			
8	x→( )	23			
9	f	15	DISPLAY		
a	STOP	41	C1	1	n
b	CONT	47			
c	GO TO ( )	44			
d	−	34			
a 0 (+)	0	00			
1	0	00			
2	↑	27			
3	b	14			
4	÷	35			
5	↓	25			
6	hyper ▾	67			
7	sin x	70			
8	↑	27			
9	2	02			
a	×	36			
b	RETURN	77			
c					
d					

Step	Key	Code	Display x	Display y	Display z
0 0 (−)	c	16			
1	↑	27			
2	×	36			
3	y→( )	40			
4	c	16			
5	1	01			
6	x→( )	23			
7	a	13			
8	↑	27			
9	π	56			
a	×	36			
b	b	14			
c	÷	35			
d	↓	25			
1 0 (−)	↑	27			
1	sin x	70			
2	↑	27			
3	×	36			
4	4	04			
5	×	36			
6	ROLL ↑	22			
7	cos x	73			
8	x⇄y	30			
9	e	12			
a	×	36			
b	d	17			
c	×	36			
d	2	02			
2 0 (−)	×	36			
1	↓	25			
2	−	34			
3	e	12			
4	↑	27			
5	×	36			
6	↓	25			
7	+	33			
8	d	17			
9	↑	27			
a	×	36			
b	↓	25			
c	+	33			
d	c	16			

Step	Key	Code	Display x	Display y	Display z
3 0 (−)	×	36			
1	f	15			
2	×	36			
3	1	01			
4	6	06			
5	÷	35			
6	y→( )	40			
7	f	15			
8	a	13			
9	↑	27			
a	b	14			
b	÷	35			
c	↑	27			
d	÷	35			
4 0 (−)	÷	35			
1	2	02			
2	÷	35			
3	↓	25			
4	−	34			
5	π	56			
6	×	36			
7	↓	25			
8	sin x	70			
9	↑	27			
a	f	15			
b	÷	35			
c	y→( )	40			
d	f	15			

Storage

Register	Value
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step		Key	Code	Display x	Display y	Display z
5	0	↓	25			
(–)	1	1	01			
	2	↑	27			
	3	b	14			
	4	÷	35			
	5	2	02			
	6	÷	35			
	7	↓	25			
	8	+	33			
	9	π	56			
	a	×	36			
	b	↓	25			
	c	sin x	70			
	d	↑	27			
6	0	f	15			
(–)	1	×	36			
	2	y→( )	40			
	3	f	15			
	4	a	13			
	5	↑	27			
	6	1	01			
	7	+	33			
	8	y→( )	40			
	9	a	13			
	a	b	14			
	b	↑	27			
	c	a	13			
	d	↑	27			
7	0	f	15	DISPLAY		
(–)	1	STOP	41	C_k or L_k	K	n
	2	CONT	47			
	3	↓	25			
	4	IF x<y	52			
	5	0	00			
	6	7	07			
	7	CLEAR	20			
	8	END	46			
	9					
	a					
	b					
	c					
	d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	Display		
			x	*y*	*z*
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	*y*	*z*
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display		
			x	*y*	*z*
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

S PARAMETER TO Y PARAMETER CONVERSION

This program converts S parameters for linear (active or passive) circuits to Y parameters. The equations used are as follows:

$$y_{11} = \frac{s_{22} + 1}{D} - \frac{1}{R_o} \qquad y_{21} = \frac{-s_{21}}{D}$$

$$y_{12} = \frac{-s_{12}}{D} \qquad y_{22} = \frac{s_{11} + 1}{D} - \frac{1}{R_o}$$

$$D = \frac{1}{2} R_o \left[(1 + s_{11}) \ (1 + s_{22}) \ - s_{12} \cdot s_{21} \right]$$

where R_o is the characteristic impendance of the S parameter measuring system.

The Matrix form is

$$R_o Y = 2 (S + I)^{-1} - I$$

thus $$S = 2 (R_o Y + I)^{-1} - I$$

The input and output format subroutines allow flexibility in the entering and outputing of data. These subroutines translate into and out of the standard internal format - polar form with magnitude expressed in natural logrithmic form. A similar program can be used for the reverse transformation; the input and output sections need be changed since S will usually be expressed in dB and degrees and Y is Siemens (Mho) in rectangular coordinators.

Similar programs could be written to convert S to Z or Z to S as follows:

$$Z = R_o \left[2 \ (I - S)^{-1} - I \right]$$

$$S = I - 2 \left(\frac{1}{R_o} Z + I \right)^{-1}$$

or convert from Z to Y or Y to Z as follows:

$$Z = Y^{-1}$$

$$Y = Z^{-1}$$

This program requires s_{11}, s_{12}, s_{21}, s_{22} in dB and degrees

and R_o in ohms

Reference: Hewlett-Packard Journal, February, 1967

USER INSTRUCTIONS

SET: [ ] DEGREES [ ] FIXED

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
11	X

ENTER DATA: $\theta^{o}_{11} \rightarrow Y$, $R_{11\ dB} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
12	X

ENTER DATA: $\theta^{o}_{12} \rightarrow Y$, $R_{12\ dB} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
21	X

ENTER DATA: $\theta^{o}_{21} \rightarrow Y$, $R_{21\ dB} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
22	X

ENTER DATA: $\theta^{o}_{22} \rightarrow Y$, $R_{22\ dB} \rightarrow X$

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $R_o \rightarrow X$

PRESS: CONTINUE

DISPLAY

11	Z
B_{11}	Y
G_{11}	X

$Y_{11} = G_{11} + jB_{11}$

PRESS: CONTINUE

DISPLAY

12	Z
B_{12}	Y
G_{12}	X

$Y_{12} = G_{12} + jB_{12}$

PRESS: CONTINUE

DISPLAY

21	Z
B_{21}	Y
G_{21}	X

$Y_{21} = G_{21} + jB_{21}$

PRESS: CONTINUE

DISPLAY

22	Z
B_{22}	Y
G_{22}	X

$Y_{22} = G_{22} + jB_{22}$

To run another case: ↑ (return to PRESS: CONTINUE)

EXAMPLE

S parameters for 2N3478 transistor in common emitter configuration, at 100 MHZ I_c = 10 ma Vce = 5V

General form

$$S_{ij} = R_{ij} \angle \theta_{ij}$$

$$S_{11} = -10.09 \angle -103^{o}$$

$$S_{12} = -29.63 \angle 60^{o}$$

$$S_{21} = 17.21 \angle 102^{o}$$

$$S_{22} = -1.67 \angle -8^{o}$$

$$R_{o} = 50\,\Omega$$

General form

$$y_{ij} = (G_{ij} + jB_{ij})$$

$$y_{11} = (14.34 + j10.97)\ \text{mS}$$

$$y_{12} = (-98.49 - j645.94)\ \mu\text{S}$$

$$y_{21} = (78.91 - j119.99)\ \text{mS}$$

$$y_{22} = (-657.70 + j1070.83)\ \mu\text{S}$$

Step	Key	Code
00	CLEAR	20
01	1	01
02	1	01
03	GO TO ( )	44
04	▲SUB▼	77
05	7	07
06	3	03
07	x→( )	23
08	b	14
09	y→( )	40
0a	a	13
0b	CLEAR	20
0c	1	01
0d	2	02
10	GO TO ( )	44
11	▲SUB▼	77
12	7	07
13	3	03
14	x→( )	23
15	9	11
16	y→( )	40
17	8	10
18	CLEAR	20
19	2	02
1a	1	01
1b	GO TO ( )	44
1c	▲SUB▼	77
1d	7	07
20	3	03
21	x→( )	23
22	−	34
23	f	15
24	y→( )	40
25	−	34
26	e	12
27	CLEAR	20
28	2	02
29	2	02
2a	GO TO ( )	44
2b	▲SUB▼	77
2c	7	07
2d	3	03

Step	Key	Code
30	x→( )	23
31	d	17
32	y→( )	40
33	c	16
34	GO TO ( )	44
35	5	05
36	a	13
37	c	16
38	↑	27
39	d	17
3a	e^x	74
3b	TO RECT	66
3c	↑	27
3d	f	15
40	+	33
41	↓	25
42	TO POLAR	62
43	ln x	65
44	x→( )	23
45	d	17
46	y→( )	40
47	c	16
48	a	13
49	↑	27
4a	b	14
4b	e^x	74
4c	TO RECT	66
4d	↑	27
50	f	15
51	+	33
52	↓	25
53	TO POLAR	62
54	ln x	65
55	x→( )	23
56	b	14
57	y→( )	40
58	a	13
59	RETURN	77
5a	1	01
5b	•	21
5c	x→( )	23
5d	f	15

Step	Key	Code	Display
60	GO TO ( )	44	
61	▲SUB▼	77	
62	3	03	
63	7	07	
64	CLEAR	20	
65	a	13	
66	↑	27	
67	c	16	
68	+	33	
69	b	14	
6a	↑	27	
6b	d	17	
6c	+	33	
6d	GO TO ( )	44	
70	−	34	
71	0	00	
72	0	00	ENTER
73	STOP	41	R ∡ 0
74	↑	27	
75	1	01	
76	0	00	
77	÷	35	
78	ln x	65	
79	×	36	
7a	2	02	
7b	÷	35	
7c	↓	25	
7d	RETURN	77	

Storage

Register	Contents
f	R_{21}
e	θ_{21}
d	R_{22}
c	θ_{22}
b	R_{11}
a	θ_{11}
9	R_{12}
8	θ_{12}
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
00	↓	25			
01	e^x	74			
02	TO RECT	66			
03	ACC +	60			
04	x←( )	67			
05	8	10			
06	↑	27			
07	x←( )	67			
08	−	34			
09	e	12			
0a	+	33			
0b	x←( )	67			
0c	9	11			
0d	↑	27			
10	x←( )	67			
11	−	34			
12	f	15			
13	+	33			
14	↓	25			
15	e^x	74			
16	TO RECT	66			
17	ACC −	63			
18	RCL	61			
19	ACC −	63			
1a	TO POLAR	62			
1b	↑	27			
1c	2	02			
1d	÷	35			
20	↓	25			
21	ln x	65			
22	ACC −	63			
23	a	13			
24	x⇄y	30			
25	−	34			
26	b	14			
27	↑	27			
28	f	15			
29	+	33			
2a	y→( )	40			
2b	b	14			
2c	c	16			
2d	ROLL ↑	22			
30	x→( )	23			
31	a	13			
32	e	12			
33	+	33			
34	d	17			
35	↑	27			
36	f	15			
37	+	33			
38	↓	25			
39	x→( )	23			
3a	d	17			
3b	y→( )	40			
3c	c	16			
3d	y⇄( )	24			
40	8	10			
41	e	12			
42	+	33			
43	x←( )	67			
44	9	11			
45	↑	27			
46	f	15			
47	+	33			
48	↓	25			
49	x→( )	23			
4a	9	11			
4b	1	01			
4c	CHG SIGN	32			
4d	arc ▾	72			
50	cos x	73			
51	+	33			
52	y→( )	40			
53	8	10			
54	y⇄( )	24			
55	−	34			
56	e	12			
57	x←( )	67			
58	−	34			
59	f	15			
5a	ACC +	60			
5b	RCL	61			
5c	x→( )	23			
5d	−	34			
60	f	15			
61	1	01			
62	CHG SIGN	32			
63	arc ▾	72			
64	cos x	73			
65	+	33			
66	y→( )	40			
67	−	34			
68	e	12			
69	CLEAR	20	ENTER		
6a	STOP	41	R_0	0	0
6b	ln x	65			
6c	↑	27			
6d	1	01			
70	ACC −	63			
71	GO TO ( )	44			
72	▲SUB▾	77			
73	+	33			
74	3	03			
75	7	07			
76	b	14			
77	↑	27			
78	e	12			
79	+	33			
7a	y→( )	40			
7b	b	14			
7c	y⇄( )	24			
7d	d	17			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
8 0	+	33			
(−) 1	y→( )	40			
2	d	17			
3	y⇄( )	24			
4	9	11			
5	+	33			
6	y→( )	40			
7	9	11			
8	y⇄( )	24			
9	−	34			
a	f	15			
b	+	33			
c	y→( )	40			
d	−	34			
9 0	f	15			
(−) 1	GO TO ( )	44			
2	9	11			
3	9	11			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	c	16			
a	↑	27			
b	d	17			
c	GO TO ( )	44			
d	▲SUB▼	77			
a 0	d	17			
(−) 1	5	05			
2	1	01			
3	1	01			
4	ROLL ↓	31	DISPLAY		
5	STOP	41	G_{11}	B_{11}	11
6	y⇄( )	24			
7	8	10			
8	x←( )	67			
9	9	11			
a	GO TO ( )	44			
b	▲SUB▼	77			
c	d	17			
d	5	05			
b 0	1	01			
(−) 1	2	02			
2	ROLL ↓	31	DISPLAY		
3	STOP	41	G_{12}	B_{12}	12
4	y⇄( )	24			
5	−	34			
6	e	12			
7	x←( )	67			
8	−	34			
9	f	15			
a	GO TO ( )	44			
b	▲SUB▼	77			
c	d	17			
d	5	05			
c 0	2	02			
(−) 1	1	01			
2	ROLL ↓	31	DISPLAY		
3	STOP	41	G_{21}	B_{21}	21
4	a	13			
5	↑	27			
6	b	14			
7	GO TO ( )	44			
8	▲SUB▼	77			
9	d	17			
a	5	05			
b	2	02			
c	2	02			
d	ROLL ↓	31	DISPLAY		
d 0	STOP	41	G_{22}	B_{22}	22
(−) 1	GO TO ( )	44			
2	+	33			
3	0	00			
4	0	00			
5	e^x	74			
6	TO RECT	66			
7	↑	27			
8	RETURN	77			
9	END	46			

Storage

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

Step	Key	Code	Display x	Display y	Display z	Step	Key	Code	Display x	Display y	Display z
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

FREQUENCY RESPONSE

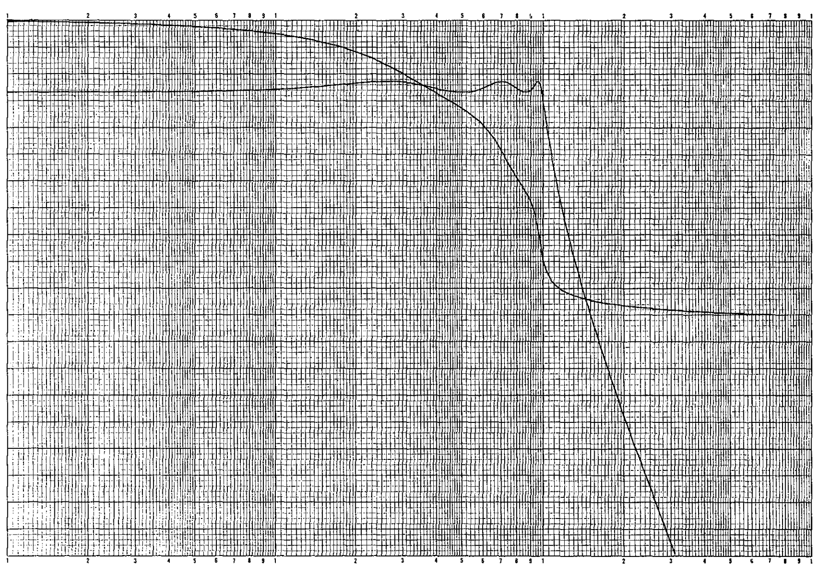
This program calculates the forced frequency response of a systems described by a ratio $\left[f(s) \right]$ of factored polynomials.

$$f(s) = \frac{(s - r_1)(s - r_2) \ldots}{(s - r_a)(s - r_b) \ldots}$$

where $r_j = \alpha + jw$

The program is limited to a maximum of six complex roots. That is, f(s) may have six zeros and no poles or six poles and no zeros, two zeros and four poles, or any other combination of poles and zeros which has a maximum of six critical frequencies. Less than 6 natural frequencies may be considered by inputting zeros of the form $0 + 0_j$. The frequency response of arbitrary mathematical functions may be calculated since the program does not require complex conjugate root pairs.

NOTE: It should be emphasized that only the forced or steady state response is obtained. All natural responses must be negligible if the calculated response is to be compared with experimental results.



USER INSTRUCTIONS

Using the Origin Controls, locate the pen as follows:

A) For Magnitude: X = 10 in., Y depends on Yscale

B) For Phase: X = 10 in., Y = 10 in.

PRESS: CLEAR

SET: | DEGREES | |

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: GO TO (-b) (8)

SET: | PROGRAM | |

PRESS: 4 if the magnitude response is desired
5 if the phase response is desired.

SET: | | RUN |

PRESS: GO TO (-c) (5)

SET: | PROGRAM | |

ENTER: SCALING CONSTANTS for Y Axis

- c	5	N
	6	N
	7	ENTER EXP
	8	N

	50 db/in	20db/in	10 db/in	100°/in	50°/in	25°/in
-c 5	1	2	5	5	1	2
6	0	5	0	0	0	0
8	2	1	1	0	1	1

SET: | | RUN |

PRESS: GO TO (-c) (d)

SET: | PROGRAM | |

ENTER: SCALING CONSTANTS for X Axis

- c	d	N
- d	0	N
	1	ENTER EXP
	2	N

USER INSTRUCTIONS (Con't)

	2.5 in/decade	5 in/decade	10 in/decade
-cd	1	2	5
-d 0	2	5	0
1	5		
2	0	3	3

SET: | | RUN |

PRESS: END

ENTER DATA: Z.P → Z*, $W_{initial}$ → Y, ΔW → X (ΔW > 1)

PRESS: CONTINUE

→ ENTER DATA: r_i from complex factors (s - r_i) where $r_i = a_i + jb_i$
Zeros are input first followed by poles.
b_i → Y, a_i → X

PRESS: CONTINUE

REPEAT ENTERING r_i until i = 6.
The magnitude (or phase) response is plotted after entry of the 6th zero or pole. To alter ΔW during execution,

PRESS: PAUSE

ENTER DATA: New ΔW → C

PRESS: CONTINUE

To plot the other parameter (phase/magnitude) do the following:

PRESS: GO TO (-b) (8)

SET: | PROGRAM | |

PRESS: 4 or 5

SET: | | RUN |

PRESS: GO TO (-c) (5)

SET: | PROGRAM | |

ENTER SCALING CONSTANTS for Y Axis

SET: | | RUN |

* Z.P represent Z(number of zeroes) and P(number of poles). For example, a filter with 2 zeroes and 4 poles would be entered as 2.4 .

USER INSTRUCTIONS (Con't)

PRESS: GO TO - (0) (0)

RE-INITIALIZE BY: Winitial → b, Z.P → a

Using Origin Controls, locate the pen as follows:

A) For Magnitude: X = 10 in., Y depends on Y scale
B) For Phase: X = 10 in., Y = 10 in.

PRESS: CONTINUE

The phase (magnitude) response is now plotted.

To run a new case, REPEAT the USER INSTRUCTIONS.

NOTE: To convert the X Axis Scaling Constants to centimeters, divide the -- db by 2.5, i.e 50 db/in. = 20 db/cm.
To convert the Y Axis Scaling Constants to centimeters, multiply by 2.5, i.e. 2.5 in./decade = 6.25 cm./decade

EXAMPLE

6 Pole Chebyshev (2 db ripple)

Z.P = 0.6

Winitial = .01

ΔW = 1.05

$r_1 = 0.1738 + j\,0.2609$

$r_2 = 0.1738 - j\,0.2609$

$r_3 = 0.1272 + j\,0.7128$

$r_4 = 0.1272 - j\,0.7128$

$r_5 = 0.0465 + j\,0.9737$

$r_6 = 0.0465 - j\,0.9737$

SCALING CONSTANTS

a. For magnitude 10 db/in.

b. For phase 100^{o}/in.

c. For frequency axis 5 in./decade

In setting the origin for the Magnitude plot, place the origin at X = 10 in. (25 cm.) and Y = 6 in. (15 cm.).

Step	Key	Code	Display x	Display y	Display z
0 0 (+)	y→( )	40	ΔW	W	Z.P
1	b	14	ENTER		
2	x→( )	23			
3	c	16			
4	↓	25			
5	y→( )	40			
6	a	13			
7	SET FLAG	54			
8	SET FLAG	54			
9	SET FLAG	54	ENTER		
a	STOP	41	α_1	W_1	0
b	x→( )	23			
c	—	34			
d	e	12			
1 0 (+)	y→( )	40			
1	—	34			
2	f	15	ENTER		
3	STOP	41	α_2	W_2	0
4	x→( )	23			
5	6	06			
6	y→( )	40			
7	7	07	ENTER		
8	STOP	41	α_3	W_3	0
9	x→( )	23			
a	8	10			
b	y→( )	40			
c	9	11	ENTER		
d	STOP	41	α_4	W_4	0
2 0 (+)	x→( )	23			
1	0	00			
2	y→( )	40			
3	1	01	ENTER		
4	STOP	41	α_5	W_5	0
5	y→( )	40			
6	3	03			
7	GO TO ( )	44			
8	d	17			
9	1	01			
a	CONT	47			
b	CONT	47			
c	CONT	47			
d	CONT	47			

Step	Key	Code	Display x	Display y	Display z
d 0 (+)					
1	x→( )	23			
2	2	02	ENTER		
3	STOP	41	α_6	W_6	0
4	x→( )	23			
5	4	04			
6	y→( )	40			
7	5	05			
8	SET FLAG	54			
9	SET FLAG	54			
a	SET FLAG	54			
b	a	13			
c	x→( )	23			
d	d	17			
0 0 (−)	CLEAR	20			
1	1	01			
2	ACC +	60			
3	↓	25			
4	GO TO ( )	44			
5	▲SUB▼	77			
6	8	10			
7	5	05			
8	x←( )	67			
9	—	34			
a	e	12			
b	↑	27			
c	x←( )	67			
d	—	34			
1 0 (−)	f	15			
1	GO TO ( )	44			
2	▲SUB▼	77			
3	6	06			
4	8	10			
5	1	01			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	8	10			
9	5	05			
a	x←( )	67			
b	6	06			
c	↑	27			
d	x←( )	67			

Step	Key	Code	Display x	Display y	Display z
2 0 (−)	7	07			
1	GO TO ( )	44			
2	▲SUB▼	77			
3	6	06			
4	8	10			
5	2	02			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	8	10			
9	5	05			
a	x←( )	67			
b	8	10			
c	↑	27			
d	x←( )	67			
3 0 (−)	9	11			
1	GO TO ( )	44			
2	▲SUB▼	77			
3	6	06			
4	8	10			
5	3	03			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	8	10			
9	5	05			
a	x←( )	67			
b	0	00			
c	↑	27			
d	x←( )	67			

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
4 0	1	01			
(−) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	6	06			
4	8	10			
5	4	04			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	8	10			
9	5	05			
a	x←( )	67			
b	2	02			
c	↑	27			
d	x←( )	67			
5 0	3	03			
(−) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	6	06			
4	8	10			
5	5	05			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	8	10			
9	5	05			
a	x←( )	67			
b	4	04			
c	↑	27			
d	x←( )	67			
6 0	5	05			
(−) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	6	06			
4	8	10			
5	GO TO ( )	44			
6	a	13			
7	7	07			
8	↑	27			
9	b	14			
a	+	33			
b	↓	25			
c	x⇄y	30			
d	TO POLAR	62			

Step	Key	Code	Display x	Display y	Display z
7 0	↑	27			
(−) 1	f	15			
2	x⇄y	30			
3	IF FLAG	43			
4	7	07			
5	c	16			
6	×	36			
7	↓	25			
8	ACC +	60			
9	x→( )	23			
a	f	15			
b	RETURN	77			
c	÷	35			
d	↓	25			
8 0	ACC −	63			
(−) 1	x→( )	23			
2	f	15			
3	SET FLAG	54			
4	RETURN	77			
5	↑	27			
6	a	13			
7	int x	64			
8	IF x>y	53			
9	x←( )	67			
a	RETURN	77			
b	IF FLAG	43			
c	a	13			
d	0	00			
9 0	0	00			
(−) 1	IF x=y	50			
2	SET FLAG	54			
3	SET FLAG	54			
4	a	13			
5	↑	27			
6	int x	64			
7	−	34			
8	1	01			
9	0	00			
a	×	36			
b	↓	25			
c	+	33			
d	y→( )	40			

Step	Key	Code	Display x	Display y	Display z
a 0	a	13			
(−) 1	IF FLAG	43			
2	SET FLAG	54			
3	RETURN	77			
4	IF x<y	52			
5	SET FLAG	54			
6	RETURN	77			
7	d	17			
8	x→( )	23			
9	a	13			
a	RCL	61			
b	log x	75			
c	↑	27			
d	2	02			
b 0	0	00			
(−) 1	×	36			
2	b	14			
3	ROLL ↓	31			
4	CONT	47			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	c	16			
8	4	04			
9	b	14			
a	↑	27			
b	c	16			
c	×	36			
d	y→( )	40			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
c 0	b	14			
(−) 1	GO TO ( )	44			
2	0	00			
3	0	00			
4	x⇄y	30			
5	5	05			
6	0	00			
7	ENTER EXP	26			
8	1	01			
9	×	36			
a	ROLL ↑	22			
b	log x	75			
c	x⇄y	30			
d	2	02			
d 0	5	05			
(−) 1	ENTER EXP	26			
2	3	03			
3	×	36			
4	↓	25			
5	FMT	42			
6	↓	25			
7	RETURN	77			
8					
9					
a					
b					
c					
d					

Storage

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	y	z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

September 1, 1969

9100B MECHANICS PROGRAM LISTING

72002 - TRANSCENDENTAL EQUATION (ARC INVOLUTE IN GEAR DESIGN)
Solves for the angle ϕ in radians in the following expression

$$INV(\phi) = TAN\,\phi - \phi$$

where the INV (ϕ) is given. $1x10^{-17} < \phi < 1x10^{6}$ (ϕ is in radians).

72003 - SPRING DESIGN - COMPRESSION AND EXTENSION SPRINGS
This program calculates one of three variables (d - diameter of wire, N - number of turns of wire, and D - mean coil diameter), the remaining two being set, and calculates the maximum allowable stress. Other inputs are set to predetermined values.

72004 - STRESS AND STRAIN FROM A RECTANGULAR ROSETTE
Calculates the principal strains and stresses given rectangular rosette and strain gauge inputs.

9100B ONLY

72501 - PROPERTIES OF AREAS
This program determines the properties of any area which can be approximated by a set of rectangles. The properties determined are:

1. Area
2. Moments of inertia
3. Distances from axes to the centroid
4. Products of inertia about the centroid

TRANSCENDENTAL EQUATION
(ARC INVOLUTE IN GEAR DESIGN)

This program solves the following equation for ϕ in radians for a given value of the involute of ϕ, Inv(ϕ).

$$INV(\phi) = TAN\,\phi - \phi$$

The method of solution is an iteration technique using the Newton-Raphson equation $\phi_{n+1} = \phi_n - \frac{f(\phi_n)}{f'(\phi_n)}$.

The range of $INV(\phi)$ is $1 \times 10^{-17} \le INV(\phi) \le 1 \times 10^{6}$

resulting in ϕ in the range $4.085 \times 10^{-6}\ RAD \le \phi \le 1.571\ RAD$

EXAMPLES	
INV(ϕ) = .35 rad	ϕ = .894 rad
INV(ϕ) = 1 x 10^{-17} rad	ϕ = 4.085 x 10^{-6} rad
INV(ϕ) = 1 x 10^{6} rad	ϕ = 1.571 rad

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

SET: [| RADIANS]

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

ENTER DATA: INV(ϕ) → X

PRESS: CONTINUE

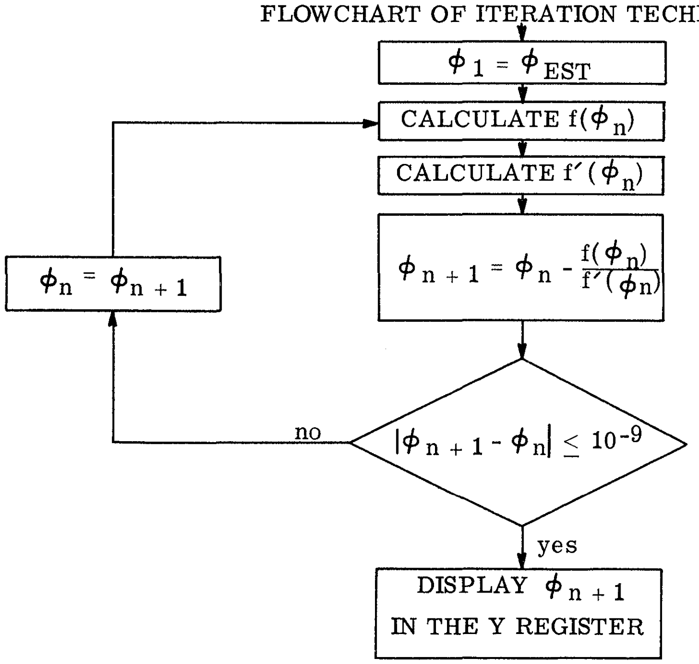
DISPLAY

$|\phi_{n+1} - \phi_n|$ — Z

ϕ_{n+1} — Y

INV(ϕ) — X

FLOWCHART OF ITERATION TECHNIQUE



Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a) / Remarks
0 0	CLEAR	20				
1	STOP	41	INV(φ)	0	0	ENTER INV(φ)
2	ACC +	60				
3	↑	27				
4	3	03				
5	×	36				
6	ROLL ↓	31				
7	ln x	65				
8	ROLL ↑	22				
9	÷	35				
a	↓	25				
b	e^x	74				
c	↑	27				
d	2	02				CALCULATE ϕ_{EST} (steps 0-2 to 1-b). IN SOME NEWTON-RAPHSON SOLUTIONS IT MAY BE NECESSARY TO MAKE A ROUGH SKETCH OF THE FUNCTION TO DETERMINE A GOOD ESTIMATE. HERE THE FIRST TWO TERMS OF THE SERIES REPRESENTING THE TANGENT WERE USED TO CALCULATE AN ESTIMATE, ϕ_{EST}, BASED ON INV(φ).
1 0	↑	27				
1	ENTER EXP	26				
2	6	06				
3	CHG SIGN	32				
4	+	33				
5	π	56				
6	$x \rightleftarrows y$	30				
7	÷	35				
8	↓	25				
9	IF $x > y$	53				
a	CONTINUE	47				
b	$x \rightleftarrows y$	30				
c (FROM 4-4)	↑	27				BEGINNING OF ITERATION LOOP
d	↑	27				
2 0	tan x	71				
1	−	34				
2	ROLL ↑	22				
3	$x \to ()$	23				CALCULATE $-f(\phi_n) = -\text{TAN}\,\phi_n + \phi_n + \text{INV}(\phi)$ (steps 1-c to 2-8)
4	e	12				
5	f	15				
6	$x \rightleftarrows y$	30				
7	ROLL ↓	31				
8	+	33				
9	↓	25				
a	ROLL ↓	31				CALCULATE $f'(\phi_n)$ (steps 2-9 to 2-b)
b	×	36				
c	↓	25				CALCULATE $-\dfrac{f(\phi_n)}{f'(\phi_n)}$ (steps 2-c to 2-d)
d	÷	35				

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	e	12	CALCULATE $\phi_{n+1} = \phi_n - \frac{f(\phi_n)}{f'(\phi_n)}$								
1	x⇄y	30									
2	+	33									
3	x⇄y	30									
4	|y|	55									
5	↑	27									
6	ENTER EXP	26									
7	0	00									
8	9	11	COMPARE $\|\phi_{n+1} - \phi_n\|$ TO 10^{-9}								
9	CHG SIGN	32	IF DIFFERENCE IS LESS THAN TOLERANCE (10^{-9})								
a	ROLL ↑	22	STOP ITERATION... IF GREATER CONTINUE.								
b	IF x<y	52									
c	4	04									
d	5	05									
4 0	ROLL ↑	22									
1	CONTINUE	47	ϕ_{n+1}	$\|\phi_{n+1} - \phi_n\|$	10^{-9}						
2	GO TO ( )( )	44	THIS MAY BE REPLACED WITH PSE IF THE								
3	1	01	CONVERGENCE IS TO BE MONITORED.								
4	c	16									
5	ROLL ↓	31									
6	f	15									
7	END	46	INV(ϕ)	ϕ_{n+1}	$\|\phi_{n+1} - \phi_n\|$	DISPLAY					
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

SPRING DESIGN - COMPRESSION & EXTENSION SPRINGS

This program calculates one of three variables (d - diameter of wire, N - number of turns of wire, and D - mean coil diameter), the remaining two being set, and calculates the maximum allowable stress. Other inputs are set to predetermined values.

Basic relationships:

$$S = \frac{8\ PD}{\pi d^3}\ Kw_2\ ; \quad Kw_2 = 1 + \frac{.5d}{D}$$

assuming the spring is all set out; Kw_2 is used as the most accurate coefficient when considering the maximum stress.

$$K = \frac{G d^4}{8\ ND^3}$$

S = Maximum stress

P = Maximum load

d = Wire diameter

D = Mean coil diameter

N = No. of active coils

G = Modulus of rigidty

K = Spring constant

P, K, G are set to predetermined or desired values for a given application.

With two of the three variables d, N, and D set to predetermined values, the 3rd is calculated from the expression

$$K = \frac{G d^4}{8\ ND^3}$$

Dimensions:

S, G in lbs/in^2 ; K in lbs/in.

N dimensionless

P in lbs.

D, d in inches

Reference: Design Handbook
Associated Spring Corporation
Bristol, Connecticut

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

→ DISPLAY

0	Z
0	Y
0	X

ENTER DATA: P → Z, K → Y, G → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

→ ENTER DATA: d → Z, D → Y, N → X

(Enter unknown as zero)

PRESS: CONTINUE

DISPLAY

(1)

S	Z
0	Y
0	X

PRESS: CONTINUE

DISPLAY

(2)

d	Z
D	Y
N	X

Note: To alter solution, enter new set of data (d, N, D) after display (2), setting unknown to zero.

To reset complete problem with new P, K, and G;

PRESS: END

PRESS: CONTINUE

EXAMPLE

We wish to use music wire ($G = 11.5 \times 10^6$ psi) to make a spring which will compress 1/4" with 5 lb. load (k = 20 lb/in.). The spring must fit over 1/2" pin. Maximum load will be 10 lbs.

P = 10

k = 20

$G = 1.15 \times 10^7$

d = 0 (d selected as the unknown)

D = .625 (Choose to allow clearance)

N = 10 (Reasonable number for maximum compression of 1/2 in.)

ANSWERS:

S = 37,955.01

d = .0763 (Calculated value)

D = .625 (Given values)

N = 10

Enter: New data for 14 gauge wire (good correlation)

d = .0736 (Standard wire size, 14 ga)

D = .625 same

N = 0 (New unknown)

ANSWERS:

S = 42,270.12

d = .0736 } Given

D = .625 }

N = 8.6387 (Calculated N)

Therefore, for 14 gauge wire with 10 lbs. maximum load and other variables as specified, the number of turns should be 8.64 and the maximum stress is 42,270.12 lbs.

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a)
0 0	CLEAR	20				
1	STOP	41	G	k	P	ENTER DATA; P,k,G
2	÷	35				
3	8	10				
4	×	36	CALCULATE $\frac{8k}{G}$ AND STORE WHICH IS USED TO CALCULATE N,D OR d			
5	↓	25				
6	x→( )	23				
7	c	16				
8	y→( )	40	STORE P			
9	b	14				
a	CLEAR	20				
b (FROM 7-5)	STOP	41	N	D	d	ENTER 2 OF 3 CONSTANTS, SET UNKNOWN TO ZERO
c	ACC +	60	STORE N AND D			
d	ROLL ↑	22				
1 0	x→( )	23	STORE d			
1	d	17				
2	CLEAR x	37				
3	IF x=y	50				
4	4	04	BRANCH TO SOLUTION FOR N IF N IS UNKNOWN			
5	7	07				
6	ROLL ↑	22				
7	IF x=y	50				
8	2	02	BRANCH TO SOLUTION FOR D IF D IS UNKNOWN			
9	b	14				
a	↑	27				
b	×	36				
c	×	36				
d	f	15				
2 0	×	36	CALCULATE d			
1	c	16				
2	×	36				
3	↓	25				
4	√x	76				
5	√x	76				
6	x→( )	23	STORE d			
7	d	17				
8	GO TO ( )( )	44				
9	5	05	BRANCH TO SOLUTION FOR S			
a	6	06				
b	d	17				
c	↑	27				
d	×	36				

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	×	36									
	1	×	36									
	2	c	16									
	3	÷	35									
	4	f	15									
	5	÷	35									
	6	1	01	CALCULATE D								
	7	↑	27									
	8	3	03									
	9	÷	35									
	a	↓	25									
	b	x⇄y	30									
	c	ln x	65									
	d	×	36									
4	0	↓	25									
	1	e^x	74									
	2	x→( )	23	STORE D								
	3	e	12									
	4	GO TO ( )( )	44									
	5	5	05	BRANCH TO SOLUTION FOR S								
	6	6	06									
FROM 1-5 →	7	d	17									
	8	ROLL ↑	22									
	9	÷	35									
	a	↓	25									
	b	↑	27									
	c	×	36	CALCULATE N								
	d	×	36									
5	0	d	17									
	1	×	36									
	2	c	16									
	3	÷	35									
	4	y→( )	40	STORE N								
	5	f	15									
FROM 2-a & 4-6 →	6	b	14									
	7	↑	27									
	8	4	04									
	9	×	36									
	a	π	56									
	b	÷	35									
	c	d	17									
	d	÷	35									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6 0	÷	35									
1	↑	27									
2	↓	25									
3	÷	35									
4	2	02	CALCULATE S								
5	×	36									
6	e	12									
7	×	36									
8	↓	25									
9	+	33									
a	CLEAR x	37									
b	↑	27	0	0	S	DISPLAY					
c	STOP	41									
d	d	17									
7 0	ROLL ↓	31									
1	RCL	61	N	D	d	DISPLAY					
2	ACC −	63									
3	GO TO ( )( )	44									
4	0	00									
5	b	14									
6	END	46									
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

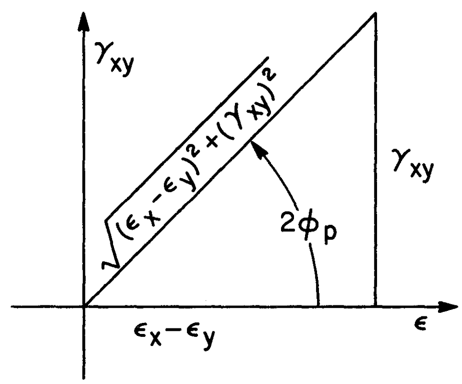
Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

The basic equation obtained from graphical representation of a set of strains is

$$(1)\quad \epsilon_\phi = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\phi + \frac{\gamma_{xy}}{2} \sin 2\phi$$

The strains can be displayed graphically in the following manner



with $2\phi_p$ is the angle from the ϵ axis to the principal strains.

By writing equations for $\sin 2\phi_p$ and $\cos 2\phi_p$ and substituting into equation (1) the following equations are obtained:

$$\epsilon_{max},\ \epsilon_{min} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\frac{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}{2}} \quad \begin{pmatrix} \text{max} & + \\ \text{min} & - \end{pmatrix}$$

$$\phi_p = 1/2 \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} \quad \text{(from the figure)}$$

$$\gamma_{max} = \sqrt{(\epsilon_x - \epsilon_y)^2 + (\gamma_{xy})^2}$$

Using a rectangular rosette gives $\phi_1 = 0^\circ$, $\phi_2 = 45^\circ$, and $\phi_3 = 90^\circ$ then from equation (1) it is found that

$\epsilon_x = \epsilon_1$, $\epsilon_y = \epsilon_3$, $\gamma_{xy} = 2\epsilon_2 - (\epsilon_1 + \epsilon_3)$.

Then writing the principal strains in terms of ϵ_1, ϵ_2, and ϵ_3 we obtain:

$$\epsilon_{max},\ \epsilon_{min} = 1/2\,(\epsilon_1 + \epsilon_3) \pm 1/2\sqrt{(\epsilon_1 - \epsilon_3)^2 + [2\,\epsilon_2 - (\epsilon_1 + \epsilon_3)]^2} \quad \left(\begin{matrix} max & + \\ min & - \end{matrix}\right)$$

$$\gamma_{max} = \sqrt{(\epsilon_1 - \epsilon_3)^2 + [2\,\epsilon_2 - (\epsilon_1 + \epsilon_3)]^2}$$

$$\phi_p = 1/2 \tan^{-1} \frac{2\,\epsilon_2 - (\epsilon_1 + \epsilon_3)}{\epsilon_1 - \epsilon_3}$$

The principal stresses are then related by the following equations which may be obtained from investigation of a Mohr's circle for strain.

$$\sigma_{max} = \frac{E}{1 - \mu^2}\,(\epsilon_{max} + \mu\,\epsilon_{min})$$

$$\sigma_{min} = \frac{E}{1 - \mu^2}\,(\epsilon_{min} + \mu\,\epsilon_{max})$$

$$\tau_{max} = \frac{E}{2\,(1 + \mu)}\,\gamma_{max}$$

ϵ = strain (in. per in.)
σ = stress (psi)
γ_{max} = maximum shearing strain (radian)
τ_{max} = maximum shearing stress (psi)
ϕ_p = angle between the ϵ axis and the principal strain axis. (degrees)
μ = Poisson's ratio (dimensionless)
E = Modulus of elasticity (psi)

Reference: The Strain Gage Primer
Perry & Lissner

McGraw-Hill
New York, 1962

USER INSTRUCTIONS

SET: | DEGREES | |

ENTER PROGRAM (Starting Address 0 - 0)

PRESS: GO TO 3 - d

SET: | PROGRAM | |

ENTER: Program steps for constant μ

SET: | | RUN |

PRESS: GO TO 5 - c

SET: | PROGRAM | |

ENTER: Program steps for constant E

SET: | | RUN |

▲ PRESS: GO TO (0) (0) or END

→ PRESS: CONTINUE

DISPLAY

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: $\epsilon_3 \rightarrow Z$, $\epsilon_2 \rightarrow Y$, $\epsilon_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

ϵ min ——— Z
ϵ max ——— Y
γ max ——— X

PRESS: CONTINUE

DISPLAY

ϕ_p ——— Z
0 ——— Y
0 ——— X

PRESS: CONTINUE

DISPLAY

σ min ——— Z
σ max ——— Y
τ max ——— X

Note; μ and E are included in the program as constants dependent on the materials being used.

▲ Denotes Revision

EXAMPLE

$\mu = .3$, $E = 30 \times 10^6$ psi

Given: $\epsilon_1 = 285 \times 10^{-6}$ in. per in.
$\epsilon_2 = 65 \times 10^{-6}$ in. per in.
$\epsilon_3 = 102 \times 10^{-6}$ in. per in.

Result of calculations:

ϵ min $= 3.5752 \times 10^{-5}$ in. per in.
ϵ max $= 3.5125 \times 10^{-4}$ in. per in.
γ max $= 3.1550 \times 10^{-4}$ radian
$\phi_p = -27.2734^{\circ}$
σ min $= 4.65251 \times 10^3$ psi
σ max $= 11.93320 \times 10^3$ psi
τ max $= 3.64034 \times 10^3$ psi

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	STOP	41	ϵ_1	ϵ_2	ϵ_3	ENTER $\epsilon_3, \epsilon_2, \epsilon_1$					
2	y→( )	40	STORE ϵ_2								
3	f	15									
4	ROLL ↑	22									
5	↑	27									
6	↓	25	CALCULATE AND STORE $\epsilon_1 - \epsilon_3$								
7	−	34									
8	y→( )	40									
9	e	12									
a	ROLL ↑	22									
b	+	33									
c	f	15									
d	↑	27									
1 0	2	02									
1	×	36	CALCULATE $(\epsilon_1 + \epsilon_3)$ AND $[2\epsilon_2 - (\epsilon_1 + \epsilon_3)]$								
2	↓	25									
3	x⇄y	30	AND STORE $[2\epsilon_2 - (\epsilon_1 + \epsilon_3)]$								
4	−	34									
5	e	12									
6	y→( )	40									
7	f	15									
8	TO POLAR	62	CALCULATE $\gamma_{MAX.}$								
9	ROLL ↑	22									
a	↑	27									
b	↓	25									
c	+	33									
d	ROLL ↑	22									
2 0	−	34									
1	x→( )	23									
2	d	17									
3	2	02	CALCULATE $\epsilon_{MIN.}$ AND $\epsilon_{MAX.}$								
4	÷	35	AND STORE $\gamma_{MAX.}$, $\epsilon_{MIN.}$, AND $\epsilon_{MAX.}$								
5	y→( )	40									
6	c	16									
7	ROLL ↑	22									
8	x⇄y	30									
9	÷	35									
a	y→( )	40									
b	b	14									
c	d	17									
d	STOP	41	$\gamma_{MAX.}$	$\epsilon_{MAX.}$	$\epsilon_{MIN.}$	DISPLAY					

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	f	15									
1	↑	27									
2	e	12									
3	÷	35									
4	↓	25									
5	arc ▾	72	CALCULATE θ_p								
6	tan x	71									
7	↑	27									
8	2	02									
9	÷	35									
a	CLEAR x	37									
b	↑	27									
c	STOP	41	0	0	θ_p	DISPLAY					
d	CONTINUE	47									
4 0	CONTINUE	47	ENTER μ AS A CONSTANT								
1	CONTINUE	47									
2	CONTINUE	47									
3	x→( )	23									
4	f	15									
5	↑	27									
6	↑	27									
7	b	14									
8	×	36									
9	c	16									
a	+	33									
b	y→( )	40	CALCULATE AND STORE $\epsilon_{MAX.} + \mu\epsilon_{MIN.}$								
c	e	12	AND $\epsilon_{MIN.} + \mu\epsilon_{MAX.}$								
d	ROLL ↑	22									
5 0	×	36									
1	↑	27									
2	b	14									
3	ROLL ↑	22									
4	+	33									
5	y→( )	40									
6	c	16									
7	↓	25									
8	1	01									
9	+	33									
a	2	02									
b	×	36									
c	CONTINUE	47									
d	CONTINUE	47									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 0	CONTINUE	47									
1	CONTINUE	47									
2	CONTINUE	47									
3	CONTINUE	47									
4	$x \to ()$	23	ENTER E, STORE, AND CALCULATE $\tau_{MAX.}$								
5	f	15									
6	$x \rightleftarrows y$	30									
7	÷	35									
8	d	17									
9	×	36									
a	$y \to ()$	40									
b	b	14									
c	ROLL ↑	22									
d	↑	27									
7 0	×	36									
1	1	01									
2	$x \rightleftarrows y$	30									
3	−	34									
4	f	15									
5	$x \rightleftarrows y$	30									
6	÷	35									
7	↓	25									
8	↑	27	CALCULATE $\sigma_{MAX.}$ AND $\sigma_{MIN.}$								
9	↑	27									
a	c	16									
b	×	36									
c	e	12									
d	ROLL ↑	22									
8 0	×	36									
1	b	14									
2	ROLL ↑	22									
3	$x \rightleftarrows y$	30									
4	END	46	$\tau_{MAX.}$	$\sigma_{MAX.}$	$\sigma_{MIN.}$	DISPLAY					
5											
6											
7											
8											
9											
a											
b											
c											
d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

PROPERTIES OF AREAS

This program determines properties of any area which can be represented (or approximated) by a set of rectangles. The properties determined are;

A — Area

$I_{x'}$ — Second Moment of the Area with Respect to Axis X'

$I_{y'}$ — Second Moment of the Area with Respect to Y' Axis

I_x — Second Moment of the Area with Respect to X Axis

I_y — Second Moment of the Area with Respect to Y Axis

X_c — Distance From Y Axis to Centroid Parallel to the X Axis

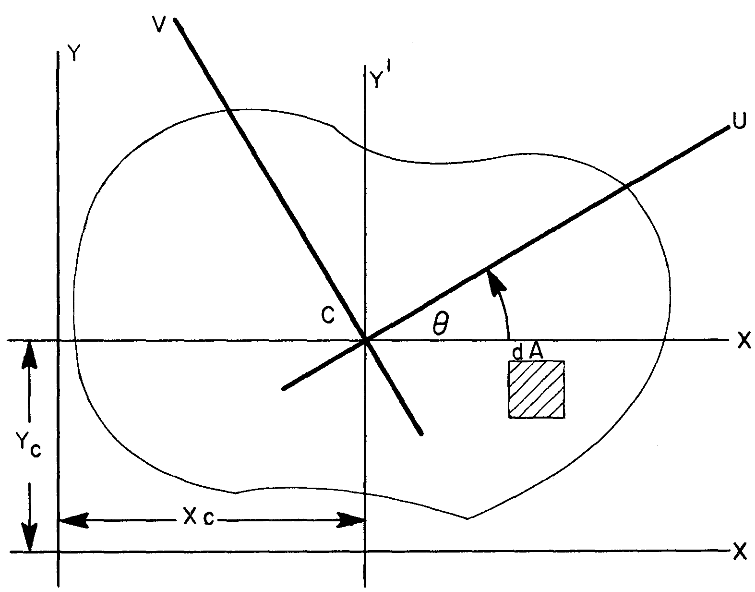
Y_c — Distance From X Axis to Centroid Parallel to the Y Axis

$I_{x'y'}$ — Product of Inertia of Area About the Centroid

I_u — Moment of Inertia About the Principle Axis U

I_v — Moment of Inertia About the Principle Axis V

The axes are illustrated below:



The formulas used in the program are given below where X_1, X_2, Y_1, and Y_2 describe the boundaries of each rectangular component of the composite area as seen in the illustration below.

$$B_i = X_2 - X_1$$

$$D_i = Y_2 - Y_1$$

$$W_i = \frac{X_2 + X_1}{2}$$

$$V_i = \frac{Y_2 + Y_1}{2}$$

$$A = \sum_i B_i D_i$$

$$X_c = \frac{1}{A} \sum_i (B_i D_i V_i)$$

$$Y_c = \frac{1}{A} \sum_i (B_i D_i W_i)$$

$$I_{x'} = \frac{1}{12} \sum_i (B_i D_i^3) + \sum_i (B_i D_i V_i^2) - A(X_c)^2$$

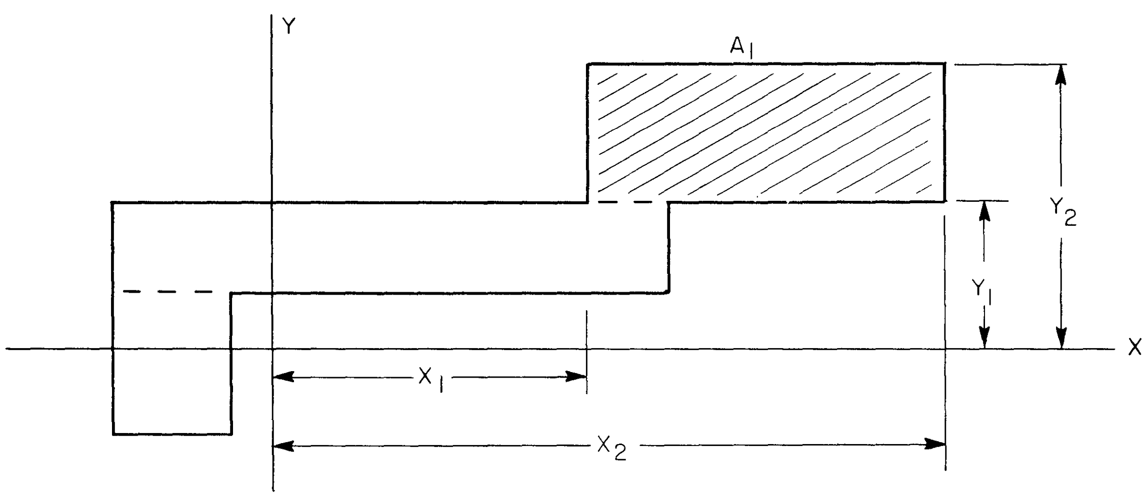
$$I_{y'} = \frac{1}{12} \sum_i (D_i B_i^3) + \sum_i (B_i D_i W_i^2) - A(Y_c)^2$$

$$I_{x'y'} = \sum_i (D_i B_i V_i W_i) - A(X_c)(Y_c)$$

$$I_u = I_{x'} \cos^2 \theta + I_{y'} \sin^2 \theta - I_{x'y'} \sin 2\theta$$

$$\theta = \frac{1}{2} \arctan \left(\frac{2 I_{x'y'}}{I_{y'} - I_{x'}} \right)$$

$$I_v = I_{y'} \cos^2 \theta + I_{x'} \sin^2 \theta + I_{x'y'} \sin 2\theta$$



USER INSTRUCTIONS

SET: [ ] DEGREES

PRESS: END

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

→ ENTER: DATA: $X_2 \rightarrow Y$, $X_1 \rightarrow X$ (of Area i)

PRESS: CONTINUE

DISPLAY

1	Z
1	Y
1	X

ENTER DATA: $Y_2 \rightarrow Y$, $Y_1 \rightarrow X$ (of Area i)

Is this the last rectangular component?

NO: PRESS: CONTINUE (return to ENTER: DATA)

YES:

PRESS. SET FLAG

PRESS: CONTINUE

DISPLAY

I_x	Z
Y_c	Y
A	X

(Print)

PRESS: CONTINUE

DISPLAY

I_y	Z
X_c	Y
A	X

(Print)

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

I_v	Z
I_u	Y
θ	X

(Print)

PRESS: CONTINUE

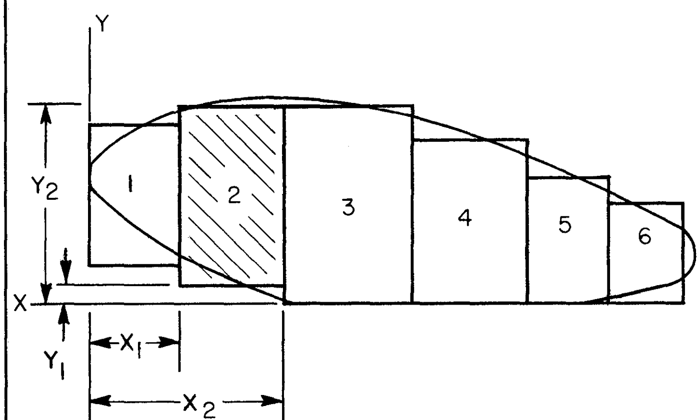
DISPLAY

$I_{x'y'}$	Z
$I_{y'}$	Y
$I_{x'}$	X

(Print)

PRESS: END for a new case

EXAMPLE



Area	X_1	X_2	Y_1	Y_2
1	0	3	1.875	5.563
2	3	12	.563	7.188
3	12	21	0	7.375
4	21	30	0	5.125
5	30	36	0	2.688
6	36	37.75	0	1.250

1	2926.4	I_x	2	73199.1	I_y
	3.267	Y_c		16.615	X_c
	201.5	A		201.5	A
3	17674.2	I_v	4	-1323.5	$I_{x'y'}$
	672.6	I_u		17570.5	$I_{y'}$
	-4.478			776.3	$I_{x'}$

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20			
01	x→( )	23			
02	d	17			
03	x→( )	23			
04	c	16			
05	x→( )	23			
06	b	14			
07	x→( )	23			
08	a	13			
09	x→( )	23			
0a	−	34			
0b	e	12			
0c	x→( )	23			
0d	−	34			
10	f	15	ENTER		
11	STOP	41	X_1	X_2	0
12	CONT	47			
13	−	34			
14	↑	27			
15	↓	25			
16	+	33			
17	+	33			
18	2	02			
19	÷	35			
1a	↓	25			
1b	x→( )	23			
1c	9	11			
1d	y→( )	40			
20	8	10			
21	1	01			
22	↑	27			
23	↑	27	ENTER		
24	STOP	41	Y_1	Y_2	0
25	CONT	47			
26	−	34			
27	↑	27			
28	↓	25			
29	+	33			
2a	+	33			
2b	2	02			
2c	÷	35			
2d	↓	25			
30	x⇄y	30			
31	y→( )	40			
32	−	34			
33	c	16			
34	↑	27			
35	y→( )	40			
36	7	07			
37	x←( )	67			
38	8	10			
39	×	36			
3a	↓	25			
3b	×	36			
3c	ROLL ↓	31			
3d	×	36			
40	ACC +	60			
41	y⇄( )	24			
42	−	34			
43	c	16			
44	x←( )	67			
45	9	11			
46	×	36			
47	ROLL ↓	31			
48	x⇄y	30			
49	×	36			
4a	ROLL ↓	31			
4b	y⇄( )	24			
4c	d	17			
4d	+	33			
50	y→( )	40			
51	d	17			
52	↓	25			
53	c	16			
54	+	33			
55	y→( )	40			
56	c	16			
57	y⇄( )	24			
58	7	07			
59	↓	25			
5a	×	36			
5b	×	36			
5c	b	14			
5d	+	33			
60	y→( )	40			
61	b	14			
62	y⇄( )	24			
63	8	10			
64	↓	25			
65	×	36			
66	×	36			
67	a	13			
68	+	33			
69	GO TO ( )	44			
6a	−	34			
6b	0	00			
6c	0	00			
6d–7c	↓	↓			
7d	0	00			

Storage

Register	+		−
f	ΣDBV	I_{pp}	ΣDBW
e	ΣDBV^2	θ	ΣDBW^2
d	ΣDBWV	$\sin 2\theta$	
c	ΣDB	Ix' x'	V
b	ΣBD^3	Iy' y'	
a	ΣDB^3		
9	W		
8	B		
7	D		
6			
5			
4			
3			
2			
1			
0			

Step	Key	Code	Display x	Display y	Display z
00	y→( )	40			
01	a	13			
02	y⇄( )	24			
03	9	11			
04	↓	25			
05	×	36			
06	ROLL ↓	31			
07	y⇄( )	24			
08	−	34			
09	f	15			
0a	+	33			
0b	y→( )	40			
0c	−	34			
0d	f	15			
10	ROLL ↑	22			
11	×	36			
12	x←( )	67			
13	−	34			
14	e	12			
15	+	33			
16	y→( )	40			
17	−	34			
18	e	12			
19	0	00			
1a	↑	27			
1b	↑	27			
1c	IF FLAG	43			
1d	2	02			
20	5	05			
21	GO TO ( )	44			
22	+	33			
23	1	01			
24	1	01			
25	b	14			
26	↑	27			
27	1	01			
28	2	02			
29	÷	35			
2a	e	12			
2b	+	33			
2c	x←( )	67			
2d	f	15			

Step	Key	Code	Display x	Display y	Display z
30	↑	27			
31	c	16			
32	÷	35	DISPLAY		
33	STOP	41	A	X_c	I_x
34	CONT	47			
35	x←( )	67			
36	f	15			
37	×	36			
38	↓	25			
39	−	34			
3a	y→( )	40			
3b	b	14			
3c	a	13			
3d	↑	27			
40	1	01			
41	2	02			
42	÷	35			
43	x←( )	67			
44	−	34			
45	e	12			
46	+	33			
47	x←( )	67			
48	−	34			
49	f	15			
4a	↑	27			
4b	c	16			
4c	÷	35	DISPLAY		
4d	STOP	41	A	Y_c	I_y
50	CONT	47			
51	x←( )	67			
52	−	34			
53	f	15			
54	×	36			
55	↓	25			
56	−	34			
57	y→( )	40			
58	a	13			
59	d	17			
5a	↑	27			
5b	x←( )	67			
5c	−	34			
5d	f	15			

Step	Key	Code	Display x	Display y	Display z
60	↑	27			
61	f	15			
62	×	36			
63	c	16			
64	÷	35			
65	↓	25			
66	−	34			
67	y→( )	40			
68	c	16			
69	2	02			
6a	×	36			
6b	a	13			
6c	↑	27			
6d	b	14			
70	−	34			
71	↓	25			
72	÷	35			
73	↓	25			
74	arc ▾	72			
75	tan x	71			
76	↑	27			
77	sin x	70			
78	x→( )	23			
79	d	17			
7a	2	02			
7b	÷	35			
7c	↓	25			
7d	x→( )	23			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
8 (−) 0	e	12			
1	cos x	73			
2	↑	27			
3	×	36			
4	b	14			
5	×	36			
6	e	12			
7	sin x	70			
8	↑	27			
9	×	36			
a	a	13			
b	×	36			
c	↓	25			
d	+	33			
9 (−) 0	d	17			
1	↑	27			
2	c	16			
3	×	36			
4	↓	25			
5	−	34			
6	y→( )	40			
7	f	15			
8	↑	27			
9	e	12			
a	cos x	73			
b	↑	27			
c	×	36			
d	a	13			
a (−) 0	×	36			
1	↓	25			
2	+	33			
3	e	12			
4	sin x	70			
5	↑	27			
6	×	36			
7	b	14			
8	×	36			
9	↓	25			
a	+	33			
b	f	15			
c	↑	27			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
b (−) 0	STOP	41	θ	I_u	I_v
1	CONT	47	DISPLAY		
2	c	16			
3	↑	27			
4	a	13			
5	↑	27			
6	b	14	DISPLAY		
7	STOP	41	$I_{x'}$	$I_{y'}$	$I_{x'y'}$
8	CONT	47			
9	END	46			
a					
b					
c					
d					

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

73001 - ANNUAL INTEREST

Calculates i that satisfies the equation:

$$R = \frac{P\ i(1+i)^n}{(1+i)^n - 1}$$

73008 - AMORTIZED LOAN

Calculates the monthly payment on the principal of a loan for a specified term, the amount of payments toward principal and interest, and a running total of the amount of payments toward principal and interest to date.

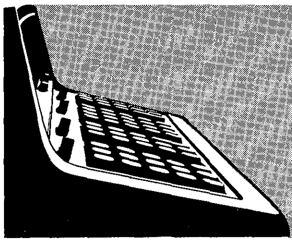
9100B ONLY

73101 - RETURN ON INVESTMENT FOR UP TO 16 VARYING CASH FLOWS

This program solves for the value of interest per period for up to sixteen varying cash flows. The program solves the equation:

$$\text{PV (Present Value)} = \sum_{k=1}^{16} \frac{C_k}{(1+i)^k}$$

where C_k represents a cash flow per period.



PART NO.
09100-73001

ANNUAL INTEREST

This program calculates i that satisfies the equation

$$R = P \frac{i(1+i)^n}{(1+i)^n - 1}$$

for a given R, P , and n .

where R = monthly payment
P = principal amount
n = number of monthly payments
i = monthly interest rate

The program uses this i and calculates and displays an annual interest.

The solution is based on a Newton-Raphson iteration technique which converges to a point which satisfies the condition

$$\left| i_K - i_{K-1} \right| \leq 10^{-9}$$

Note i_K is the K^{th} estimate of i and i_{K-1} is the $(K-1)^{th}$ estimate of i . The initial estimate of i is R/P unless otherwise specified. If the solution does not converge it may be necessary to enter a better initial estimate.

The value of i that satisfies the equation

$$f(i) = (1+i)^n (R - Pi) - R = 0$$

is found. The annual interest is displayed.

Reference: Principles of Engineering Economy
by E. L. Grant

Ronald Press, N.Y. 1950

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: P → Z, R → Y, n → X

Initial Estimate for i to be Entered? (NO / YES)

YES:

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: (estimated i percent)
i_{est}% → X

(NO joins here)

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
i	——	X

(i is annual interest)

EXAMPLES

P = $10,000
R = $276.45
n = 40 months

i = 5.999% (Note: This is annual interest)

P = $2500
R = $81.95
n = 36 months

i = 11.087%

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a) / Remarks
0 0	CLEAR	20				
1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY			
2	STOP	41	N	R	P	ENTER N,R, AND P
3	ACC +	60				
4	ROLL ↑	22	BRANCH FOR i_{est} ENTRY			
5	$x \rightarrow ( )$	23				
6	d	17				
7	IF FLAG	43				
8	1	01				
9	3	03				(to 1-3)
a	ROLL ↑	22				
b	$x \rightleftarrows y$	30	GENERATE $i_{est} = \frac{R}{P}$			
c	÷	35				
d	↓	25				
1 0	GO TO ( )( )	44				
1	2	02	BRANCH TO ITERATION			
2	1	01				(to 2-1)
3	CLEAR x	37				
4	↑	27				
5	↑	27				
6	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY			
7	STOP	41	i_{est}	0	0	ENTER i_{est} IN PERCENT
8	↑	27				
9	1	01				
a	2	02	CONVERT ANNUAL INTEREST RATE IN PERCENT TO MONTHLY RATE.			
b	0	00				
c	0	00				
d	÷	35				
2 0	↓	25				
1	$x \rightarrow ( )$	23				
2	b	14				(FROM 6-c)
3	↑	27				
4	1	01				
5	+	33				
6	↓	25				
7	↑	27				
8	ln x	65				
9	ROLL ↑	22	CALCULATE AND STORE $(i_K + 1)^n$			
a	f	15				
b	×	36				
c	↓	25				
d	e^x	74				

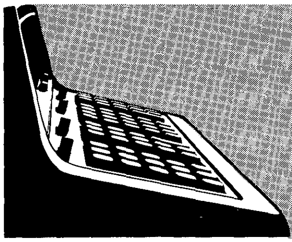
Step		Key	Code	Display x	y	z	Storage f	e	d	c	b	a
3	0	$x \rightarrow ()$	23									
	1	c	16									
	2	1	01									
	3	$x \rightleftarrows y$	30									
	4	÷	35									
	5	d	17									
	6	ROLL ↑	22									
	7	b	14									
	8	×	36									
	9	e	12									
	a	$x \rightleftarrows y$	30									
	b	−	34	CALCULATE $f'(i_K) = (1+i_K)^n \left(\frac{n(R-Pi_K)}{1+i_K} - P \right)$								
	c	↓	25									
	d	×	36									
4	0	ROLL ↓	31									
	1	$x \rightleftarrows y$	30									
	2	f	15									
	3	×	36									
	4	d	17									
	5	−	34									
	6	c	16									
	7	×	36									
	8	$x \rightleftarrows y$	30									
	9	ROLL ↓	31									
	a	×	36	CALCULATE $f(i_K)$								
	b	e	12									
	c	−	34									
	d	ROLL ↓	31									
5	0	$x \rightleftarrows y$	30	CALCULATE $\frac{f(i_K)}{f'(i_K)}$								
	1	÷	35									
	2	b	14									
	3	$x \rightleftarrows y$	30	CALCULATE $i_{K+1} = i_K - \frac{f(i_K)}{f'(i_K)} = i_K - \Delta_K$								
	4	−	34									
	5	$x \rightleftarrows y$	30									
	6	$x \rightarrow ()$	23									
	7	b	14									
	8	↑	27									
	9	1	01	CONVERT i_{K+1} TO ANNUAL PERCENTAGE FOR DISPLAY								
	a	2	02									
	b	0	00									
	c	0	00									
	d	×	36									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	CLEAR x	37									
	1	ROLL ↓	31									
	2	PAUSE	57	i_{K+1} %	Δ_K	0	DISPLAY EACH ITERATION					
	3	$\|y\|$	55									
	4	ROLL ↑	22									
	5	ENTER EXP	26									
	6	0	00									
	7	9	11	COMPARE $\|i_{K+1} - i_K\|$ TO 10^{-9} TOLERANCE. IF LESS THAN 10^{-9} BRANCH TO DISPLAY. IF GREATER THAN 10^{-9} CONTINUE ITERATION.								
	8	CHG SIGN	32									
	9	ROLL ↑	22									
	a	IF $x > y$	53									
	b	2	02									
	c	2	02									
	d	CLEAR x	37	CLEAR AND POSITION i_{K+1}								
7	0	$x \rightleftarrows y$	30									
	1	CLEAR x	37									
	2	ROLL ↑	22									
	3	END	46	i	0	0	DISPLAY					
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

HEWLETT · PACKARD

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



AMORTIZED LOAN

This program calculates the monthly payment necessary to amortize a loan over a specified number of periods. The principal, annual interest rate, and term of the loan in months are provided by the user.

The program displays (or prints) the calculated monthly payment, showing the amount paid toward principal and the amount of interest for each payment. After each payment, the total principal paid, total interest paid and the remaining loan balance may be displayed. After displaying the last payment, the program ends displaying the total principal paid, the total interest paid and the total interest and principal paid.

The monthly payment is constant. The amount paid in interest is one-twelfth the annual interest rate times the remaining principal. The remainder of the monthly payment is subtracted from the principal balance.

The following equations are used:

$$\text{Percent Monthly Interest } (i') = \frac{\text{Annual interest rate } (i)}{12}$$

$$\text{Monthly Payment } (P') = \frac{\frac{S_i}{1200}\,(1 + i/1200)^n}{(1 + i/1200)^n - 1}$$

where S is the amount of the loan (original principal) and n is the term of the loan in periods.

$P = P'$ rounded up

Monthly Payment Paid on Principal $(P_M) = P - I$

Monthly Interest Payment $(I) = P_R \times i'$

Remaining Loan Balance $(P_R) = S - P_T$

$$\text{Total Principal Paid to date } (P_T) = \sum_{a=0}^{n} P_{M_a}$$

$$\text{Total Interest Paid to date } (I_T) = \sum_{a=0}^{n} I_a$$

Total Amount Paid on loan to date $= P_T + I_T$

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY:

0 ——— Z
0 ——— Y
0 ——— X

ENTER DATA: Loan principal in Dollars → Z
Interest Rate in Percent → Y
Term of loan in Monts → X

PRESS: CONTINUE

DISPLAY:

0 ——— Z
0 ——— Y
Calculated Monthly Payment raised to nearest cent ——— X

Is a Different monthly payment desired?

YES → ENTER DATA: Desired monthly payment → X

NO →

PRESS: CONTINUE

DISPLAY: For each month the following brief display occurs:

Amount of Payment toward Principal ——— Z
Amount of Payment toward Interest ——— Y
Payment Number ——— X

To stop display - PRESS: PAUSE

If desired, while stopped,

PRESS: SET FLAG

PRESS: CONTINUE

USER INSTRUCTIONS (con't)

DISPLAY:

Total Principal Paid to date ——— Z
Total Interest Paid to date ——— Y
Remaining Loan Balance ——— X

To continue incrementing payments -

PRESS: CONTINUE

Has the entire loan been paid?

NO → (return to DISPLAY above)

YES →

DISPLAY:

Amount of Last Payment to Principal ——— Z
Amount of Last Payment to Interest ——— Y
Payment Number ——— X

PRESS: CONTINUE

DISPLAY:

Total Principal Paid ——— Z
Total Interest Paid ——— Y
Total Interest and Principal paid ——— X

Note: To print out, substitute program steps noted in program listing.

EXAMPLES

Example 1

Borrow $1,000 at 6.25%
for 5 years and 3 months.

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA:

1000 ⟶ Z
6.25 ⟶ Y
63 ⟶ X

PRESS: CONTINUE
DISPLAY:

0 —— Z
0 —— Y
18.67 —— X

Calculated monthly payment = $18.67

PRESS: CONTINUE

PRESS: PAUSE when display for 36 months is reached.

DISPLAY:

16.15 — Z - Payment toward Principal
2.52 — Y - Payment toward Interest
36 — X - Payment Number

PRESS: SET FLAG and CONTINUE
DISPLAY:

531.51 — Z - Total Principal Paid to Date
140.61 — Y - Total Interest Paid to Date
468.49 — X - Loan Balance

PRESS: CONTINUE

PROGRAM STOPS TO DISPLAY

17.88 — Z - Final Payment - Principal
.09 — Y - Final Payment - Interest
63 — X - Payment Number

PRESS: CONTINUE

DISPLAY:

1000 —— Z - Total Paid - Principal
175.51 —— Y - Total Paid - Interest
1175.51 —— X - Total Paid

Example 2

Borrow $1,000 at 6.25%
for 5 years and 3 months.

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

ENTER DATA:

1000 ⟶ Z
6.25 ⟶ Y
63 ⟶ X

PRESS: CONTINUE
DISPLAY:

0 —— Z
0 —— Y
18.67 —— X

ENTER: $19.00 into X register.

PRESS: CONTINUE

PRESS: PAUSE when display for 36 months is reached.

DISPLAY:

16.54 —— Z - Payment - Principal
2.46 —— Y - Payment - Interest
36 —— X - Monthly Payment

PRESS: SET FLAG and CONTINUE
DISPLAY:

544.54 —— Z - Total Principal Paid to Date
139.46 —— Y - Total Interest Paid to Date
455.46 —— X - Loan Balance

PRESS: CONTINUE

PROGRAM STOPS TO DISPLAY

12.72 —— Z - Final Payment to Principal
.07 —— Y - Final Payment to Interest
62 —— X - Payment Number

PRESS: CONTINUE

DISPLAY:

1000 —— Z - Total Paid - Principal
171.79 —— Y - Total Paid - Interest
1171.79 —— X - Total Paid

9616

Step	Key	Code	Display x	Display y	Display z	Storage / Notes
0 0	CLEAR	20				
1	$x \to ( )$	23				
2	9	11				
3	STOP	41	n	i	S	ENTER AMOUNT OF THE LOAN, (S), INTEREST, (i), AND PERIODS, (n).
4	CONTINUE	47				
5	$x \to ( )$	23	STORE n			
6	d	17				
7	1	01	CALCULATE MONTHLY INTEREST AS PERCENT			
8	2	02				
9	÷	35				
a	$y \to ( )$	40				
b	c	16				
c	ENTER EXP	26	CALCULATE MONTHLY INTEREST AS FRACTION			
d	2	02				
1 0	÷	35				
1	↓	25				
2	$y \to ( )$	40	STORE S			
3	b	14				
4	×	36	CALCULATE $\left(1+\frac{i}{1200}\right)^n$			
5	↑	27				
6	1	01				
7	+	33				
8	d	17				
9	$x \rightleftarrows y$	30				
a	ln x	65				
b	×	36				
c	↓	25				
d	e^x	74				
2 0	×	36	CALCULATE MONTHLY PAYMENT			$P' = \dfrac{S\frac{i}{1200}\left(1+\frac{i}{1200}\right)^n}{\left(1+\frac{i}{1200}\right)^n - 1}$
1	↑	27				
2	1	01				
3	−	34				
4	ROLL ↓	31				
5	÷	35				
6	ENTER EXP	26	ROUND UP TO NEAREST CENT			
7	2	02				
8	×	36				
9	↓	25				
a	↑	27				
b	int x	64				
c	$x \rightleftarrows y$	30				
d	IF $x > y$	53				

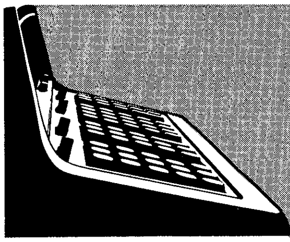
Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	↓	25									
	1	+	33									
	2	ENTER EXP	26									
	3	2	02									
	4	÷	35									
	5	CLEAR x	37	CLEAR AND DISPLAY P MONTHLY PAYMENT ROUNDED UP								
	6	↑	27									
	7	ROLL ↑	22									
	8	CONTINUE	47									
	9	STOP	41	P	0	0						
	a	CONTINUE	47									
	b	x→( )	23	STORE P								
	c	a	13									
	d	y⇄( )	24									
4	0	9	11	RECALL COUNTER AND N								
	1	d	17									
	2	IF x = y	50	BRANCH WHEN CALCULATION IS COMPLETE								
	3	8	10									
	4	7	07									
	5	1	01	INCREMENT COUNTER								
	6	+	33									
	7	y→( )	40									
	8	9	11									
	9	b	14	CALCULATE MONTHLY INTEREST PAYMENT (I) AND ROUND UP								
	a	↑	27									
	b	c	16									
	c	×	36									
	d	.	21									
5	0	5	05									
	1	+	33									
	2	↓	25									
	3	int x	64									
	4	↑	27									
	5	ENTER EXP	26									
	6	2	02									
	7	÷	35									
	8	a	13	CALCULATE P-I = AMOUNT OF PAYMENT PAID ON PRINCIPAL								
	9	x⇄y	30									
	a	−	34									
	b	ROLL ↓	31									
	c	y⇄( )	24									
	d	b	14	RECALL AMOUNT OWED ON PRINCIPAL								

FROM 7-6, 8-0 (→ step 3d)

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a) / Remarks
6 0	IF x>y	53	BRANCH WHEN THROUGH i.e., AMOUNT PAID ON PRINCIPAL EXCEEDS AMOUNT OWED.			
1	8	10				
2	1	01				
3	−	34	CALCULATE & STORE CURRENT AMOUNT OWED ON PRINCIPAL			
4	y⇄( )	24				
5	b	14				
6	ROLL ↑	22	ACCUMULATE AMOUNT PAID ON PRINCIPAL AND INTEREST			
7	ACC +	60				
8	ROLL ↑	22				
9	PAUSE	57	n	I	P_M	DISPLAY PAYMENT NUMBER, AMOUNT OF PAYMENT FOR INTEREST & PRINCIPAL
a						
b						
c						
d						
7 0	PAUSE	57				
1	IF FLAG	43	BRANCH WHEN FLAG SET TO DISPLAY TOTAL AMOUNT FOR PRINCIPAL & INTEREST PAID IN, & AMOUNT OWED ON PRINCIPAL			
2	7	07				
3	7	07				
4	GO TO ( )( )	44	BRANCH TO CONTINUE CALCULATION			
5	3	03				
6	d	17				
7	RCL	61				
8	↑	27				
9	b	14				
a	STOP	41	P_R	I_T	P_T	DISPLAY REMAINING LOAN BALANCE, TOTAL INTEREST PAID AND TOTAL PRINCIPAL PAID
b	CONTINUE	47				
c	GO TO ( )( )	44				
d	3	03				
8 0	d	17				
1	b	14				
2	ROLL ↓	31	ACCUMULATE TOTAL INTEREST PAID AND TOTAL PAID ON PRINCIPAL			
3	x⇄y	30				
4	ACC +	60				
5	ROLL ↑	22				
6	STOP	41	n	I	P_M	DISPLAY FINAL PAYMENT TO INTEREST, AND FINAL PRINCIPAL PAYMENT
7	RCL	61	CALCULATE TOTAL AMOUNT PAID IN, TOTAL INTEREST, AND TOTAL PRINCIPAL AND DISPLAY			
8	↑	27				
9	↓	25				
a	+	33				
b	x⇄y	30				
c	CONTINUE	47				
d	END	46	T	I_T	P_T	

FROM 4-4

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



RETURN ON INVESTMENT FOR UP TO 16 VARYING CASH FLOWS

9100B ONLY
PART NO.
09100-73101

This program solves for the value of interest per period for up to 16 varying cash flows. The program solves the equation:

$$\text{Present Value (P. V.)} = \sum_{k=1}^{N} \frac{C_k}{(1+i)^k} \qquad N \leq 16$$

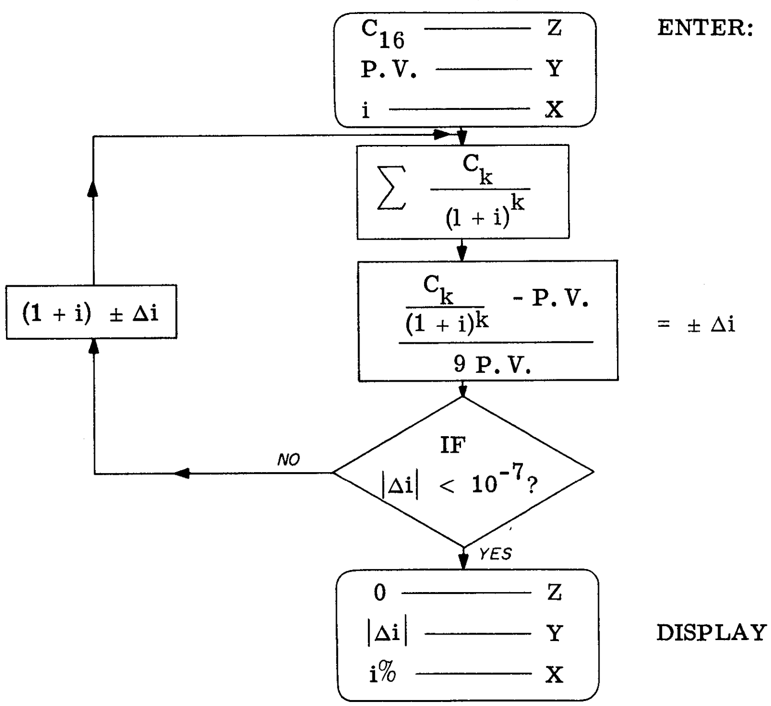
for i to an accuracy better than 10^{-7}.

C_k = cash flow per period

i = discount rate per period

FLOWCHART

$$\text{Present Value} = \sum_{k=1}^{N} \frac{C_k}{(1+i)^k}$$



Reference: Financial Management - Analytical Approach, by J. Robert Lindsay and Arnold W. Sametz, Richard D. Irwin, Inc. 1967.

9100B ONLY
PART NO.
09100-73101

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $C_3 \rightarrow Z$, $C_2 \rightarrow Y$, $C_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $C_6 \rightarrow Z$, $C_5 \rightarrow Y$, $C_4 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: $C_9 \rightarrow Z$, $C_8 \rightarrow Y$, $C_7 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

ENTER DATA: $C_{12} \rightarrow Z$, $C_{11} \rightarrow Y$, $C_{10} \rightarrow X$

PRESS: CONTINUE

USER INSTRUCTIONS (Con't)

DISPLAY

0	Z
0	Y
4	X

ENTER DATA: $C_{15} \rightarrow Z$, $C_{14} \rightarrow Y$, $C_{13} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
5	X

→ ENTER DATA: $C_{16} \rightarrow Z$, P. V. (present value) $\rightarrow Y$
i or 0 $\rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
$\|\Delta i\|$	Y
i%	X

To re-run with different i or **P. V.**

To re-run with different values of C, repeat USER INSTRUCTIONS

EXAMPLES

Example # 1

Present Value = $9.4 x 10^6 (Note: 10^6 is factored out of the basic equation as a constant.)

C_1 = .55
C_2 = .34
C_3 = 1.20
C_4 = -.34
C_5 = .82
C_6 = -.57
C_7 = .76
C_8 = .76
C_9 = .76
C_{10} = .82
C_{11} = .82

EXAMPLE (Con't)

C_{12} = 1.00
C_{13} = 1.24
C_{14} = 1.35
C_{15} = 1.46
C_{16} = 0.00

i = 1.598%

Example #2

Present Value = $150.00
C_1 = 100
C_2 = 100

i = 21.525%

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20	ENTER		
(+) 1	STOP	41	C_1	C_2	C_3
2	x→( )	23			
3	d	17			
4	y→( )	40			
5	c	16			
6	↓	25			
7	y→( )	40			
8	b	14			
9	CLEAR	20			
a	1	01			
b	STOP	41	ENTER		
c	x→( )	23	C_4	C_5	C_6
d	a	13			
1 0	y→( )	40			
(+) 1	9	11			
2	↓	25			
3	y→( )	40			
4	8	10			
5	CLEAR	20			
6	2	02	ENTER		
7	STOP	41	C_7	C_8	C_9
8	x→( )	23			
9	7	07			
a	y→( )	40			
b	6	06			
c	↓	25			
d	y→( )	40			
2 0	5	05			
(+) 1	CLEAR	20			
2	3	03	ENTER		
3	STOP	41	C_{10}	C_{11}	C_{12}
4	x→( )	23			
5	4	04			
6	y→( )	40			
7	−	34			
8	f	15			
9	↓	25			
a	y→( )	40			
b	−	34			
c	e	12			
d	CLEAR	20			

Step	Key	Code	Display x	Display y	Display z
3 0	4	04	ENTER		
(+) 1	STOP	41	C_{13}	C_{14}	C_{15}
2	x→( )	23			
3	0	00			
4	y→( )	40			
5	1	01			
6	↓	25			
7	y→( )	40			
8	2	02			
9	CLEAR	20			
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00	ENTER		
0 0	5	05	i	PV	C_{16}
(−) 1	STOP	41	i %	|Δi|	0
2	y→( )	40	DISPLAY		
3	e	12			
4	ROLL ↑	22			
5	x→( )	23			
6	3	03			
7	ENTER EXP	26			
8	2	02			
9	÷	35			
a	1	01			
b	+	33			
c	d	17			
d	↑	27			
1 0	↓	25			
(−) 1	GO TO ( )	44			
2	▲SUB▼	77			
3	a	13			
4	8	10			
5	c	16			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	a	13			
9	8	10			
a	b	14			
b	GO TO ( )	44			
c	▲SUB▼	77			
d	a	13			

Step	Key	Code	Display x	Display y	Display z
2 0	8	10			
(−) 1	a	13			
2	GO TO ( )	44			
3	▲SUB▼	77			
4	a	13			
5	8	10			
6	x←( )	67			
7	9	11			
8	GO TO ( )	44			
9	▲SUB▼	77			
a	a	13			
b	8	10			
c	x←( )	67			
d	8	10			
3 0	GO TO ( )	44			
(−) 1	▲SUB▼	77			
2	a	13			
3	8	10			
4	x←( )	67			
5	7	07			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	a	13			
9	8	10			
a	x←( )	67			
b	6	06			
c	GO TO ( )	44			
d	▲SUB▼	77			

Storage

	+	−
f		
e	PV	C_{11}
d	C_1	C_{12}
c	C_2	
b	C_3	
a	C_4	
9	C_5	
8	C_6	
7	C_7	
6	C_8	
5	C_9	
4	C_{10}	
3	C_{16}	
2	C_{15}	
1	C_{14}	
0	C_{13}	

Step	Key	Code	Display x	Display y	Display z
4 0	a	13			
(−) 1	8	10			
2	x←( )	67			
3	5	05			
4	GO TO ( )	44			
5	▲SUB▼	77			
6	a	13			
7	8	10			
8	x←( )	67			
9	4	04			
a	GO TO ( )	44			
b	▲SUB▼	77			
c	a	13			
d	8	10			
5 0	x←( )	67			
(−) 1	−	34			
2	f	15			
3	GO TO ( )	44			
4	▲SUB▼	77			
5	a	13			
6	8	10			
7	x←( )	67			
8	−	34			
9	e	12			
a	GO TO ( )	44			
b	▲SUB▼	77			
c	a	13			
d	8	10			
6 0	x←( )	67			
(−) 1	0	00			
2	GO TO ( )	44			
3	▲SUB▼	77			
4	a	13			
5	7	07			
6	x←( )	67			
7	1	01			
8	GO TO ( )	44			
9	▲SUB▼	77			
a	a	13			
b	8	10			
c	x←( )	67			
d	2	02			

Step	Key	Code	Display x	Display y	Display z
7 0	GO TO ( )	44			
(−) 1	▲SUB▼	77			
2	a	13			
3	8	10			
4	x←( )	67			
5	3	03			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	a	13			
9	8	10			
a	e	12			
b	↑	27			
c	x⇄y	30			
d	f	15			
8 0	x⇄y	30			
(−) 1	−	34			
2	÷	35			
3	9	11			
4	÷	35			
5	ROLL ↓	31			
6	+	33			
7	ROLL ↑	22			
8	ENTER EXP	26			
9	7	07			
a	CHG SIGN	32			
b	|y|	55			
c	IF x>y	53			
d	9	11			
9 0	8	10			
(−) 1	CLEAR x	37			
2	x→( )	23			
3	f	15			
4	ROLL ↓	31			
5	GO TO ( )	44			
6	0	00			
7	c	16			
8	x⇄y	30			
9	ROLL ↓	31			
a	1	01			
b	−	34			
c	ENTER EXP	26			
d	2	02			

Step	Key	Code	Display x	Display y	Display z
a 0	×	36			
(−) 1	CLEAR x	37			
2	ROLL ↓	31			
3	CONT	47			
4	CONT	47			
5	GO TO ( )	44			
6	0	00			
7	1	01			
8	ROLL ↑	22			
9	÷	35			
a	x⇄y	30			
b	y⇄( )	24			
c	f	15			
d	+	33			
b 0	y⇄( )	24			
(−) 1	f	15			
2	x⇄y	30			
3	ROLL ↑	22			
4	×	36			
5	ROLL ↑	22			
6	RETURN	77			
7					
8					
9					
a					
b					
c					
d					

Storage

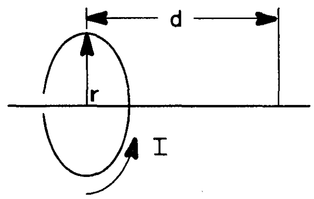
Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

September 1, 1969

9100B PHYSICS PROGRAM LISTING

73202 - FLUX DENSITY

Calculates the flux density along an axis of a circular loop for a specified radius r, current I, and distance d.



73203 - VENTURI METER

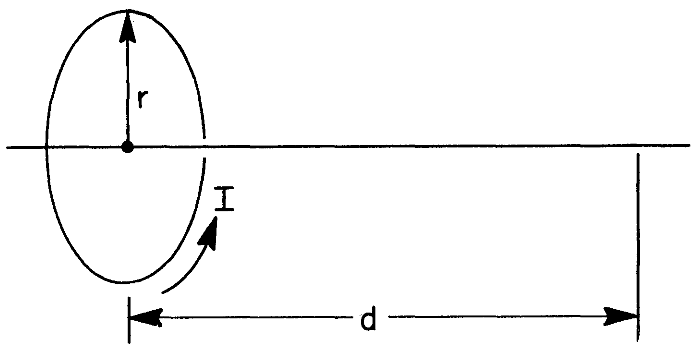
Calculates flow Q by using Bernoulli's equation.

73204 - ORBITAL MECHANICS

Calculates the velocity of a satellite at apogee and perigee and the orbital time in seconds of a satellite orbiting a body in space.

FLUX DENSITY

This program solves for the Flux density along axis of a circular loop.



r = radius of circular loop

I = current flowing around loop

d = distance from center of loop

A = area of loop (πr^2)

In free space $\mu = \mu_0 = 4\pi \times 10^{-7}$

$$B = \frac{2\mu_0 AI}{4\pi (r^2 + d^2)^{3/2}}$$

$$B = \frac{2(4\pi \times 10^{-7})\,\pi r^2 I}{4\pi (r^2 + d^2)^{3/2}} = \frac{2\pi r^2 I \times 10^{-7}}{(r^2 + d^2)^{3/2}}$$

If d and r are given in meters, I in amps, then B is given in webers/meter^2

This formula for flux density is good for thin-wire, circular loops; as d becomes larger and larger the equation may be approximated by $B = \frac{2\pi r^2 I}{d^3} \times 10^{-7}$ which becomes increasingly accurate as the ratio $\frac{d}{r}$ increases.

Reference: University Physics, 3rd edition
Sears & Zemanship

Addison - Vesley
1964

USER INSTRUCTIONS	EXAMPLES

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: r → Z, I → Y, d → X

PRESS: CONTINUE

DISPLAY

B	Z
0	Y
0	X

(A)

r = .2m

d = 0 m   B = 6.283×10^{-6} w/m^2

I = 2 amps

(B)

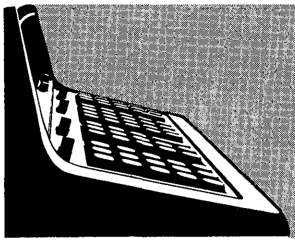
r = .2m

d = .05m   B = 5.737×10^{-6} w/m^2

I = 2 amps

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a)
0 0	CLEAR	20				
1	1	01	DISPLAY 1 TO INDICATE FIRST DATA ENTRY			
2	STOP	41	d	I	r	ENTER r,I,d
3	y→( )	40	STORE I (steps 03–04)			
4	a	13				
5	ROLL ↑	22	CALCULATE $(d^2+r^2)^{3/2}$ (steps 05–10)			
6	↑	27				
7	×	36				
8	y→( )	40				
9	b	14				
a	ROLL ↑	22				
b	TO POLAR	62				
c	↑	27				
d	×	36				
1 0	×	36				
1	2	02	CALCULATE $2\pi r^2 I \times 10^{-7}$ (steps 11–1d)			
2	↑	27				
3	π	56				
4	×	36				
5	b	14				
6	×	36				
7	a	13				
8	×	36				
9	ENTER EXP	26				
a	7	07				
b	CHG SIGN	32				
c	×	36				
d	↓	25				
2 0	x⇄y	30	CALCULATE B (steps 20–21)			
1	÷	35				
2	CLEAR x	37				
3	↑	27				
4	END	46	0	0	B	DISPLAY
5						
6						
7						
8						
9						
a						
b						
c						
d						

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



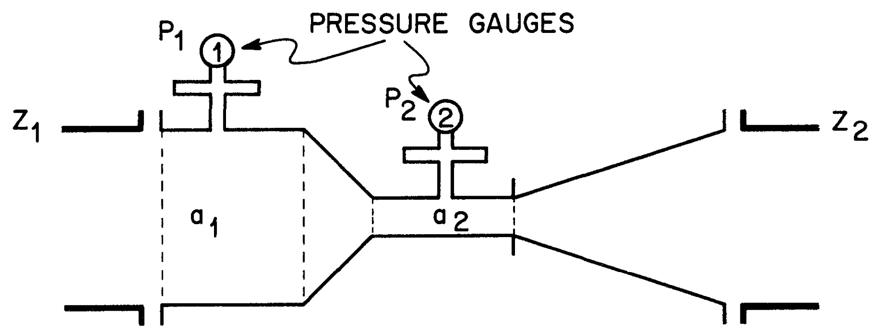
PART NO.
09100-73203

VENTURI METER

This program measures the rate of flow in a uniform flow tube by use of a Venturi meter.

A venturi meter is simply a constriction in a pipe which causes a pressure variation directly related to flow rate.

The following diagram applies:



Q (rate of flow) is defined by $Q = a_1V_1 = a_2V_2$ where a_1, a_2 are measured areas and V_1, V_2 are fluid velocities at the respective points.

Bernoulli's equation relates the flow characteristics at points one (1) and (2).

$$P_1/\gamma + V_1^2/2g + Z_1 = P_2/\gamma + V_2^2/2g + Z_2$$

Assuming the heights $Z_1 = Z_2$

then, $P_1/\gamma + V_1^2/2g = P_2/\gamma + V_2^2/2g$ where P_1, P_2 = pressure
γ = density
g = force of gravity

Solving for V_1 using the substitution $V_2^2 = (a_1/a_2)^2 V_1^2$

we obtain $$V_1 = \sqrt{\frac{2g\ (P_1 - P_2)}{\gamma\ \left[(a_1/a_2)^2 - 1\right]}}$$

or $$Q = a_1 V_1 = a_1 \sqrt{\frac{2g\ (P_1 - P_2)}{\gamma\ \left[(a_1/a_2)^2 - 1\right]}}$$

Dimensions:

a_1, a_2 given in inches^2

P_1, P_2 given in lbs/in.^2

γ given in lbs/in.^{3}
g given in inches/sec.^{2}
Then the dimensions of Q are in^3/sec.

Reference: Elementary Fluid Mechanics
by John K. Vennard

John Wiley & Sons
New York

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $a_1 \rightarrow Y$, $a_2 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: $\gamma \rightarrow Z$, $P_1 \rightarrow Y$, $P_2 \rightarrow X$

PRESS: CONTINUE

DISPLAY

Q	Z
0	Y
0	X

To enter new density or pressure; Press CONTINUE

To reset problem and/or calculate flow for different areas (a_1 & a_2); Press END, Press CONTINUE after display of Q.

EXAMPLE

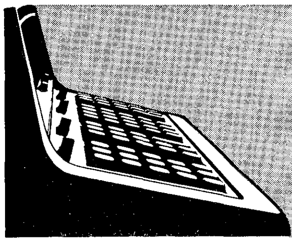
a_1 and a_2 have respective areas of 0.5 ft.^{2} & 0.25 ft.^{2} with pressure drop of 10 psi when water flows in the pipe (γ = 62.4 lb /ft.^{3} or 62.4/1728 lbs/in.^{3})

a_1 = 72 in.^{2}
a_2 = 36 ft.^{2}
γ = .03611 lb./in.^{3}
P_1 = 12 psi
P_2 = 2 psi

Answer: 19,221.589 in.^{3}/sec.

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	a_2	a_1	0	ENTER a_1, a_2					
	2	↑	27									
	3	↓	25									
	4	÷	35	CALCULATE $\frac{a_1}{a_2}$	AND STORE $\frac{a_1}{a_2}$	AND	a_1					
	5	↓	25									
	6	ACC +	60									
	7	CLEAR x	37									
	8	↑	27									
	9	↑	27									
	a	STOP	41	P_2	P_1	γ	ENTER γ, P_1, P_2					
	b	−	34	CALCULATE	$P_1 - P_2$							
	c	7	07									
	d	7	07									
1	0	2	02									
	1	·	21									
	2	0	00									
	3	8	10									
	4	×	36									
	5	↓	25									
	6	$x \rightleftarrows y$	30	CALCULATE	$\frac{2g(P_1-P_2)}{\gamma\left[\left(\frac{a_1}{a_2}\right)^2-1\right]}$							
	7	÷	35									
	8	f	15									
	9	↑	27									
	a	×	36									
	b	1	01									
	c	−	34									
	d	e	12									
2	0	ROLL ↓	31									
	1	÷	35									
	2	↓	25									
	3	$\sqrt{x}$	76									
	4	×	36	CALCULATE	AND POSITION Q							
	5	CLEAR x	37									
	6	↑	27									
	7	STOP	41	0	0	Q	DISPLAY					
	8	GO TO ( )( )	44									
	9	0	00	BRANCH FOR	NEW DATA	ENTRY						
	a	a	13									
	b	END	46									
	c											
	d											

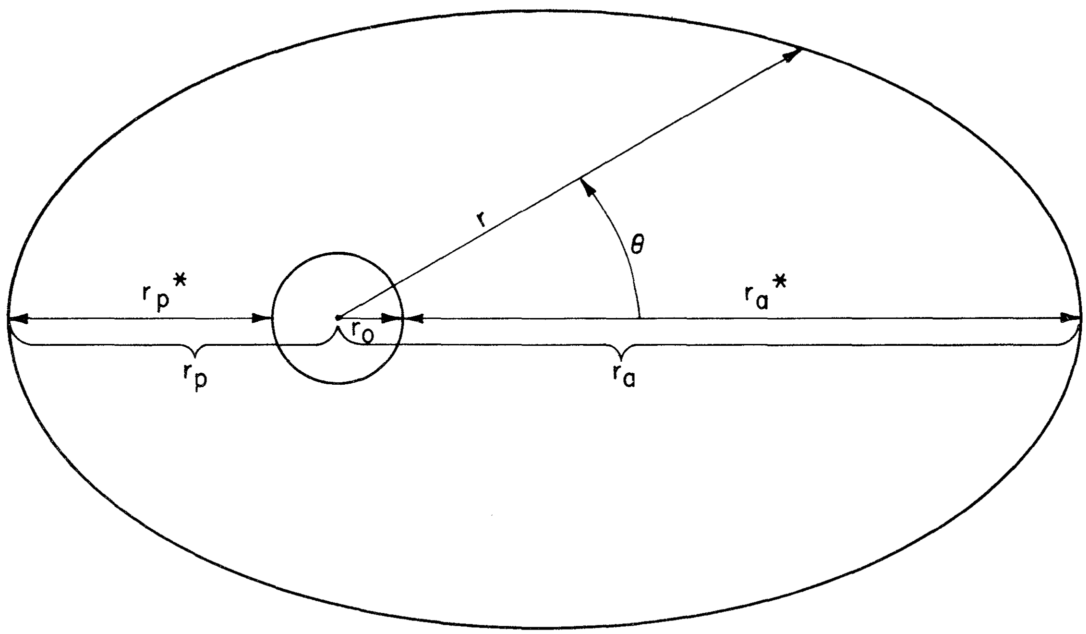
Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-73204

ORBITAL MECHANICS

This program computes the velocity of a satellite at apogee, the velocity at perigee, and the orbital period in seconds of a satellite orbiting a body in space.



Given:

r_a^* = radius at apogee measured above r_o $(r_a = r_a^* + r_o)$

r_p^* = radius at perigee measured above r_o $(r_p = r_p^* + r_o)$

r_o = radius of orbited body

g_o = acceleration of gravity at r_o

Find:

$$V_a = \sqrt{\frac{2\, g_o (r_o)^2 r_p}{r_a\,(r_a + r_p)}} \quad = \text{ velocity at apogee}$$

$$V_p = V_a \cdot \frac{r_a}{r_p} \quad = \text{ velocity at perigee}$$

$$p = \pi \sqrt{\frac{(r_a + r_p)^3}{2\, g_o (r_o)^2}} \quad = \text{ orbital period}$$

USER INSTRUCTIONS

SET: | Radians | |

ENTER PROGRAM: (Starting Address is 0-0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: $g_o \rightarrow Y$, $r_o \rightarrow X$

→ PRESS: CONTINUE

DISPLAY

1	———	Z
1	———	Y
1	———	X

ENTER DATA: $r_p^* \rightarrow Y$, $r_a^* \rightarrow X$

PRESS: CONTINUE

DISPLAY

p	———	Z
V_p	———	Y
V_a	———	X

To reset problem for same r_o and g_o.

EXAMPLE

$$g_o = \frac{(32.2)\,(3600)^2}{5280} = 7.904 \times 10^4$$

for units of miles and hours

r_o = 1,080 miles (moon)

r_a^* = 8,000 miles

r_p^* = 2,000 miles

▲ p = 9.810 hours

V_p = 6685.948 mi./hr.

V_a = 2267.921 mi./hr.

▲ Denotes Revision

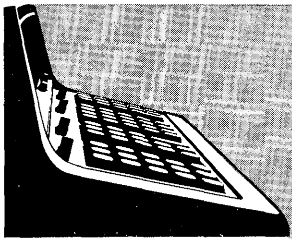
Step		Key	Code	Display x	Display y	Display z	Storage / Remarks
0	0	CLEAR	20				
	1	STOP	41	r_o	g_o		ENTER g_o AND r_o
	2	$x\rightarrow()$	23	STORE g_o AND r_o			
	3	a	13				
	4	$y\rightarrow()$	40				
	5	b	14				
	6	1	01	(FROM 4-d)			
	7	↑	27				
	8	↑	27				
	9	STOP	41	r_a^*	r_p^*		ENTER r_p^* AND r_a^*
	a	$x\rightarrow()$	23	STORE r_p^* AND r_a^*			
	b	f	15				
	c	$y\rightarrow()$	40				
	d	e	12				
1	0	a	13				
	1	↑	27				
	2	ACC +	60				
	3	b	14				
	4	↑	27				
	5	+	33				
	6	a	13				
	7	×	36				
	8	×	36				
	9	e	12				
	a	×	36				
	b	↑	27				
	c	f	15				
	d	+	33				
2	0	$y\rightarrow()$	40	CALCULATE $V_a = \sqrt{\dfrac{2 g_o r_o^2 r_p}{r_a (r_a + r_p)}}$			
	1	d	17				
	2	×	36				
	3	↓	25				
	4	÷	35				
	5	↓	25				
	6	$\sqrt{x}$	76				
	7	$x\rightarrow()$	23	STORE V_a			
	8	c	16				
	9	↑	27	CALCULATE $V_p = V_a \cdot \dfrac{r_a}{r_p}$			
	a	f	15				
	b	×	36				
	c	e	12				
	d	÷	35				

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	d	17									
	1	y→( )	40	STORE V_p								
	2	d	17									
	3	↑	27	CALCULATE $P = \pi\sqrt{\frac{(r_a + r_p)^3}{2 g_o (r_o)^2}}$								
	4	×	36									
	5	×	36									
	6	b	14									
	7	÷	35									
	8	2	02									
	9	÷	35									
	a	a	13									
	b	÷	35									
	c	÷	35									
	d	π	56									
4	0	x⇄y	30									
	1	√x	76									
	2	×	36									
	3	a	13									
	4	↑	27									
	5	ACC −	63									
	6	y⇄( )	24									
	7	d	17									
	8	c	16									
	9	STOP	41	V_a	V_p	P	DISPLAY					
	a	CLEAR x	37									
	b	GO TO ( )( )	44	BRANCH TO ENTER NEW r_a^* AND r_p^*								
	c	0	00									
	d	6	06									
5	0	END	46									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

73851 - HEAT TRANSFER THROUGH A MULTILAYERED CYLINDER
This program calculates the rate of heat flow through a multilayered cylinder. Both convective and conductive layers are considered.

9100B ONLY

73852 - TRANSIENT CONDUCTION IN A SLAB
Given the initial temperature of a slab and a fluid, this program determines the temperature (as a function of time and distance) of the slab as it is immersed into the fluid.



PART NO.
09100-73851

HEAT TRANSFER THROUGH A MULTILAYERED CYLINDER

This program determines the rate of heat flow per unit length through a multilayered cylinder. It has practical application wherever fluids of different temperatures are separated by a cylindrical surface since both convective and conductive layers are considered.

The basic equation is:

$$q = U_n A (\Delta T)$$

where

$$A = 2\pi \cdot r_o L ,$$

and

$$U_n = \frac{1}{\left(\frac{r_o}{r_{c1}h_1} + \frac{r_o}{r_{c2}h_2} + \cdots\cdots\right) + \left(\frac{r_o \ln\left(\frac{r_{o1}}{r_{i1}}\right)}{k_1} + \frac{r_o \ln\left(\frac{r_{o2}}{r_{i2}}\right)}{k_2} + \cdots\cdots\right)}$$

Nomenclature

- U_n - Outside over-all heat transfer coefficient
- r_o - Outside radius of the cylinder
- k_n - Thermal conductivity of a conductive layer
- r_{on} - Outside radius of a conductive layer
- r_{in} - Inside radius of a conductive layer
- h_n - Film coefficient of a convective layer
- r_{cn} - Radius to a convective surface
- ΔT - Temperature difference (inside - outside)
- q - Heat transfer rate

Reference: Heat Transfer, Alan J. Chapman, Macmillan Company, New York, 1967.

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: (Starting Address is 0-0)

PRESS: END

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
0	———	X

ENTER DATA: $r_o \rightarrow Z$, $T_o \rightarrow Y$, $T_i \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
n	———	X

n is the layer to be entered

ENTER DATA: For conductive layer
$k \rightarrow Z$, $r_{on} \rightarrow Y$, $r_{in} \rightarrow X$

For convective layer
$0 \rightarrow Z$, $h \rightarrow Y$, $r_{cn} \rightarrow X$

NOTE: A convective layer is considered very thin and only the radius to the convection surface is entered.

PRESS: CONTINUE

Have all layers been entered? (NO → return to DISPLAY; YES → continue)

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	———	Z
q/L	———	Y
0	———	X

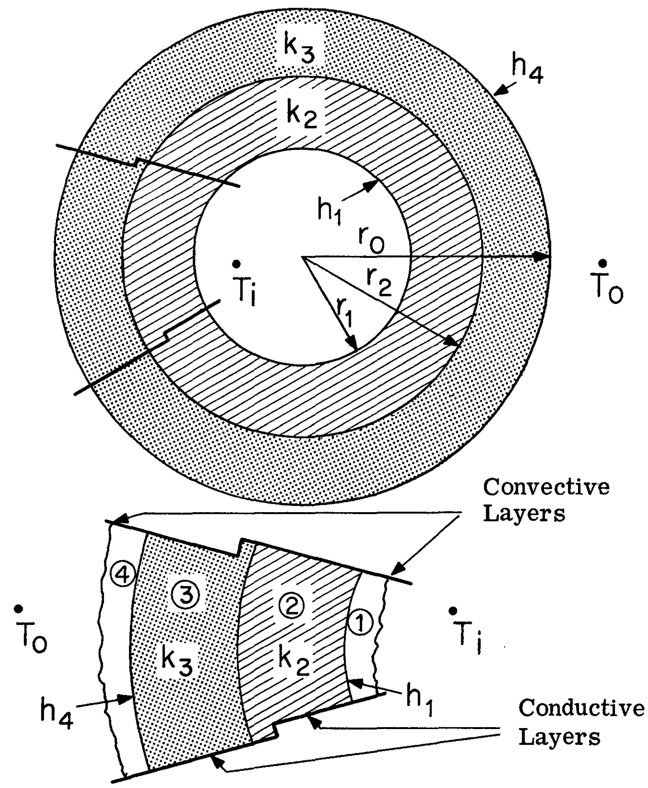
To run another problem (return to PRESS: CONTINUE)

EXAMPLES

A.

r_o = .833 ft. T_o = 100^o F
r_2 = .667 ft. T_i = 500^o F
r_1 = .500 ft.
h_1 = 300 Btu/hr. ft.^{2} ^{o}F
k_2 = 40 Btu/hr. ft. ^{o}F
k_3 = .10 Btu/hr. ft. ^{o}F
h_4 = 6 Btu/ hr. ft.^{2} ^{o}F

4 LAYER PROBLEM



DATA ENTRY: .833 → Z, 100 → Y, 500 → X

Layer	X	Y	Z
1	.500	300	0
2	.500	.667	40
3	.667	.833	.10
4	.833	6	0

Results: q/L = 1031.558 Btu/hr. ft.

B. Same as above except k_3 = .01 Btu/hr. ft. ^{o}F (Better insulation).

Results: q/L = 112.008 Btu/ hr. ft.

Step	Key	Code	Display x	Display y	Display z
00	CLEAR	20	ENTER		
01	STOP	41	T_i	T_o	r_o
02	x⇄y	30			
03	−	34			
04	y→( )	40			
05	b	14			
06	↓	25			
07	y→( )	40			
08	a	13			
09	CLEAR	20			
0a	1	01			
0b	ACC +	60	ENTER		
0c	f	15	r_{cn}	h	0
0d	STOP	41	r_{in}	r_{on}	k
10	IF FLAG	43			
11	3	03			
12	b	14			
13	x→( )	23			
14	d	17			
15	0	00			
16	ROLL ↑	22			
17	IF x=y	50			
18	2	02			
19	d	17			
1a	ROLL ↓	31			
1b	d	17			
1c	÷	35			
1d	a	13			
20	x⇄y	30			
21	ln x	65			
22	×	36			
23	↓	25			
24	x⇄y	30			
25	÷	35			
26	0	00			
27	ACC +	60			
28	↑	27			
29	↑	27			
2a	GO TO ( )	44			
2b	0	00			
2c	a	13			
2d	a	13			
30	ROLL ↑	22			
31	÷	35			
32	d	17			
33	÷	35			
34	0	00			
35	ACC +	60			
36	↑	27			
37	↑	27			
38	GO TO ( )	44			
39	0	00			
3a	a	13			
3b	1	01			
3c	↑	27			
3d	e	12			
40	÷	35			
41	2	02			
42	×	36			
43	π	56			
44	×	36			
45	a	13			
46	×	36			
47	b	14			
48	×	36			
49	CLEAR x	37			
4a	CONT	47	DISPLAY		
4b	END	46	0	q/L	0

Storage

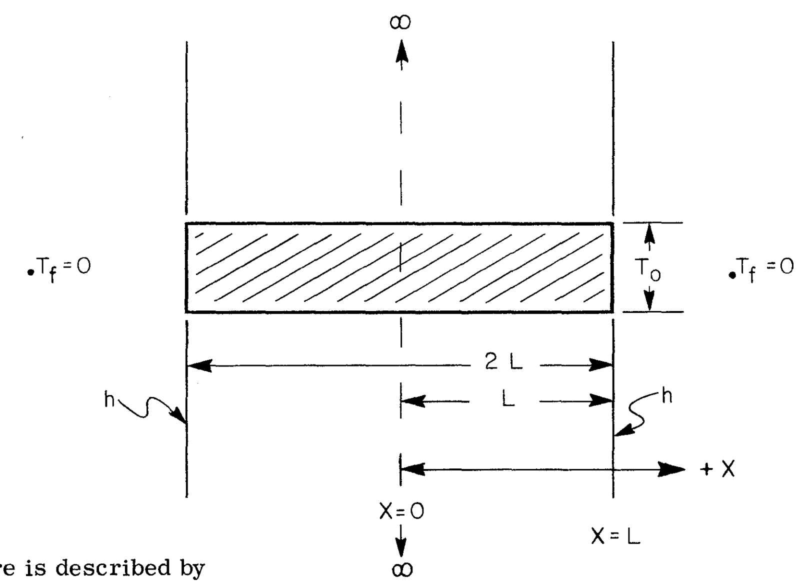
Step	Key	Code	Display			Step	Key	Code	Display			Step	Key	Code	Display		
			x	y	z				x	y	z				x	y	z
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0											
1						1											
2						2											
3						3											
4						4											
5						5											
6						6											
7						7											
8						8											
9						9											
a						a											
b						b											
c						c											
d						d											

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

TRANSIENT CONDUCTION IN A SLAB

This program calculates, as a function of time and position, the temperature of a plane in a slab which has been heated to an initial temperature and then placed in a fluid of a different temperature. This is equivalent to plunging a heated bar into a quenching medium. The program assumes the slab is infinite, therefore, the problem involves one dimensional transient conduction.

The temperature of the fluid is taken to be zero and used as a reference. For example, if the temperature of the fluid is actually $40^{O}F$ and the slab is initially at $100^{O}F$, the user would input $60^{O}F$ (100 - 40) as the initial temperature of the slab. The actual temperature of a plane in the slab at a later time is then the calculated value plus $40^{O}F$ (see example B).



The temperature is described by

$$T = 2T_o \sum_{n=1}^{\infty} e^{-\lambda_n^2 \alpha \tau} \frac{\sin(\lambda_n L)}{\lambda_n L + \sin(\lambda_n L)\cos(\lambda_n L)} \cdot \cos(\lambda_n X)$$

where λ_n is the nth root of

$$(\lambda_n L)\tan(\lambda_n L) - N_{Bi} = 0$$

and N_{Bi} is the Biot modulus ($N_{Bi} = h\,L/k$) .

Reference: Heat Transfer, Alan J. Chapman
Macmillan Company, New York, 1967.

Nomenclature:

T_o - Initial temperature of the slab

α - Thermal diffusivity of the slab

k - Thermal conductivity of the slab

h - Film coefficient

L - One-half the width of the slab

τ - Independent time variable

X - Independent space variable

T - Dependent temperature variable

T_f - Temperature of the fluid

NOTES: 1. The program performs the summation until adding the next term changes it less than 10^{-5}. To change this tolerance go to location (-5)(d) and enter the desired exponent.

2. The calculation of the roots ($\lambda_n L$) may be seen by placing a PAUSE at location (1) (c). The root ($\lambda_n L$) is displayed in the X register as the solution converges. Place a STOP at location (4)(1) for a display of the calculated root in the X register.

3. The execution time is about two minutes.

USER INSTRUCTIONS

SET: RADIANS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $N_{Bi} \rightarrow Z$, $T_o \rightarrow Y$, $L \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: $\alpha \rightarrow Z$, $\tau \rightarrow Y$, $X \rightarrow X$

PRESS: CONTINUE

DISPLAY

T	Z
n	Y
X	X

n is the number of terms used in the summation

To re-run problem with different X, τ, or α :

PRESS: CONTINUE

To run new problem:

PRESS: END

NOTE: Any consistent set of units may be used.

EXAMPLES

A.

$N_{Bi} = 20$

$T_o = 120^oF$ $(T_f = 0^oF)$

$L = .4$ ft.

$\alpha = .02$ ft.^{2}/hr.

$\tau = 1$ hr.

$X = .1$ ft.

Results

$T = 105.49^oF$

$n = 4$

$X = .1$ ft.

B.

$N_{Bi} = 20$

Temperature of the slab initially is 180^oF

Temperature of the fluid is 30^oF

$T_o = 180^o - 30^o = 150^oF$

$L = .4$ ft.

$\alpha = .02$ ft.^{2}/hr.

$\tau = 1$ hr.

$X = .1$ ft.

Results

$T = 131.87^oF$

$n = 4$

$X = .1$ ft.

Actual temperature is $T + 30^oF = 161.87^oF$

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	GO TO ( )	44			
(+) 1	8	10			
2	0	00			
3	•	21			
4	1	01			
5	x→( )	23			
6	7	07			
7	c	16			
8	GO TO ( )	44			
9	0	00			
a	c	16			
b	SET FLAG	54			
c	x→( )	23			
d	d	17			
1 0	↑	27			
(+) 1	tan x	71			
2	×	36			
3	b	14			
4	−	34			
5	↓	25			
6	↑	27			
7	y⇄( )	24			
8	6	06			
9	×	36			
a	↑	27			
b	d	17			
c	CONT	47			
d	IF FLAG	43			
2 0	4	04			
(+) 1	1	01			
2	ROLL ↓	31			
3	CLEAR x	37			
4	IF x<y	52			
5	3	03			
6	0	00			
7	IF x=y	50			
8	ROLL ↑	22			
9	GO TO ( )	44			
a	0	00			
b	b	14			
c	CHG SIGN	32			
d	•	21			

Step	Key	Code	Display x	Display y	Display z
3 0	1	01			
(+) 1	y⇄( )	24			
2	7	07			
3	×	36			
4	y→( )	40			
5	7	07			
6	d	17			
7	+	33			
8	IF x=y	50			
9	0	00			
a	b	14			
b	↓	25			
c	GO TO ( )	44			
d	0	00			
4 0	c	16			
(+) 1	CONT	47			
2	↑	27			
3	x←( )	67			
4	c	16			
5	↑	27			
6	π	56			
7	+	33			
8	y→( )	40			
9	c	16			
a	↓	25			
b	CHG SIGN	32			
c	x→( )	23			
d	6	06			
5 0	↓	25			
(+) 1	RETURN	77			
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
8 0	CLEAR	20			
(+) 1	1	01	ENTER		
2	STOP	41	L	T_0	N_{Bi}
3	x→( )	23			
4	−	34			
5	f	15			
6	y→( )	40			
7	−	34			
8	c	16			
9	ROLL ↑	22			
a	x→( )	23			
b	b	14			
c	CLEAR	20			
d	x→( )	23			
9 0	−	34			
(+) 1	a	13			
2	x→( )	23			
3	−	34			
4	b	14			
5	x→( )	23			
6	c	16			
7	GO TO ( )	44			
8	−	34			
9	0	00			
a	0	00			
b	CONT	47			
c	CONT	47			
d	CONT	47			

Storage

	+	−
f		L
e		α τ
d		X
c	n π	T_0
b	N_{Bi}	n
a		Σ
9		
8		
7		
6		
5		
4		
3		
2		
1		
0		

Step	Key	Code	Display x	Display y	Display z
00	2	02			
(−) 01	CHG SIGN	32			
02	x→( )	23			
03	6	06			
04	CHG SIGN	32	ENTER		
05	STOP	41	X	τ	a
06	x→( )	23			
07	−	34			
08	d	17			
09	ROLL ↓	31			
0a	×	36			
0b	y→( )	40			
0c	−	34			
0d	e	12			
10	x←( )	67			
(−) 11	−	34			
12	b	14			
13	↑	27			
14	1	01			
15	+	33			
16	y→( )	40			
17	−	34			
18	b	14			
19	GO TO ( )	44			
1a	▲SUB▼	77			
1b	+	33			
1c	0	00			
1d	3	03			
20	x→( )	23			
(−) 21	−	34			
22	9	11			
23	CLEAR x	37			
24	↓	25			
25	cos x	73			
26	x⇄y	30			
27	sin x	70			
28	×	36			
29	ROLL ↓	31			
2a	+	33			
2b	↓	25			
2c	÷	35			
2d	y⇄( )	24			

Step	Key	Code	Display x	Display y	Display z
30	−	34			
(−) 31	9	11			
32	x←( )	67			
33	−	34			
34	f	15			
35	÷	35			
36	↓	25			
37	↑	27			
38	×	36			
39	x⇄y	30			
3a	↑	27			
3b	x←( )	67			
3c	−	34			
3d	e	12			
40	×	36			
(−) 41	↓	25			
42	CHG SIGN	32			
43	e^x	74			
44	y⇄( )	24			
45	−	34			
46	9	11			
47	×	36			
48	x←( )	67			
49	−	34			
4a	d	17			
4b	ROLL ↑	22			
4c	×	36			
4d	CLEAR x	37			
50	↓	25			
(−) 51	cos x	73			
52	×	36			
53	x←( )	67			
54	−	34			
55	a	13			
56	+	33			
57	y→( )	40			
58	−	34			
59	a	13			
5a	−	34			
5b	|y|	55			
5c	ENTER EXP	26			
5d	5	05			

Step	Key	Code	Display x	Display y	Display z
60	CHG SIGN	32			
(−) 61	IF x>y	53			
62	6	06			
63	7	07			
64	GO TO ( )	44			
65	1	01			
66	0	00			
67	x←( )	67			
68	−	34			
69	a	13			
6a	↑	27			
6b	2	02			
6c	×	36			
6d	x←( )	67			
70	−	34			
(−) 71	c	16			
72	×	36			
73	x←( )	67			
74	−	34			
75	b	14			
76	↑	27			
77	x←( )	67			
78	−	34			
79	d	17			
7a	CONT	47	DISPLAY		
7b	STOP	41	X	n	T
7c	GO TO ( )	44			
7d	+	33			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step		Key	Code	Display x	Display y	Display z
8	0	8	10			
(–)	1	C	16			
	2	END	46			
	3					
	4					
	5					
	6					
	7					
	8					
	9					
	a					
	b					
	c					
	d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	Display *x*	*y*	*z*	Step	Key	Code	Display *x*	*y*	*z*	Step	Key	Code	Display *x*	*y*	*z*
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					
0						0						0					
1						1						1					
2						2						2					
3						3						3					
4						4						4					
5						5						5					
6						6						6					
7						7						7					
8						8						8					
9						9						9					
a						a						a					
b						b						b					
c						c						c					
d						d						d					

Step	Key	Code	*x*	*y*	*z*	Step	Key	Code	*x*	*y*	*z*
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
a						a					
b						b					
c						c					
d						d					

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

74003 - INVERSE TRAVERSE FROM COORDINATES
Calculates bearing, distance, and quadrant code from end-point coordinates.

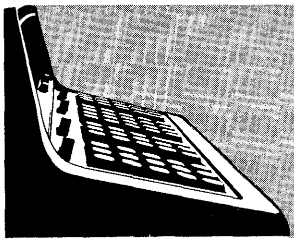
74004 - THREE POINT PROBLEM
Calculates the coordinates of an observer's position given coordinates of three other points and two reference angles to the observer's position.

9100B ONLY

74101 - TRAVERSE WITH COMPASS RULE ADJUSTMENT OPTION
Traverses by bearing and distance, calculates coordinates, closure error, total traverse distance and precision ratio. The program may then be used to distribute the closure error by the Compass Rule.

74102 - COORDINATE GEOMETRY AND ENCLOSED AREA
Traverses by bearing and distance and calculates coordinates, enclosed area, and the following curve data:

1. Arc length
2. Chord length
3. Central angle in decimal degrees
4. Coordinates of tangent points



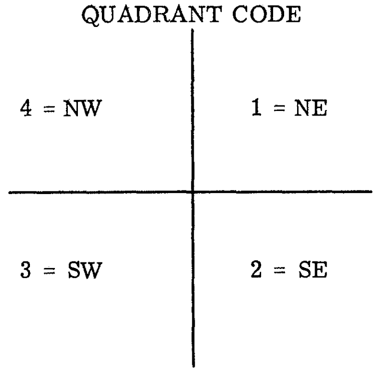
PART NO.
09100-74003

INVERSE TRAVERSE FROM COORDINATES

This program will calculate bearings and distances between given coordinates. If it is desired to continue on to a new point from the last point entered the operator has only to press CONTINUE and enter coordinates for the new forward point.

The quadrants are denoted by NE = 1, SE = 2, SW = 3 and NW = 4.

QUADRANT CODE



USER INSTRUCTIONS

SET: DEGREES

PRESS: GO TO (0) (0) [or END]

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: END

PRESS: CONTINUE

DISPLAY

- 0 — Z
- 0 — Y
- 1 — X

ENTER DATA: Point A Northing → Y

Point A Easting → X

PRESS: CONTINUE

→ DISPLAY

- 0 — Z
- 0 — Y
- 2 — X

ENTER DATA: Point B Nothing → Y

Point B Easting → X

PRESS: CONTINUE

DISPLAY

- Degrees — Z
- Minutes — Y
- Seconds — X

PRESS: CONTINUE

DISPLAY

- 0 — Z
- Distance — Y
- Quadrant Code — X

PRESS: CONTINUE to enter a new course

EXAMPLES

1100 North
800 East

1300 North
900 East

Answer:

26° 33' 54.184"

223.607'

1 (Quadrant Code)

1300 North
1200 East

Answer:

90° 00' 00"

300.000'

2 (Quadrant Code)

1000 North
1000 East

Answer:

33° 41' 24.243"

360.555'

3 (Quadrant Code)

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
2	STOP	41	A_E	A_N	0	ENTER A_N, A_E					
3	ACC +	60	STORE STARTING COORDINATES								
4	↓	25									
5 (FROM 6-7)	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
6	STOP	41	B_E	B_N	0	ENTER B_N, B_E					
7	↑	27									
8	$y \rightleftarrows ()$	24									
9	f	15									
a	$x \rightleftarrows y$	30									
b	−	34									
c	e	12									
d	CHG SIGN	32									
1 0	ROLL ↑	22	STORE FORWARD COORDINATES FOR FUTURE INVERSE PROBLEM. *(NOTE: IN THE NEXT PROBLEM THESE COORDINATES WILL BE THE STARTING POINT.)*								
1	$x \to ()$	23									
2	e	12									
3	+	33									
4	↓	25									
5	$x \rightleftarrows y$	30									
6	TO POLAR	62									
7	$x \to ()$	23									
8	d	17									
9	0	00									
a	IF $x<y$	52	DETERMINE QUADRANT								
b	3	03									
c	2	02									
d	$\lvert y \rvert$	55									
2 0	9	11									
1	0	00									
2	IF $x>y$	53									
3	2	02									
4	a	13									
5	−	34									
6	3	03									
7	GO TO ( )( )	44									
8	4	04									
9	1	01									
a	$x \rightleftarrows y$	30									
b	−	34									
c	2	02									
d	GO TO ( )( )	44									

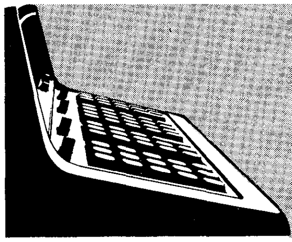
FROM 1-c → 3-2; FROM 2-9 → 3-c; FROM 3-6 → 4-1

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	4	04									
1	1	01									
2	9	11									
3	0	00									
4	IF x > y	53									
5	3	03									
6	c	16									
7	−	34									
8	4	04									
9	GO TO ( )( )	44									
a	4	04									
b	1	01									
c	x⇄y	30									
d	−	34									
4 0	1	01									
1	x→( )	23									
2	c	16									
3	↓	25									
4	↑	27									
5	int x	64									
6	−	34									
7	x→( )	23									
8	b	14									
9	6	06									
a	0	00									
b	×	36									
c	↓	25									
d	↑	27	CONVERT DECIMAL DEGREES TO DEGREES, MINUTES, SECONDS								
5 0	int x	64									
1	−	34									
2	x→( )	23									
3	a	13									
4	6	06									
5	0	00									
6	×	36									
7	b	14									
8	↑	27									
9	a	13									
a	ROLL ↑	22									
b	STOP	41	SECONDS	MINUTES	DEGREES	DISPLAY					
c	0	00									
d	↑	27									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	d	17	RECALL AND POSITION DISTANCE AND QUADRANT CODE								
	1	↑	27									
	2	c	16									
	3	STOP	41	QUAD. CODE	DISTANCE	0	DISPLAY					
	4	↓	25									
	5	GO TO ( )( )	44	BRANCH FOR NEW FORWARD POINT								
	6	0	00									
	7	5	05									
	8	END	46									
	9											
	a											
	b											
	c											
	d											

HEWLETT · PACKARD

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-74004

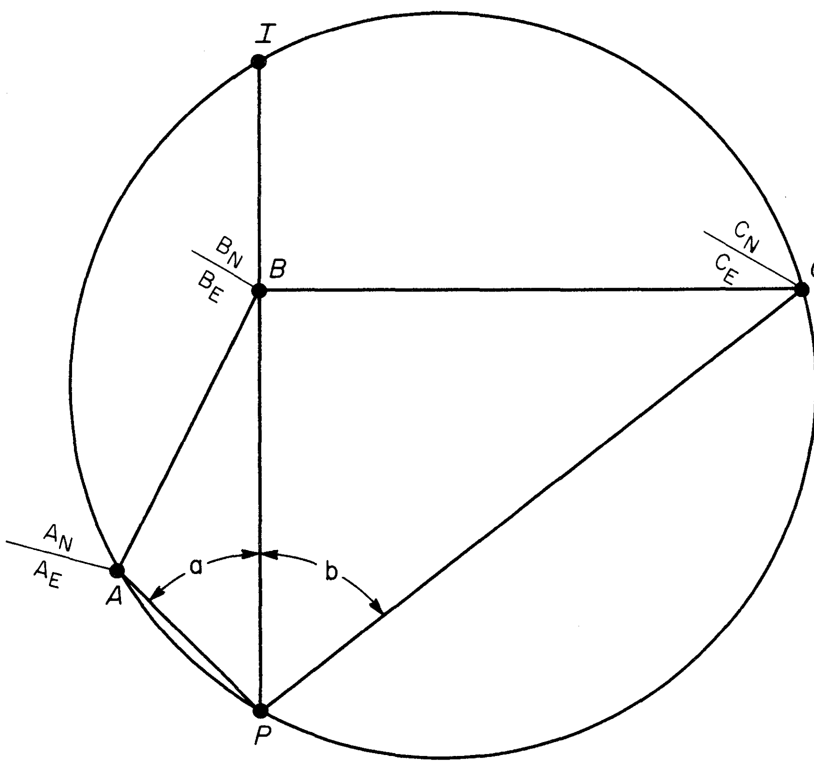
THREE POINT PROBLEM

The three point problem, sometimes known as Italian Resection, involves the determination of coordinates of an observers position (Point P) by measuring only angles a & b. The coordinates of points A, B & C must be known.

To make the solution possible it is necessary:

1. To construct a circle through points A, C, & P.
2. Project line P - B to intersect the circle at point I
   (Note: No solution possible if "B" on circle)

See figure below:



USER INSTRUCTIONS

SET: | DEGREES | |

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $A_N \rightarrow Y$, $A_E \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: $C_N \rightarrow Y$, $C_E \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

ENTER DATA: ANGLE a

$a^o \rightarrow Z$, $a' \rightarrow Y$, $a'' \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
4	X

USER INSTRUCTIONS (con't)

ENTER DATA: ANGLE b

$b^o \rightarrow Z$, $b' \rightarrow Y$, $b'' \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
5	X

ENTER DATA: $B_N \rightarrow Y$, $B_E \rightarrow X$

PRESS: CONTINUE

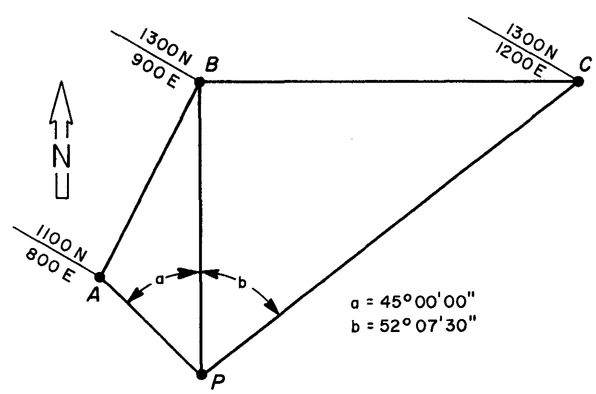
DISPLAY

0	Z
Pn	Y
Pe	X

TO ENTER ANOTHER PROBLEM, PRESS: CONTINUE

EXAMPLE

DATA	ANSWERS
Coordinates, Point A 1100N, 800E	Observers Point (P) Coordinates 1000.00N, 1000.00E
Coordinates, Point C 1300N, 1200E	
Angle a 45^o 00' 00"	
Angle b 52^o 07' 30"	
Coordinates, Point B 1300N, 900E	

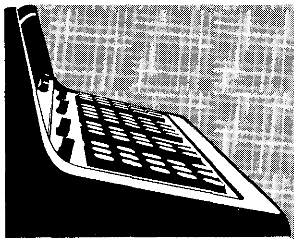


Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
2	STOP	41	A_E	A_N	0	ENTER A_N, A_E					
3	x→( )	23	STORE DATA (steps 3–6)								
4	d	17									
5	y→( )	40									
6	c	16									
7	ACC −	63									
8	↓	25									
9	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
a	STOP	41	C_E	C_N	0	ENTER C_N, C_E					
b	ACC +	60	CALCULATE $C_N - A_N$ AND $C_E - A_E$								
c	↓	25									
d	3	03	DISPLAY 3 TO INDICATE THIRD ENTRY								
1 0	STOP	41	SECONDS	MINUTES	DEGREES	ENTER DEGREES, MINUTES AND SECONDS					
1	x→( )	23	CONVERT TO DECIMAL DEGREES (steps 1 1 – 1 d)								
2	b	14									
3	6	06									
4	0	00									
5	÷	35									
6	ROLL ↓	31									
7	+	33									
8	b	14									
9	ROLL ↑	22									
a	÷	35									
b	÷	35									
c	↓	25									
d	+	33									
2 0	IF FLAG	43									
1	2	02									
2	d	17									
3	SET FLAG	54									
4	y→( )	40	STORE ANGLE (steps 4–5)								
5	a	13									
6	CLEAR x	37	CLEAR DISPLAY (steps 6–8)								
7	↑	27									
8	↑	27									
9	4	04	DISPLAY 4 TO INDICATE FOURTH ENTRY								
a	GO TO ( )( )	44	BRANCH FOR b ANGLE ENTRY (steps a–c)								
b	1	01									
c	0	00									
d	a	13	RECALL ANGLE a								

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	y→( )	40	STORE ANGLE b								
	1	b	14									
	2	+	33									
	3	↓	25	CALCULATE sin OF a AND b								
	4	sin x	70									
	5	ROLL ↓	31									
	6	RCL	61	RECALL ΔE AND ΔN OF BASELINE A→C								
	7	TO POLAR	62	AND CONVERT TO POLAR								
	8	y→( )	40	STORE AZIMUTH θ A→C								
	9	e	12									
	a	x⇄y	30									
	b	a	13	CALCULATE sin θ_a								
	c	sin x	70									
	d	y→( )	40	STORE BASELINE DISTANCE A→C								
4	0	f	15									
	1	×	36									
	2	x⇄y	30	COMPLETE LAW OF SINES FOR DISTANCE A→I								
	3	ROLL ↑	22									
	4	÷	35									
	5	e	12									
	6	↑	27									
	7	b	14									
	8	+	33									
	9	c	16									
	a	ROLL ↓	31									
	b	x⇄y	30									
	c	TO RECT	66	CALCULATE AZIMUTH A→I (FROM A→C) AND								
	d	ROLL ↓	31	CALCULATE COORDINATES POINT I								
5	0	+	33									
	1	0	00									
	2	y→( )	40									
	3	b	14									
	4	ROLL ↓	31									
	5	d	17									
	6	+	33									
	7	y→( )	40									
	8	e	12									
	9	↓	25									
	a	5	05	DISPLAY 5 TO INDICATE FIFTH ENTRY								
	b	STOP	41	B_E	B_N	0	ENTER B_N, B_E					
	c	↑	27									
	d	e	12									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 0	−	34									
1	b	14									
2	ROLL ↑	22	CALCULATE LINE EQUATION B → I								
3	x⇄y	30									
4	−	34									
5	↓	25									
6	x⇄y	30									
7	TO POLAR	62									
8	y⇄( )	24	STORE AZIMUTH B → I								
9	e	12									
a	d	17									
b	x⇄y	30									
c	−	34									
d	c	16									
7 0	↑	27	CALCULATE AZIMUTH AND DISTANCE A → I								
1	b	14									
2	−	34									
3	↓	25									
4	x⇄y	30									
5	TO POLAR	62									
6	↑	27									
7	a	13									
8	sin x	70									
9	÷	35									
a	c	16	COMPLETE LAW OF SINES FOR DISTANCE A → I								
b	ROLL ↑	22									
c	−	34									
d	↓	25									
8 0	sin x	70									
1	×	36									
2	e	12									
3	↑	27									
4	a	13									
5	+	33	CALCULATE ΔE AND A_N OF LINE A → P								
6	↓	25									
7	x⇄y	30									
8	TO RECT	66									
9	↑	27									
a	d	17									
b	+	33									
c	ROLL ↑	22									
d	x⇄y	30									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	c	16	SUM COORDINATES FOR POSITION OF P								
	1	+	33									
	2	CLEAR x	37									
	3	ROLL ↓	31									
	4	$x \rightleftarrows y$	30									
	5	END	46	P_E	P_N	0	DISPLAY					
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



9100B ONLY
PART NO.
09100-74101

TRAVERSE WITH COMPASS RULE ADJUSTMENT OPTION

This program traverses by bearing and distance, calculates coordinates, closure error, total traverse distance and the precision ratio. The precision ratio is the ratio of the total distance traversed to the closure error distance from a fixed point.

The program may then be used to distribute the closure error by the Compass Rule. The Compass Rule assumes:

1. Course corrections are proportional to course lengths.
2. Angular measurement errors are equal to linear measurement errors.

The Compass Adjustment formulas used are:

$$\text{easting correction} = \left[\frac{\text{course distance}}{\text{total traverse distance}}\right] \times \left[\text{easting closure error}\right]$$

$$\text{northing correction} = \left[\frac{\text{course distance}}{\text{total traverse distance}}\right] \times \left[\text{northing closure error}\right]$$

QUADRANT CODE

4 = NW	1 = NE
3 = SW	2 = SE

USER INSTRUCTIONS

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER COORDINATES:
Northing → Y
Easting → X

PRESS: ACC+

ENTER BEARING ANGLE:
Degrees → Z
Minutes → Y
Seconds → X

PRESS: CONTINUE

DISPLAY: Bearing Angle

Decimal Degrees	Z
2	Y
2	X

ENTER DATA: Quadrant Code → X
(NE = 1, SE = 2, SW = 3, NW = 4)

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

Exit from curve? NO → (return to ENTER BEARING ANGLE) / YES ↓

ENTER DATA: Distance → Y, Distance → X

PRESS: CONTINUE

DISPLAY

Arc Length	Z
Chord	Y
Radius	X

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY: Central Angle

Degrees	Z
Minutes	Y
Seconds	X

PRESS: CONTINUE

ENTER DATA: Distance → X

PRESS: CONTINUE

DISPLAY

0	Z
Northing	Y
Easting	X

Is traverse complete? NO → (return to ENTER BEARING ANGLE) / YES ↓

ENTER FIXED COORDINATES:
Northing → Y
Easting → X

PRESS: GO TO ( ) ( )

PRESS: d

PRESS: 1

PRESS: CONTINUE

DISPLAY

Precision Ratio	Z
Closure Error	Y
Distance Traversed	X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER STARTING COORDINATES:
Northing → Y
Easting → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER Bearing Angles:
Degrees → Z
Minutes → Y
Seconds → X

PRESS: CONTINUE

DISPLAY: Bearing Angle

Decimal Degrees	Z
2	Y
2	X

ENTER DATA: Quadrant Code → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

ENTER DATA: Distance → X

PRESS: CONTINUE

DISPLAY

0	Z
Corrected Distance	Y
Quadrant Code	X

PRESS: CONTINUE

DISPLAY: Corrected Bearing Angle

Degrees	Z
Minutes	Y
Seconds	X

USER INSTRUCTIONS (Con't)

PRESS: CONTINUE

DISPLAY: CORRECTED COORDINATES

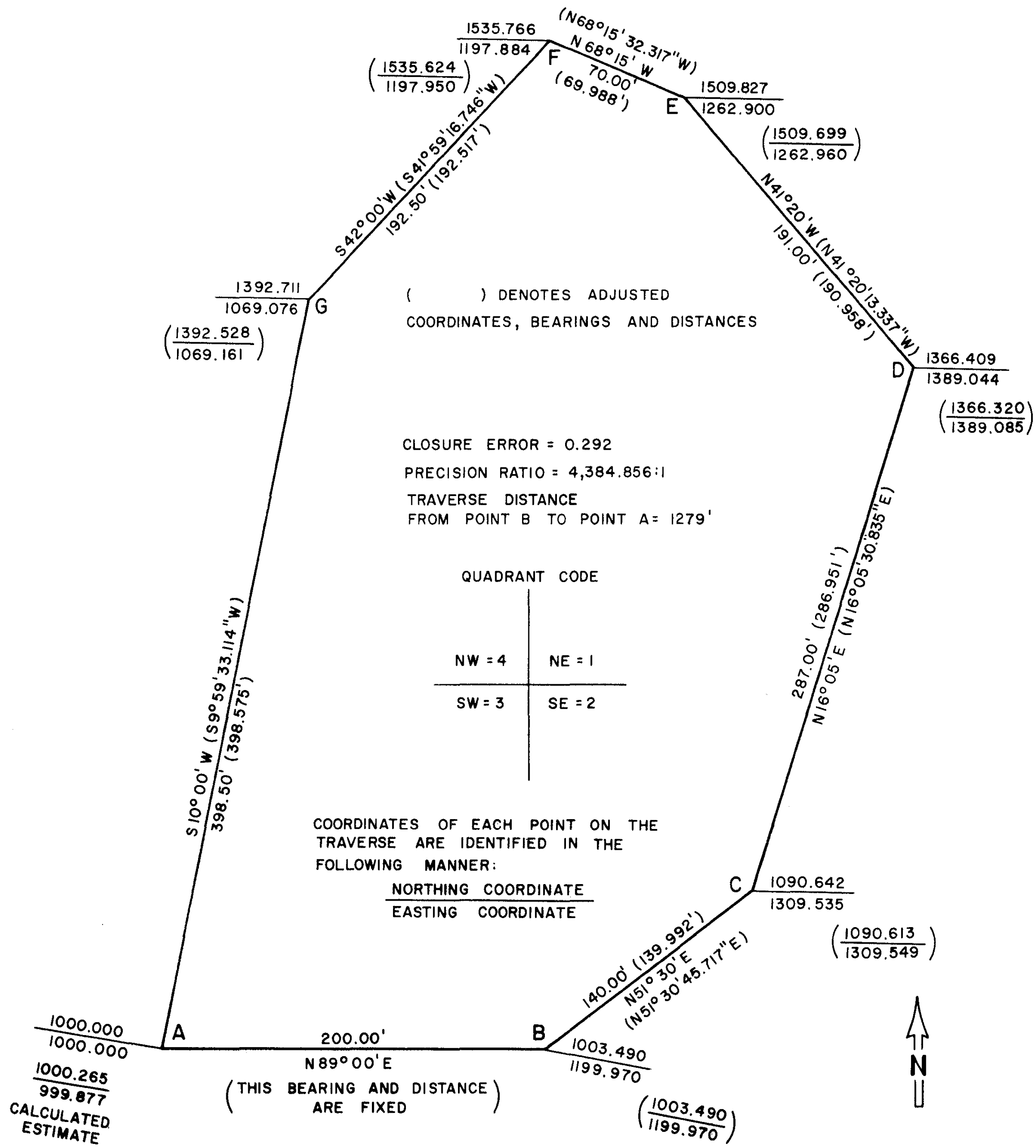
0	Z
Northing	Y
Easting	X

To continue with corrected traverse.

To enter new traverse data:

PRESS: END

EXAMPLE



POINT A IS FIXED

POINT B IS CALCULATED FROM FIXED BEARING AND DISTANCE
THIS POINT (B) IS THE STARTING POINT FOR TRAVERSE FROM B TO A IN A COUNTER-CLOCKWISE DIRECTION

Step	Key	Code	Display x	Display y	Display z
0 0 (+)	CLEAR	20			
1	x→( )	23			
2	−	34	ENTER		
3	f	15	SEC.	MIN.	DEG.
4	STOP	41	EA.	NO.	0
5	ROLL ↑	22	DISPLAY		
6	x→( )	23			
7	−	34			
8	e	12			
9	6	06			
a	0	00			
b	÷	35			
c	↓	25			
d	+	33			
1 0 (+)	6	06			
1	0	00			
2	÷	35			
3	x←( )	67			
4	−	34			
5	e	12			
6	+	33			
7	2	02			
8	↑	27	ENTER		
9	STOP	41	QUAD CODE	0	0
a	x⇄y	30			
b	IF x=y	50			
c	c	16			
d	1	01			
2 0 (+)	3	03			
1	IF x=y	50			
2	c	16			
3	0	00			
4	SET FLAG	54			
5	1	01			
6	IF x=y	50			
7	c	16			
8	0	00			
9	ROLL ↑	22			
a	×	36			
b	9	11			
c	0	00			
d	IF FLAG	43			
3 0 (+)	c	16			
1	5	05			
2	−	34			
3	CLEAR x	37			
4	y→( )	40			
5	−	34			
6	e	12			
7	↑	27			
8	ROLL ↑	22			
9	3	03	ENTER		
a	STOP	41	DIST.	0	0
b	IF x=y	50	DIST.	DIST.	0
c	SET FLAG	54			
d	CONT	47			
4 0 (+)	x⇄y	30			
1	x←( )	67			
2	−	34			
3	e	12			
4	x⇄y	30			
5	TO RECT	66			
6	ACC +	60			
7	TO POLAR	62			
8	IF FLAG	43			
9	5	05			
a	a	13			
b	y→( )	40			
c	b	14			
d	x⇄y	30			
5 0 (+)	x←( )	67			
1	−	34			
2	f	15			
3	+	33			
4	y→( )	40			
5	−	34			
6	f	15			
7	GO TO ( )	44			
8	a	13			
9	7	07			
a	x⇄y	30			
b	y→( )	40			
c	b	14			
d	−	34			
6 0 (+)	1	01			
1	8	10			
2	0	00			
3	CHG SIGN	32			
4	+	33			
5	IF x>y	53			
6	c	16			
7	a	13			
8	|y|	55			
9	y→( )	40			
a	−	34			
b	e	12			
c	÷	35			
d	π	56			
7 0 (+)	CHG SIGN	32			
1	×	36			
2	b	14			
3	×	36			
4	y⇄( )	24			
5	−	34			
6	f	15			
7	−	34			
8	x←( )	67			
9	−	34			
a	e	12			
b	↑	27			
c	2	02			
d	÷	35			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
8 0	x⇄y	30			
(+) 1	sin x	70			
2	×	36			
3	b	14			
4	×	36			
5	ROLL ↓	31			
6	+	33			
7	y⇄( )	24			
8	−	34			
9	f	15			
a	ROLL ↑	22	DISPLAY		
b	STOP	41	raduis	chord	arc length
c	x←( )	67			
d	−	34			
9 0	e	12			
(+) 1	↑	27			
2	int x	64			
3	−	34			
4	x→( )	23			
5	−	34			
6	e	12			
7	6	06			
8	0	00			
9	×	36			
a	x⇄y	30			
b	↑	27			
c	int x	64			
d	−	34			
a 0	ROLL ↓	31			
(+) 1	×	36			
2	x←( )	67			
3	−	34			
4	e	12			
5	ROLL ↓	31	DISPLAY		
6	STOP	41	SEC.	MIN.	DEG.
7	CLEAR x	37			
8	ROLL ↓	31			
9	RCL	61			
a	GO TO ( )	44			
b	0	00			
c	4	04			
d	CONT	47			

Step	Key	Code	Display x	Display y	Display z
b 0	CONT	47			
(+) 1	↓	↓			
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d	↓	↓			
c 0	CHG SIGN	32			
(+) 1	1	01			
2	GO TO ( )	44			
3	2	02			
4	9	11			
5	+	33			
6	+	33			
7	GO TO ( )	44			
8	3	03			
9	2	02			
a	−	34			
b	−	34			
c	GO TO ( )	44			
d	6	06			
d 0	8	10			
(+) 1	ACC −	63			
2	RCL	61			
3	CHG SIGN	32			
4	x→( )	23			
5	f	15			
6	1	01			
7	CHG SIGN	32			
8	×	36			
9	y→( )	40			
a	e	12			
b	f	15			
c	TO POLAR	62			
d	x⇄y	30			

Step	Key	Code	Display x	Display y	Display z
0 0	x←( )	67			
(−) 1	−	34			
2	f	15			
3	x⇄y	30			
4	÷	35			
5	↑	27			
6	x←( )	67			
7	−	34			
8	f	15	DISPLAY		
9	STOP	41	dist. trav.	error	prec. ratio
a	RCL	61			
b	↑	27			
c	x←( )	67			
d	−	34			
1 0	f	15			
(−) 1	÷	35			
2	ROLL ↑	22			
3	x⇄y	30			
4	÷	35			
5	ROLL ↓	31			
6	y→( )	40			
7	−	34			
8	f	15			
9	x→( )	23			
a	−	34			
b	e	12			
c	CLEAR	20			
d	1	01			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
20	STOP	41	EA.	NO.	0
21	ACC +	60		ENTER	
22	↓	25			
23	2	02		ENTER	
24	STOP	41	SEC.	MIN.	DEG.
25	ROLL ↑	22			
26	x→( )	23			
27	b	14			
28	6	06			
29	0	00			
2a	÷	35			
2b	↓	25			
2c	+	33			
2d	6	06			
30	0	00			
31	÷	35			
32	b	14			
33	+	33			
34	2	02			
35	↑	27		ENTER	
36	STOP	41	QUAD CODE	0	0
37	x⇄y	30			
38	IF x=y	50			
39	c	16			
3a	1	01			
3b	3	03			
3c	IF x=y	50			
3d	c	16			
40	0	00			
41	4	04			
42	SET FLAG	54			
43	1	01			
44	IF x=y	50			
45	c	16			
46	0	00			
47	ROLL ↑	22			
48	×	36			
49	9	11			
4a	0	00			
4b	IF FLAG	43			
4c	c	16			
4d	5	05			

Step	Key	Code	Display x	Display y	Display z
50	−	34			
51	CLEAR x	37			
52	y→( )	40			
53	b	14			
54	↑	27			
55	↑	27			
56	3	03		ENTER	
57	STOP	41	DIST.	0	0
58	↑	27			
59	ROLL ↑	22			
5a	x←( )	67			
5b	−	34			
5c	e	12			
5d	×	36			
60	x←( )	67			
61	−	34			
62	f	15			
63	ROLL ↑	22			
64	×	36			
65	y⇄( )	24			
66	b	14			
67	TO RECT	66			
68	ROLL ↓	31			
69	+	33			
6a	b	14			
6b	ROLL ↑	22			
6c	+	33			
6d	ROLL ↓	31			
70	ACC +	60			
71	TO POLAR	62			
72	x→( )	23			
73	b	14			
74	CLEAR x	37			
75	IF x<y	52			
76	8	10			
77	b	14			
78	|y|	55			
79	9	11			
7a	0	00			
7b	IF x>y	53			
7c	8	10			
7d	5	05			

Step	Key	Code	Display x	Display y	Display z
80	−	34			
81	3	03			
82	GO TO ( )	44			
83	9	11			
84	a	13			
85	x⇄y	30			
86	−	34			
87	2	02			
88	GO TO ( )	44			
89	9	11			
8a	a	13			
8b	9	11			
8c	0	00			
8d	IF x>y	53			
90	9	11			
91	7	07			
92	−	34			
93	4	04			
94	GO TO ( )	44			
95	9	11			
96	a	13			
97	x⇄y	30			
98	−	34			
99	1	01			
9a	ROLL ↑	22			
9b	CLEAR x	37			
9c	ROLL ↓	31			
9d	y⇄( )	24			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
a 0	b	14	DISPLAY		
(–) 1	STOP	41	QUAD CODE	DIST.	0
2	b	14			
3	↑	27			
4	int x	64			
5	—	34			
6	x→( )	23			
7	b	14			
8	6	06			
9	0	00			
a	×	36			
b	x⇄y	30			
c	↑	27			
d	int x	64			
b 0	—	34			
(–) 1	ROLL ↓	31			
2	×	36			
3	b	14			
4	ROLL ↓	31	DISPLAY		
5	STOP	41	SEC.	MIN.	DEG.
6	CLEAR x	37			
7	ROLL ↓	31			
8	RCL	61	DISPLAY		
9	STOP	41	EA.	NO.	0
a	↓	25			
b	GO TO ( )	44			
c	2	02			
d	3	03			
c 0	CHG SIGN	32			
(–) 1	1	01			
2	GO TO ( )	44			
3	4	04			
4	7	07			
5	+	33			
6	+	33			
7	GO TO ( )	44			
8	5	05			
9	0	00			
a	END	46			
b					
c					
d					

Storage

COORDINATE GEOMETRY AND ENCLOSED AREA

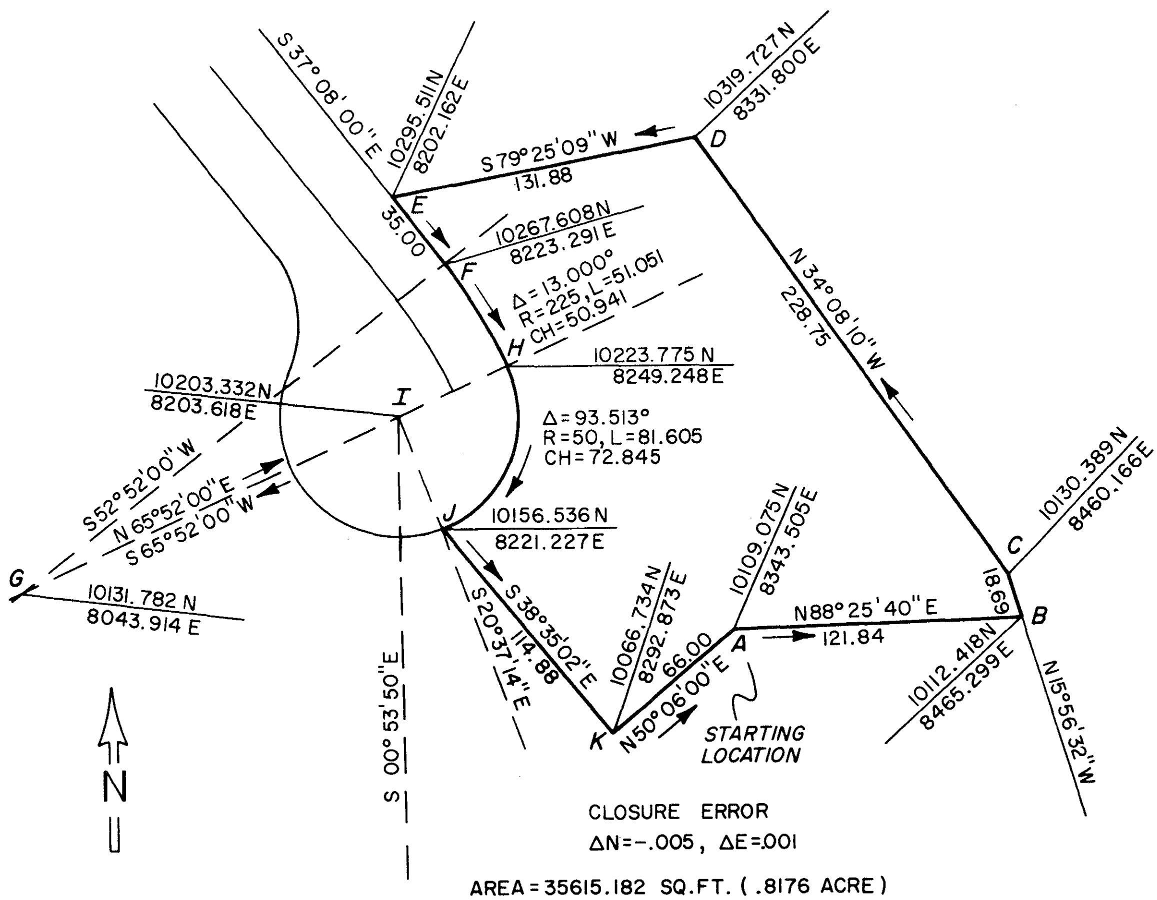
This program traverses by bearing and distance and calculates coordinates, enclosed area, and the following curve data:

1. Length
2. Chord
3. Central angle in decimal degrees
4. Coordinates of tangent points

This program is designed to traverse from point to point on a survey map indicating coordinates of successive points on the survey and also indicating the descriptive features of tangent curved portions. If direction of traverse is given in degrees and lengths in feet, then the area is given in square feet. The four quadrants are represented as follows: NE = 1, SE = 2, SW = 3, NW = 4.

QUADRANT CODE

4 = NW	1 = NE
3 = SW	2 = SE



SET: | DEGREES | |

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

ENTER COORDINATES:
Northing → Y
Easting → X

PRESS: CONTINUE

DISPLAY

1	Z
Northing	Y
Easting	X

ENTER BEARING ANGLE DATA:
Degrees → Z
Minutes → Y
Seconds → X

Note: Decimal Degrees, Decimal Minutes or Decimal Seconds may be entered provided they are entered in the proper register.

PRESS: CONTINUE

DISPLAY

Decimal Bearing	Z
2	Y
2	X

NO

ENTER DATA: Quadrant Code (1 = NE, 2 = SE, 3 = SW, 4 = NW) in X register

PRESS: CONTINUE

DISPLAY: Intermediate results

Inter. Res.	Z
Inter. Res.	Y
3	X

ENTER DATA: Distance → X (for normal traverse line)

Leaving a Center Point?

YES:

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

	— Z
(360° - α°) or α°	— Y
360	— X

Angle in Y correct?

NO:

PRESS: $x \rightleftarrows y$

PRESS: -

YES:

PRESS: CONTINUE

DISPLAY

arc length	— Z
chord length	— Y
radius of arc	— X

Note: 1. If progressing counterclockwise and arc is bowed inward with respect to the area then -- PRESS: CHG SIGN. Otherwise proceed.

2. If progressing clockwise and arc is bowed outward with respect to the area then -- PRESS: CHG SIGN. Otherwise proceed.

NO:

PRESS: CONTINUE

DISPLAY

1	— Z
North Coord.	— Y
East Coord.	— X

Path Closed?

NO: (return to ENTER DATA)

YES:

PRESS: RCL

DISPLAY

1	— Z
Δ N	— Y
Δ E	— X

Note: This display enables a closure error to be obtained between the initial coordinate point and that point as seen by the calculator after the path has been traversed.

PRESS: ∂ (∂ contains enclosed area)

DISPLAY

1	— Z
Δ N	— Y
Area	— X

To restart new problem, PRESS: END

EXAMPLES

STARTING COORDINATES = Coordinates of A

Coordinates of A:

10109.075 Northing
8343.505 Easting

NE 88° 25' 40"
Distance 121.84'

Coordinates of B:

10112.418 Northing *
8465.299 Easting *

NW 15° 56' 32"
Distance 18.69'

Coordinates of C:

10130.389 Northing *
8460.166 Easting *

NW 34° 08' 10"
Distance 228.75'

Coordinates of D:

10319.727 Northing *
8331.800 Easting *

SW 79° 25' 09"
Distance 131.88'

Coordinates of E:

10295.511 Northing *
8202.162 Easting *

SE 37° 08' 00"
Distance 35.00'

Coordinates of F:

10267.608 Northing *
8223.291 Easting *

START OF CURVE:

SW 52° 52' 00"
Distance 225.00'

Coordinates of G:

10131.782 Northing *
8043.914 Easting *

N. E. 65° 52' 00"
Distance 225.00'

Central Angle 13.000

51.051 --- Arc Length
50.941 --- Chord Length
225.000 --- Radius

EXAMPLES (con't)

Coordinates of H:

10223.775 Northing *
8249.248 Easting *

SW 65° 52' 00"
Distance 50.00'

Coordinates of I:

10203.332 Northing *
8203.618 Easting *

SE 20° 37' 14"
Distance 50.00'

Central angle 93.513

81.605 --- Arc Length
72.845 --- Chord Length
50.00 --- Radius

Coordinates of J:

10156.536 Northing *
8221.227 Easting *

SE 38° 35' 02"
Distance 114.88'

Coordinates of K:

10066.734 Northing *
8292.873 Easting *

NE 50° 06' 00"
Distance 66.00'

Coordinates of A:
10109.070 Northing*
8343.506 Easting *

CLOSURE ERROR

N = -.005

E = +.001

ENCLOSED AREA:

35,615.182 sq. feet
.8176 acre

*Coordinate Answers

Step	Key	Code	Display (x y z)
0 0	CLEAR	20	ENTER
(+) 1	STOP	41	E N O
2	x→( )	23	
3	d	17	
4	y→( )	40	
5	c	16	
6	CLEAR	20	
7	y→( )	40	
8	a	13	
9	RCL	61	
a	c	16	
b	+	33	
c	f	15	
d	↑	27	
1 0	d	17	
(+) 1	+	33	
2	1	01	ENTER
3	ROLL ↓	31	SEC MIN DEG
4	STOP	41	NO. EA. I
5	ROLL ↑	22	DISPLAY
6	x→( )	23	
7	b	14	
8	6	06	
9	0	00	
a	÷	35	
b	↓	25	
c	+	33	
d	6	06	
2 0	0	00	
(+) 1	÷	35	
2	b	14	
3	+	33	
4	2	02	
5	↑	27	ENTER
6	STOP	41	QUAD CODE
7	x⇄y	30	
8	IF x=y	50	
9	9	11	
a	6	06	
b	3	03	
c	IF x=y	50	
d	9	11	

Step	Key	Code	Display (x y z)
3 0	5	05	
(+) 1	SET FLAG	54	
2	1	01	
3	IF x=y	50	
4	9	11	
5	5	05	
6	ROLL ↑	22	
7	×	36	
8	9	11	
9	0	00	
a	IF FLAG	43	
b	9	11	
c	0	00	
d	−	34	
4 0	3	03	ENTER
(+) 1	STOP	41	Distance
2	IF FLAG	43	
3	5	05	
4	c	16	
5	y→( )	40	
6	−	34	
7	a	13	
8	TO RECT	66	
9	ACC +	60	
a	↑	27	
b	e	12	
c	×	36	
d	f	15	
5 0	ROLL ↑	22	
(+) 1	×	36	
2	↓	25	
3	−	34	
4	CHG SIGN	32	
5	2	02	
6	÷	35	
7	a	13	
8	+	33	
9	GO TO ( )	44	
a	0	00	
b	7	07	
c	x→( )	23	
d	b	14	

Step	Key	Code	Display (x y z)
6 0	↓	25	
(+) 1	↑	27	
2	y⇄( )	24	
3	−	34	
4	a	13	
5	−	34	
6	1	01	
7	8	10	
8	0	00	
9	+	33	
a	|y|	55	
b	↑	27	
c	+	33	
d	ROLL ↓	31	
7 0	IF x<y	52	
(+) 1	arc ▾	72	
2	−	34	DISPLAY
3	STOP	41	360 | α or 360−α | 180
4	↓	25	
5	↑	27	
6	ROLL ↑	22	
7	÷	35	
8	π	56	
9	ACC +	60	
a	ACC −	63	
b	×	36	
c	b	14	
d	×	36	

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
8 (+) 0	ROLL ↑	22			
1	$x \rightleftarrows y$	30			
2	2	02			
3	÷	35			
4	$x \rightleftarrows y$	30			
5	sin x	70			
6	×	36			
7	b	14			
8	×	36	DISPLAY		
9	STOP	41	arc length	chord length	radius of arc
a	GO TO ( )	44			
b	−	34			
c	0	00			
d	0	00			
9 (+) 0	+	33			
1	+	33			
2	GO TO ( )	44			
3	3	03			
4	d	17			
5	CHG SIGN	32			
6	1	01			
7	GO TO ( )	44			
8	3	03			
9	6	06			
a	CONT	47			
b	CONT	47			
c	CONT	47			
d	CONT	47			
0 (−) 0	ROLL ↑	22			
1	×	36			
2	2	02			
3	÷	35			
4	a	13			
5	+	33			
6	$y \to ( )$	40			
7	a	13			
8	$y \rightleftarrows ( )$	24			
9	−	34			
a	a	13			
b	b	14			
c	GO TO ( )	44			
d	+	33			

Step	Key	Code	Display x	Display y	Display z
1 (−) 0	4	04			
1	5	05			
2	END	46			

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

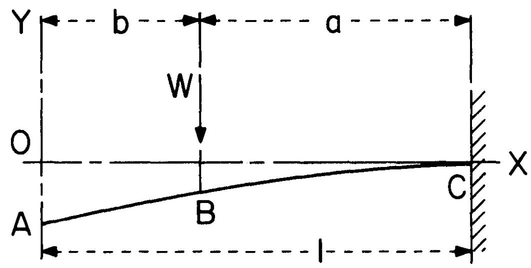
74203 - CANTILEVER BEAM - INTERMEDIATE LOAD
Calculates shear, moment, and deflection.

74204 - CANTILEVER BEAM - TRIANGULAR LOAD
Calculates shear, moment, and deflection.

74205 - COORDINATES OF EQUALLY SPACED POINTS ON A CIRCLE
Computes the rectangular coordinates of equally spaced points on a circle given the center point coordinates, radius and offset angle of the circle, and the number of coordinate points desired on the circle.

CANTILEVER BEAM
INTERMEDIATE LOAD

This program computes the shear, moment, and deflection reactions for a cantilever beam with a concentrated intermediate load as given by:



Reaction R_2 and Vertical Shear V	Bending Moment M and max bending moment	Deflection y, maximum deflection, and end slope θ
$R_2 = +W$	(A to B) $M = 0$	(A to B) $y = -\frac{1}{6}\frac{W}{EI}(-a^3 + 3a^2l - 3a^2x)$
(A to B) $V = 0$	(B to C) $M = -W(x-b)$	(B to C) $y = -\frac{1}{6}\frac{W}{EI}\left[(x-b)^3 - 3a^2(x-b) + 2a^3\right]$
(B to C) $V = -W$	Max $M = -Wa$ at C	Max $y = -\frac{1}{6}\frac{W}{EI}(3a^2l - a^3)$
		$\theta = +\frac{1}{2}\frac{Wa^2}{EI}$ (A to B)

NOTATION:

W = Load (lb)

M = Positive when clockwise

V = Positive when upward

Y = Positive when upward

θ = Radians = Tan θ

All forces are in pounds, all moments in inch-pounds, all deflections and dimensions in inches.

Reference: Formula for Stress and Strain
by R. J. Roark

McGraw - Hill Book Company 1965

USER INSTRUCTIONS

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	—	Z
0	—	Y
1	—	X

ENTER: l → Z, a → Y, b → X

PRESS: CONTINUE

DISPLAY

0	—	Z
0	—	Y
2	—	X

ENTER: E → Z, I → Y, W → X

PRESS: CONTINUE

DISPLAY

0	—	Z
0	—	Y
3	—	X

ENTER: x → X

PRESS: CONTINUE

DISPLAY

R_2	—	Z
V_{AB}	—	Y
V_{BC}	—	X

PRESS: CONTINUE

DISPLAY

M_{AB}	—	Z
M_{BC}	—	Y
M_{MAX}	—	X

PRESS: CONTINUE

DISPLAY

0	—	Z
Y_{AB}	—	Y
Y_{BC}	—	X

PRESS: CONTINUE

DISPLAY

0	—	Z
Y_{MAX}	—	Y
θ	—	X

EXAMPLES

LET

l	=	100
a	=	60
b	=	40
E	=	10^6
I	=	20
W	=	1000
x	=	24

THEN

R_2	=	1000
V_{AB}	=	0
V_{BC}	=	-1000
M_{AB}	=	0
M_{BC}	=	16000
M_{MAX}	=	-60000
Y_{AB}	=	-5.040
Y_{BC}	=	-5.006
Y_{MAX}	=	-7.200
θ	=	.090

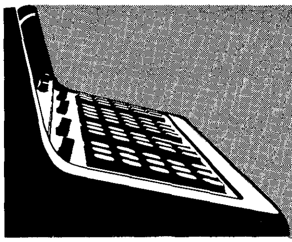
Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	STOP	41	b	a	l	ENTER l, a, b					
	3	ACC +	60									
	4	CLEAR x	37									
	5	ROLL ↓	31									
	6	$y \to ()$	40									
	7	d	17									
	8	↓	25									
	9	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
	a	STOP	41	W	I	E	ENTER E, I, W					
	b	$x \to ()$	23									
	c	c	16									
	d	ROLL ↓	31									
1	0	×	36									
	1	CLEAR x	37									
	2	ROLL ↓	31									
	3	÷	35									
	4	$y \to ()$	40									
	5	b	14									
	6	↓	25									
	7	3	03	DISPLAY 3 TO INDICATE THIRD ENTRY								
	8	STOP	41	X	O	O	ENTER X					
	9	$x \to ()$	23									
	a	a	13									
	b	c	16									
	c	↑	27	CALCULATE R_2, V								
	d	CLEAR x	37									
2	0	↑	27									
	1	c	16									
	2	CHG SIGN	32									
	3	STOP	41	V_{BC}	V_{AB}	R_2	DISPLAY					
	4	a	13									
	5	↑	27									
	6	f	15									
	7	−	34									
	8	c	16									
	9	CHG SIGN	32	CALCULATE M_{AB}, M_{BC}, M_{MAX}								
	a	×	36									
	b	↑	27									
	c	e	12									
	d	×	36									

 PART NO. 09100-74203

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	CLEAR x	37									
	1	ROLL ↓	31									
	2	STOP	41	M_{MAX}	M_{BC}	M	DISPLAY					
	3	e	12									
	4	↑	27									
	5	×	36									
	6	×	36									
	7	↑	27									
	8	×	36									
	9	3	03									
	a	×	36									
	b	d	17									
	c	×	36									
	d	↓	25									
4	0	x⇄y	30									
	1	−	34									
	2	e	12									
	3	↑	27									
	4	×	36									
	5	a	13									
	6	×	36									
	7	3	03									
	8	×	36									
	9	↓	25									
	a	−	34									
	b	b	14									
	c	×	36									
	d	6	06									
5	0	CHG SIGN	32	CALCULATE Y_{AB}, Y_{BC}								
	1	÷	35									
	2	y⇄( )	24									
	3	a	13									
	4	f	15									
	5	−	34									
	6	e	12									
	7	↑	27									
	8	×	36									
	9	3	03									
	a	CHG SIGN	32									
	b	×	36									
	c	↓	25									
	d	x⇄y	30									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	×	36									
	1	↑	27									
	2	×	36									
	3	×	36									
	4	↓	25									
	5	+	33									
	6	e	12									
	7	↑	27									
	8	×	36									
	9	×	36									
	a	2	02									
	b	×	36									
	c	↓	25									
	d	+	33									
7	0	b	14									
	1	×	36									
	2	6	06									
	3	CHG SIGN	32									
	4	÷	35									
	5	CLEAR x	37									
	6	ROLL ↓	31									
	7	y⇄( )	24									
	8	a	13									
	9	STOP	41	Y_{BC}	Y_{AB}	0	DISPLAY					
	a	e	12									
	b	↑	27									
	c	×	36									
	d	3	03									
8	0	×	36									
	1	d	17									
	2	×	36									
	3	e	12									
	4	↑	27	CALCULATE Y_{MAX}								
	5	×	36									
	6	×	36									
	7	↓	25									
	8	−	34									
	9	b	14									
	a	×	36									
	b	6	06									
	c	CHG SIGN	32									
	d	÷	35									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9 0	b	14	CALCULATE θ								
1	↑	27									
2	e	12									
3	×	36									
4	×	36									
5	2	02									
6	÷	35									
7	CLEAR x	37									
8	ROLL ↓	31									
9	END	46	θ	Y_{MAX}	0	DISPLAY					
a											
b											
c											
d											

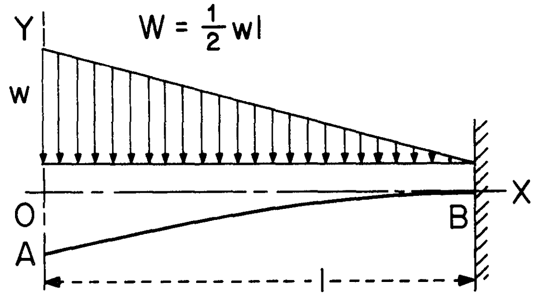


PART NO.
09100-74204

CANTILEVER BEAM
TRIANGULAR LOAD

This program computes the shear, moment, and deflection reactions for a cantilever beam with a triangular load as given by:

CANTILEVER, TRIANGULAR LOAD



Reaction R_2 and Vertical Shear V	Bending Moment M and Maximum Bending Moment	Deflection y, maximum deflection and end slope θ
$R_2 = +W$ $V = -W\left(\frac{2lx - x^2}{l^2}\right)$	$M = -\frac{1}{3}\frac{W}{l^2}(3lx^2 - x^3)$ Max $M = -\frac{2}{3}Wl$ at B	$y = -\frac{1}{60}\frac{W}{EIl^2}(-x^5 - 15l^4x + 5lx^4 + 11l^5)$ Max $y = -\frac{11}{60}\frac{Wl^3}{EI}$ at A $\theta = +\frac{1}{4}\frac{W}{EI}l^2$ at A

NOTATION:
- w = Unit load (lb per linear in.)
- W = Load (LB)
- M = Positive when clockwise
- V = Positive when upward
- Y = Positive when upward
- θ = Radians = Tan θ

All forces are in pounds, all moments in inch-pounds, all deflections and dimensions in inches.

Reference: Formula for Stress and Strain
by R. J. Roark

McGraw-Hill Book Company 1965

USER INSTRUCTIONS	EXAMPLES

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: W → Z, l → Y, x → X

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: E → Y, I → X

PRESS: CONTINUE

DISPLAY

0	——	Z
R_2	——	Y
V	——	X

PRESS: CONTINUE

DISPLAY

0	——	Z
M	——	Y
M_{MAX}	——	X

PRESS: CONTINUE

DISPLAY

Y	——	Z
Y_{MAX}	——	Y
θ	——	X

EXAMPLES

LET W = 1000

l = 100

x = 24

E = 10^6

I = 20

THEN R_2 = 1000

V = -422.4

M = -5299.2

M_{MAX} = -66666.6

Y = -6.180

Y_{MAX} = -9.167

θ = .125

 PART NO. 09100-74204

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	1	01	DISPLAY 1 TO INDICATE FIRST ENTRY								
	2	STOP	41	X	I	W	ENTER W, I, X					
	3	ACC +	60									
	4	CLEAR x	37									
	5	ROLL ↓	31									
	6	y→( )	40									
	7	d	17									
	8	↓	25									
	9	2	02	DISPLAY 2 TO INDICATE SECOND ENTRY								
	a	STOP	41	I	E	O	ENTER E, I					
	b	×	36									
	c	d	17									
	d	x⇄y	30									
1	0	÷	35									
	1	y→( )	40									
	2	c	16									
	3	e	12									
	4	↑	27									
	5	2	02									
	6	×	36									
	7	f	15									
	8	×	36									
	9	↑	27									
	a	×	36									
	b	↓	25									
	c	−	34									
	d	e	12									
2	0	↑	27	CALCULATE R_2, V								
	1	×	36									
	2	↓	25									
	3	÷	35									
	4	d	17									
	5	CHG SIGN	32									
	6	×	36									
	7	d	17									
	8	x⇄y	30									
	9	↑	27									
	a	CLEAR x	37									
	b	ROLL ↓	31									
	c	STOP	41	V	R_2	O	DISPLAY					
	d	e	12									

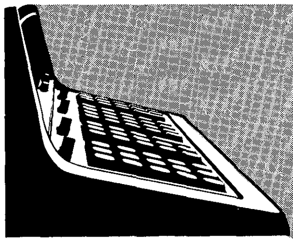
PAGE 2 PART NO. 09100-74204

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	↑	27									
	1	3	03									
	2	×	36									
	3	f	15									
	4	×	36									
	5	×	36									
	6	↑	27									
	7	×	36									
	8	×	36									
	9	↓	25									
	a	—	34									
	b	d	17									
	c	×	36									
	d	3	03									
4	0	÷	35	CALCULATE M, M_{MAX}								
	1	e	12									
	2	↑	27									
	3	×	36									
	4	↓	25									
	5	CHG SIGN	32									
	6	÷	35									
	7	d	17									
	8	↑	27									
	9	e	12									
	a	×	36									
	b	2	02									
	c	×	36									
	d	3	03									
5	0	CHG SIGN	32									
	1	÷	35									
	2	CLEAR x	37									
	3	ROLL ↓	31									
	4	STOP	41	M_{MAX}	M	O	DISPLAY					
	5	f	15									
	6	↑	27									
	7	×	36									
	8	×	36									
	9	×	36									
	a	×	36									
	b	↑	27									
	c	1	01									
	d	5	05									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	×	36									
	1	e	12									
	2	×	36									
	3	×	36									
	4	×	36									
	5	×	36									
	6	↓	25									
	7	+	33									
	8	y→( )	40									
	9	b	14									
	a	f	15									
	b	↑	27									
	c	×	36									
	d	×	36									
7	0	×	36									
	1	e	12	CALCULATE Y								
	2	×	36									
	3	5	05									
	4	×	36									
	5	e	12									
	6	↑	27									
	7	×	36									
	8	×	36									
	9	×	36									
	a	×	36									
	b	1	01									
	c	1	01									
	d	×	36									
8	0	↓	25									
	1	+	33									
	2	b	14									
	3	−	34									
	4	c	16									
	5	×	36									
	6	6	06									
	7	0	00									
	8	CHG SIGN	32									
	9	÷	35									
	a	e	12									
	b	÷	35									
	c	÷	35									
	d	y→( )	40									

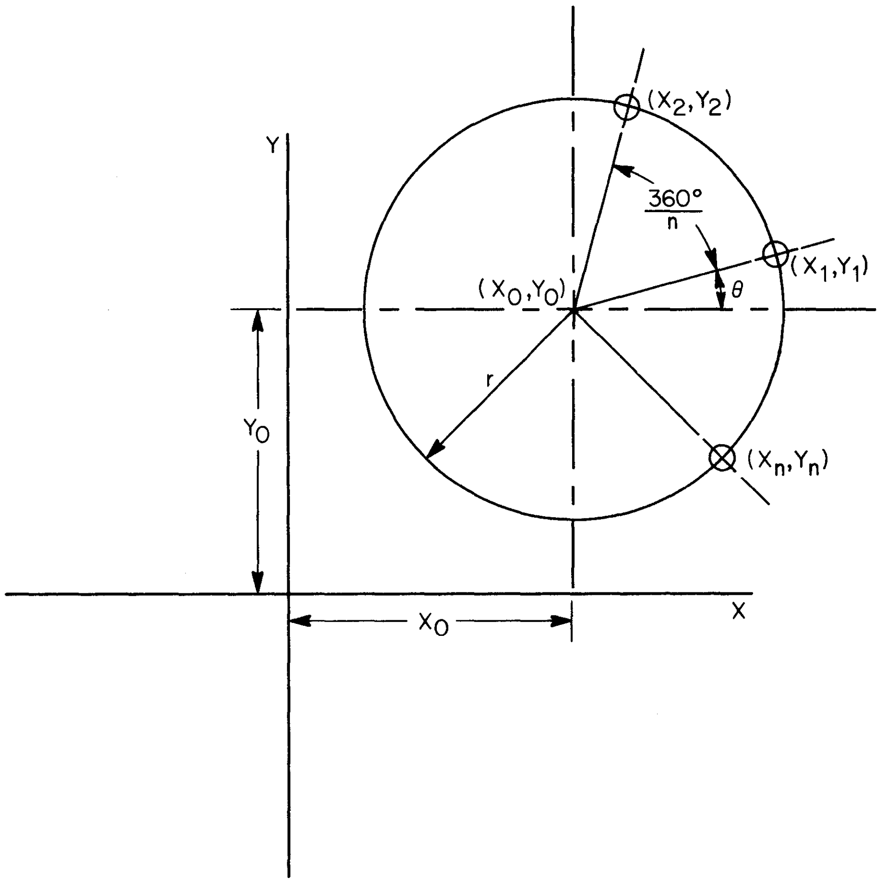
 PART NO. 09100-74204

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	b	14									
	1	c	16									
	2	↑	27									
	3	e	12									
	4	×	36									
	5	×	36									
	6	×	36									
	7	1	01	} CALCULATE Y_{MAX} (steps 9-1 to 9-d)								
	8	1	01									
	9	×	36									
	a	6	06									
	b	0	00									
	c	CHG SIGN	32									
	d	÷	35									
a	0	c	16									
	1	↑	27									
	2	e	12									
	3	×	36	} CALCULATE θ (steps a-0 to a-6)								
	4	×	36									
	5	4	04									
	6	÷	35									
	7	b	14									
	8	ROLL ↓	31									
	9	END	46	θ	Y_{MAX}	Y	DISPLAY					
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



COORDINATES OF EQUALLY SPACED POINTS ON A CIRCLE

This program computes the coordinates (X_i, Y_i) $i = 1, \cdots, n$ of equally spaced points on a circle of radius r. The user specifies the number of points n, and offset θ, and coordinates of the center of the circle X_o and Y_o. See the figure below.



The equations used are:

$$X_i = r \cos\left((i-1)\frac{360^o}{n} + \theta \right) + X_o \qquad i = 1, \cdots, n$$

$$Y_i = r \sin\left((i-1)\frac{360^o}{n} + \theta \right) + Y_o$$

θ - offset angle in degrees

USER INSTRUCTIONS

ENTER PROGRAM (Starting address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
1	——	X

ENTER DATA: $Y_o \rightarrow Y$, $X_o \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
2	——	X

ENTER DATA: $\theta \rightarrow Z$, $n \rightarrow Y$, $r \rightarrow X$

PRESS: CONTINUE

DISPLAY

i	——	Z
Y_i	——	Y
X_i	——	X

PRESS CONTINUE AFTER i REACHES n TO RESTART ANOTHER SOLUTION

EXAMPLES

X_o = 2 in.

Y_o = 1 in.

r = 2 in.

n = 10

$\theta = 0^o$

i	X_i	Y_i
1	3.618033989	2.175570505
2	2.618033989	2.902113033
3	1.381966011	2.902113033
4	0.381966011	2.175570505
5	0.000000000	1.000000000
6	0.381966011	- 0.175570505
7	1.381966011	- .902113033
8	2.618033989	- .902113033
9	3.618033989	- .175570505
10	4.000000000	1.000000000

Step		Key	Code	Display x	Display y	Display z	Storage	Remarks
0	0	CLEAR	20					← FROM 4-1
	1	1	01					DISPLAY 1 TO INDICATE FIRST ENTRY AND INITIALIZE COUNTER
	2	$x\to()$	23					
	3	9	11					
	4	STOP	41	X_0	Y_0	0	ENTER Y_0, X_0	
	5	$x\to()$	23					STORE DATA
	6	d	17					
	7	$y\to()$	40					
	8	c	16					
	9	CLEAR	20					
	a	2	02					DISPLAY 2 TO INDICATE SECOND ENTRY
	b	STOP	41	r	n	θ°	ENTER θ°, n, r	
	c	ACC +	60					STORE DATA
	d	ROLL ↑	22					
1	0	$x\to()$	23					
	1	b	14					
	2	3	03					CALCULATE $\Delta\theta$
	3	6	06					
	4	0	00					
	5	ROLL ↑	22					
	6	÷	35					
	7	$y\to()$	40					
	8	a	13					
	9	b	14					CALCULATE θ_1
	a	+	33					
	b	$y\to()$	40					STORE θ_i ← FROM 4-5
	c	b	14					
	d	1	01					CALCULATE X_i AND Y_i
2	0	TO RECT	66					
	1	↑	27					
	2	f	15					
	3	×	36					
	4	ROLL ↑	22					
	5	×	36					
	6	c	16					
	7	+	33					
	8	d	17					
	9	ROLL ↑	22					
	a	+	33					
	b	ROLL ↑	22					
	c	$y\rightleftarrows()$	24					
	d	9	11					

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3 0	ROLL ↑	22									
1	STOP	41	X	Y	i	DISPLAY					
2	1	01									
3	ROLL ↑	22									
4	+	33	INCREMENT i								
5	$y\to( )$	40									
6	9	11									
7	a	13									
8	↑	27	CALCULATE θ_{i+1}								
9	b	14									
a	+	33									
b	e	12									
c	ROLL ↑	22									
d	IF $x>y$	53	BRANCH FOR NEW DATA ENTRY WHEN $n>N$								
4 0	0	00									
1	0	00									
2	ROLL ↓	31									
3	GO TO ( )( )	44	POSITION θ_{i+1} AND BRANCH FOR								
4	1	01	NEXT COORDINATE CALCULATION								
5	b	14									
6	END	46									
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

September 1, 1969

9100B FLUID MECHANICS PROGRAM LISTING

75001 - CHEZY-MANNING EQUATION
Finds channel flow when channel is circular pipe and flowing full.

75003 - RECTANGULAR WEIR
Calculates the rate of fluid flow over a rectangular weir.

75004 - WATER FLOW IN PIPE
Calculates the loss factor (f) and the head loss for a pipe of specified dimensions with a known flow rate.

CHEZY - MANNING EQUATION

This program calculates the flow (Q) of an open channel.

The basic equation for open channel flow with any designated flow depth in a container (pipe, etc.) with any cross-sectional area is the Chezy-Manning equation given below:

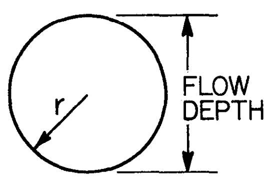
(1)

$$Q = \frac{1.49\ AR^{2/3}\ S^{1/2}}{n}$$

where Q = Flow in ft.^{3}/sec.
A = Cross-sectional area of container in ft.^{2}
R = Hydraulic radius in ft.
S = Slope of pipe in feet/ft.
n = Roughness coefficient (dimensionless)

This program is specialized for open-channel flow in a circular pipe with flow depth designated at full.

Situation:



Then R = hydralic radius $= \dfrac{\text{area of flow cross-section}}{\text{wetted perimeter}} = \dfrac{\pi r^2}{2\pi r} = \dfrac{r}{2} = \dfrac{D}{4}$ (D = diameter of container)

substituting in equation (1) gives:

$$Q = 1.49 \left(\frac{\pi D^2}{4}\right)\left(\frac{D}{4}\right)^{2/3} \frac{S^{1/2}}{n}$$

$$= \frac{(1.49)\ (\pi/4)}{(4)^{2/3}}\ \frac{D^{8/3}\ S^{1/2}}{n} = \frac{.46441\ D^{8/3}\ S^{1/2}}{n}$$ (equation to be used)

Variables to be entered for flow calculation are D, S, n.

Reference: Elementary Fluid Mechanics, 3rd. edition, Feb., 1959.
by John K. Vennard
John Wiley & Sons, Inc.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
0	——	X

ENTER DATA: n → Z, D → Y, S → X

PRESS: CONTINUE

DISPLAY

Q	——	Z
0	——	Y
0	——	X

EXAMPLE

Find Q_{FULL} in a circular pipe of diameter 18 in., if the slope is .001 feet/ft. and n = .015

Input: n = .015

D = 1.5 $\left(\frac{18''}{12''} = 1.5'\right)$

S = .001 ft./ft.

Answer: Q = 2.8866 $ft.^3$/sec.

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	STOP	41	S	D	n	ENTER n, D, S					
	2	$\sqrt{x}$	76									
	3	ROLL ↓	31									
	4	y→( )	40									
	5	f	15									
	6	ln x	65									
	7	x⇄y	30									
	8	8	10									
	9	×	36									
	a	3	03									
	b	÷	35									
	c	↓	25									
	d	e^x	74	CALCULATE Q								
1	0	×	36									
	1	f	15									
	2	÷	35									
	3	·	21									
	4	4	04									
	5	6	06									
	6	4	04									
	7	4	04									
	8	1	01									
	9	×	36									
	a	CLEAR x	37									
	b	↑	27									
	c	END	46	0	0	Q	DISPLAY					
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

(Steps 0,2 through 1,9: CALCULATE Q)

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

RECTANGULAR WEIR

This program calculates the rate of fluid flow over a rectangular weir.

The following equations are used:

$$Q = 2/3\ cb\sqrt{2g}\left[\left(H + \frac{V^2}{2g}\right)^{3/2} - \left(\frac{V^2}{2g}\right)^{3/2}\right]$$

where:
- Q = rate of fluid flow (ft.^{3}/sec.)
- b = length of weir crest (ft.)
- H = head on weir (ft.)
- V = average velocity of fluid approach (ft./sec.)
- c = correction factor

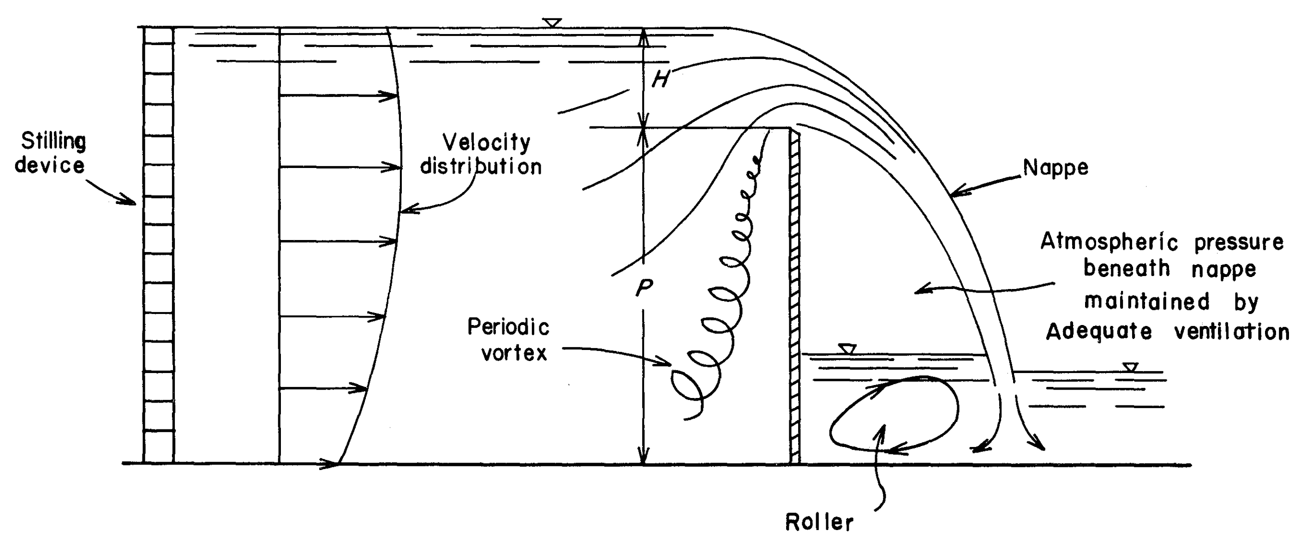
The coefficient (c) includes the effects of various phenomena, such as viscosity, turbulence, surface tension, non-uniform velocity, etc.

The program includes the ability to enter c as data, or to calculate c using Rehbock's equation, given below:

$$c = .605 + .08\ H/p + \frac{1}{305H}$$

where: p = the weir height

Diagram of a standard rectangular weir:



Reference: Elementary Fluid Mechanics - 3rd Edition
John K. Vennard
John Wiley & Sons, Inc.

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address is (0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: V → Z, b → Y, H → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

Wish to calculate c?

YES:

ENTER DATA: p → X

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	Z
2	Y
Q	X

(2 indicates that c was calculated)

NO:

ENTER DATA: c → X

PRESS: CONTINUE

DISPLAY

0	Z
1	Y
Q	X

(1 indicates that c was entered as data)

TO RESET PROBLEM: (return to PRESS: CONTINUE)

EXAMPLES

(A)

Data: H = 1 ft.
V = 1 ft./sec.
b = 3 ft.
c = .6

(c entered as data)

Solution: Q = 9.8321 ft.^{3}/sec.

(B)

Data: H = 2 ft.
V = 1 ft./sec.
b = 6 ft.
p = 6.85 ft.

(c to be calculated)

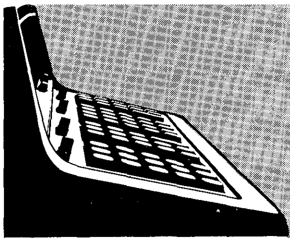
Solution: Q = 57.8007 ft.^{3}/sec.

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	1	01									
2	STOP	41	H	b	V	ENTER DATA					
3	ACC +	60									
4	x→( )	23	STORE b AND H								
5	c	16									
6	↓	25									
7	ROLL ↓	31									
8	×	36									
9	2	02									
a	↑	27									
b	3	03									
c	2	02									
d	·	21									
1 0	1	01									
1	7	07									
2	×	36									
3	↓	25									
4	÷	35									
5	√x	76									
6	↑	27									
7	2	02									
8	×	36									
9	3	03									
a	÷	35									
b	e	12	CALCULATE $Q_p =$								
c	×	36	$\frac{2}{3} b \sqrt{2g} \left[(H + \frac{V^2}{2g})^{3/2} - (\frac{V^2}{2g})^{3/2} \right]$								
d	y→( )	40									
2 0	e	12									
1	CLEAR x	37									
2	ROLL ↑	22									
3	ACC +	60									
4	↑	27									
5	√x	76									
6	×	36									
7	f	15									
8	↑	27									
9	√x	76									
a	×	36									
b	↓	25									
c	x⇄y	30									
d	−	34									

Step		Key	Code	Display (x, y, z)	Storage (f, e, d, c, b, a)
3	0	e	12		
	1	×	36		
	2	y→( )	40	STORE Q_P	
	3	d	17		
	4	CLEAR	20		
	5	2	02		
	6	STOP	41	C or P	ENTER DATA; C or P
	7	IF FLAG	43	BRANCH TO CALCULATE C IF P IS ENTERED AS DATA	
	8	4	04		
	9	6	06		
	a	↑	27	CALCULATE Q FOR DATA ENTRY OF C	
	b	d	17		
	c	×	36		
	d	CLEAR x	37		
4	0	↑	27		
	1	1	01		
	2	ROLL ↑	22		
	3	GO TO ( )( )	44	BRANCH TO DISPLAY Q	
	4	6	06		
	5	6	06		
	6	↑	27	CALCULATE C IF P IS ENTERED AS DATA	
	7	3	03		
	8	0	00		
	9	5	05		
	a	↑	27		
	b	c	16		
	c	×	36		
	d	ROLL ↑	22		
5	0	÷	35		
	1	.	21		
	2	0	00		
	3	8	10		
	4	×	36		
	5	1	01		
	6	ROLL ↑	22		
	7	÷	35		
	8	.	21		
	9	6	06		
	a	0	00		
	b	5	05		
	c	+	33		
	d	CLEAR x	37		

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	ROLL ↓	31									
	1	+	33									
	2	d	17	CALCULATE Q FOR DATA ENTRY OF P (C CALCULATED)								
	3	×	36									
	4	2	02									
	5	x⇄y	30									
FROM-4-5 →	6	END	46	Q	1 or 2		DISPLAY ; 1 or 2 INDICATES WHETHER C WAS ENTERED OR CALCULATED					
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



WATER FLOW IN PIPE

PART NO.
09100-75004

Given the roughness coefficient (ϵ), the diameter (d) of a pipe in inches, the rate of flow (Q) of water in gal./min. into the pipe, this program calculates the friction factor (f) and the head loss (h_f) in feet.

The equations used are the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.51}{R_e \sqrt{f}} + \frac{12\,\epsilon}{3.7\,d} \right)$$

used to calculate f, and the Darcy-Weisback equation

$$h_f = f \left(\frac{L}{d} \right) \left(\frac{6\,V^2}{g} \right)$$

where L = length of pipe in feet
and V = velocity in ft./sec.

R_e is Reynolds number where $R_e = \frac{Vd}{12\nu} = \frac{77\,Q}{720\pi\nu\,d}$

For water, $\nu = 1.40 \times 10^{-5}$ ft.^{2}/sec.

Therefore, $R_e = \frac{77 \times 10^5}{1008\,\pi} \times \frac{Q}{d}$

Reference: Marks Standard Handbook for Mechanical Engineers
Seventh Edition
McGraw-Hill Book Company
1967

USER INSTRUCTIONS	EXAMPLE

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: Roughness Coefficient →Y,
Diameter →X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: Length →Y, Rate of Flow →X

PRESS: CONTINUE

DISPLAY

d	Z
Q	Y
f	X

PRESS: CONTINUE

DISPLAY

d	Z
Q	Y
h_f	X

To enter new data (return to PRESS: CONTINUE)

Data:

d = 10 in.
ϵ = .00085
Q = 2000 gpm.
L = 500 ft.

Results:

f = .02035
h_f = 12.65377 ft.

Step	Key	Code	Display x	Display y	Display z	Comments
0 0	CLEAR	20				STORE 3, CLEAR AND DISPLAY 1 TO INDICATE FIRST ENTRY
1	3	03				
2	x→( )	23				
3	a	13				
4	1	01				
5	STOP	41	d	ϵ	0	ENTER ROUGHNESS COEFFICIENT AND DIAMETER
6	÷	35				CALCULATE AND STORE RELATIVE ROUGHNESS (ϵ/d)
7	x→( )	23				
8	b	14				
9	y→( )	40				
a	c	16				
b	CLEAR	20				CLEAR AND DISPLAY 2 TO INDICATE SECOND ENTRY
c	2	02				
d	STOP	41	Q	L	0	ENTER LENGTH AND RATE OF FLOW
1 0	ACC +	60				STORE Q AND L
1	↑	27				CALCULATE R_e
2	7	07				
3	7	07				
4	ENTER EXP	26				
5	6	06				
6	×	36				
7	1	01				
8	0	00				
9	0	00				
a	8	10				
b	÷	35				
c	π	56				
d	÷	35				
2 0	b	14				
1	÷	35				
2	y→( )	40				
3	d	17				
4 (FROM 5-4)	2	02				CALCULATE $\Delta = 2\left\lvert \text{LOG}\left(\frac{2.51}{R_e}\cdot\frac{1}{\sqrt{f}} + \frac{12}{3.7}\cdot\frac{\epsilon}{d}\right)\right\rvert - \frac{1}{\sqrt{f}}$
5	.	21				
6	5	05				
7	1	01				
8	↑	27				
9	d	17				
a	÷	35				
b	a	13				
c	×	36				
d	c	16				

Storage: f e d c b a

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
3 0	↑	27									
1	1	01									
2	2	02									
3	×	36									
4	3	03									
5	·	21									
6	7	07									
7	÷	35									
8	↓	25									
9	+	33									
a	↓	25									
b	log x	75									
c	↑	27									
d	|y|	55									
4 0	2	02									
1	×	36									
2	a	13									
3	−	34									
4	2	02	CALCULATE $\Delta/2$								
5	÷	35									
6	ENTER EXP	26	ENTER TOLERANCE, 10^{-3}								
7	3	03									
8	CHG SIGN	32									
9	IF $x > y$	53	BRANCH WHEN $\Delta/2$ < TOLERANCE								
a	5	05									
b	5	05									
c	a	13									
d	+	33	ADD $\left(\frac{1}{\sqrt{f}}\right)_i + \frac{\Delta_i}{2}$ AND STORE AS NEW ESTIMATE $\left(\frac{1}{\sqrt{f}}\right)_{i+1}$								
5 0	$y \rightarrow ()$	40									
1	a	13									
2	GO TO ( ) ( )	44	BRANCH FOR $\frac{\Delta_{i+1}}{2}$ CALCULATION								
3	2	02									
4	4	04									
5	1	01									
6	↑	27									
7	a	13									
8	÷	35									
9	↓	25	CALCULATE f								
a	↑	27									
b	×	36									
c	$y \rightarrow ()$	40									
d	a	13									

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
6 0	b	14	RECALL AND POSITION DISPLAY								
1	↑	27									
2	f	15									
3	ROLL ↑	22									
4	STOP	41	f	Q	d	DISPLAY f					
5	d	17	RECALL R_e								
6	↑	27									
7	1	01	CALCULATE $V^2 = \left(\frac{\nu + R_e \times 12}{d}\right)^2$								
8	4	04									
9	ENTER EXP	26									
a	5	05									
b	CHG SIGN	32									
c	×	36									
d	b	14									
7 0	÷	35									
1	1	01									
2	2	02									
3	×	36									
4	↓	25									
5	↑	27									
6	×	36									
7	b	14	CALCULATE $h_f = \frac{f L V^2}{d g}$								
8	÷	35									
9	6	06									
a	×	36									
b	3	03									
c	2	02									
d	.	21									
8 0	2	02									
1	÷	35									
2	e	12									
3	×	36									
4	a	13									
5	×	36									
6	b	14									
7	↑	27	RECALL AND POSITION DISPLAY								
8	f	15									
9	ROLL ↑	22									
a	END	46	h_f	Q	d	DISPLAY h_f					
b											
c											
d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

75201 - RADIOACTIVE DECAY

Calculates the mass loss between any two times given the half-life, initial mass and present mass; displays the decay curve; or calculates the age of the mass based upon the present mass.

PART NO.
09100-75201

RADIOACTIVE DECAY

Given a number of activity readings at various times from a Geiger-Mueller counter or similar radioactivity-measuring device, this program calculates the decay constant (λ), the half-life (τ), and, if desired, a set of M/M_o values versus time. M_o is the original mass of the sample and M is the mass remaining after a time t.

The equation for activity is:

$$1) \quad A = \frac{-dN}{dt} = \lambda N_o e^{-\lambda t}$$

Taking the natural logarithm of both sides of equation (1), the following equation results.

$$2) \quad \ln A = \ln \lambda N_o - \lambda t$$

Since equation (2) is a linear function of t, a linear regression (method of least squares) is performed on the set of ln A and t to obtain the slope ($-\lambda$) of the ln A versus t line.

The following equation is used to solve for λ (see Program 09100-70811).

$$\lambda = \frac{-\left[n \cdot \sum_{i=1}^{n} (\ln A_i)(t_i) - \left(\sum_{i=1}^{n} t_i\right)\left(\sum_{i=1}^{n} \ln A_i\right)\right]}{n \cdot \sum_{i=1}^{n} (t_i)^2 - \left(\sum t_i\right)^2}$$

Given λ, the half-life τ is calculated from the equation:

$$\tau = \ln 2 / \lambda$$

The fraction of mass remaining at time t is:

$$\frac{M}{M_o} = e^{-\lambda t}$$

The fraction of mass at any time t is easily obtained by entering the desired t in the Y register at the correct display during program execution (see "User Instructions"). M/M_o is calculated for the value of time appearing in the Y register <u>before</u> incrementing that time by the amount entered in the X register for the next calculation. A decay curve (M/M_o versus t) may also be obtained for any initial time and any increment (Δt).

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
i	——	X

ENTER DATA: t_i → Y, A_i → X

PRESS: CONTINUE

Has all data been entered? (NO: return to DISPLAY; YES: continue)

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	——	Z
τ	——	Y
λ	——	X

Wish to plot a decay curve? (NO: return to PRESS: GO TO (0) (0); YES: continue)

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
0	——	X

ENTER DATA: t(initial) → Y, Δt → X

YES

USER INSTRUCTIONS (con't)

PRESS: CONTINUE

DISPLAY:

t	——	Z
λ	——	Y
M/M_o	——	X

PRESS: CONTINUE

DISPLAY

0	——	Z
t + Δt	——	Y
Δt	——	X

YES

Wish to change t or Δt? (YES: return to ENTER DATA: t(initial) → Y, Δt → X; NO: return to PRESS: CONTINUE)

To reset problem:

EXAMPLES

A) Calculate λ, τ and the value of M/M_O at $t = \tau$ for following set of data. (M/M_O should be equal to .5 at $t = \tau$)

Data:

i	t_i (minutes)	A_i(counts/minute)
1	.1	5.6×10^7
2	.3	5.01×10^7
3	.5	4.44×10^7
4	1	3.29×10^7
5	10	1.48×10^5

Solution: τ = 1.15491

λ = .60018

M/M_o (for t = 1.15491) = .50000

B) Calculate λ, τ and a set of M/M_o values for t (initial) = 0, Δt = .2, using the following data.

Data:

i	t_i (minutes)	A_i (counts/min.)
1	1	4.01×10^5
2	1.5	3.80×10^5
3	4	2.61×10^4
4	4.5	2.11×10^4

Solution: τ = .75422
λ = .91903

Δ t	t	M/M_o
.2	0	1.00000
.2	.2	.83210
.2	.4	.69239
.2	.6	.57613
.2	.8	.47940
.2	1.0	.39891

Step		Key	Code	Display: x	y	z	Storage: f	e	d	c	b	a
0	0	CLEAR	20	CLEAR STORAGE AND DISPLAY REGISTERS								
	1	$x\to()$	23									
	2	c	16									
	3	$x\to()$	23									
	4	b	14									
	5	1	01	INITIALIZE COUNTER								
	6	$x\to()$	23									
	7	a	13									
	8	STOP	41	A_i	t_i	0	ENTER DATA					
	9	IF FLAG	43									
	a	2	02									
	b	9	11									
	c	ln x	65	CALCULATE $\Sigma \ln (A_i)$ AND Σt_i								
	d	ACC +	60									
1	0	$x \rightleftarrows y$	30	CALCULATE $\Sigma (t_i)^2$								
	1	×	36									
	2	↑	27									
	3	×	36									
	4	b	14									
	5	+	33									
	6	$y\to()$	40									
	7	b	14									
	8	c	16	CALCULATE $\Sigma t_i \cdot \ln (A_i)$								
	9	ROLL ↑	22									
	a	+	33									
	b	$y\to()$	40									
	c	c	16									
	d	CLEAR x	37									
2	0	↑	27									
	1	a	13	INCREMENT COUNTER								
	2	↑	27									
	3	1	01									
	4	+	33									
	5	↓	25									
	6	GO TO ( )( )	44	BRANCH TO ENTER NEXT DATA SET								
	7	0	00									
	8	6	06									
	9	a	13	DECREMENT n								
	a	↑	27									
	b	1	01									
	c	−	34									
	d	$y \rightleftarrows ()$	24									

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	c	16									
1	c	16									
2	×	36									
3	↑	27									
4	RCL	61									
5	x⇄y	30									
6	×	36									
7	ROLL ↓	31									
8	−	34									
9	y⇄( )	24									
a	b	14	CALCULATE λ (steps 30–47)								
b	c	16									
c	×	36									
d	↓	25									
4 0	ROLL ↓	31									
1	×	36									
2	↓	25									
3	−	34									
4	b	14									
5	x⇄y	30									
6	÷	35									
7	|y|	55									
8	2	02									
9	ln x	65	CALCULATE τ (steps 48–4b)								
a	x⇄y	30									
b	÷	35									
c	↑	27									
d	CLEAR x	37									
5 0	y→( )	40	STORE λ (steps 50–51)								
1	a	13									
2	ROLL ↓	31									
3	STOP	41	λ	τ	0	DISPLAY					
4	CLEAR	20									
5	STOP	41	Δt	t(INITIAL)	0	ENTER DATA: t(INITIAL)→Y, Δt→X					
6	↑	27									
7	↓	25									
8	x→( )	23									
9	c	16									
a	a	13	CALCULATE M/M_0 (steps 56–5d)								
b	CHG SIGN	32									
c	×	36									
d	↓	25									

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6 0	e^x	74									
1	↑	27	RECALL AND POSITION λ AND t								
2	a	13									
3	x⇄y	30									
4	STOP	41	M/M_0	λ	t	DISPLAY					
5	CLEAR x	37	INCREMENT t								
6	ROLL ↓	31									
7	c	16									
8	+	33									
9	GO TO ( )( )	44	BRANCH TO DISPLAY t AND Δt FOR NEXT M/M_0 CALCULATION.								
a	5	05									
b	5	05									
c	END	46									
d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

9100B CHEMICAL PROGRAM LISTING

75502 - ELEMENTAL PERCENTAGE AND MOLECULAR WEIGHT - 6 ELEMENT
Calculates percentages and molecular weight of compounds containing 6 elements or less.

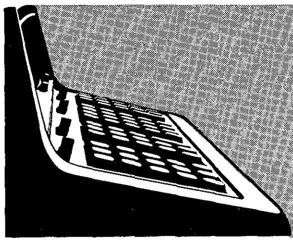
75503 - CHN ANALYSIS [K VALUES]
Calculates K values given C, H and N, blank values, and percentages for a known standard.

75504 - CHN PERCENTAGES
Given C, H, and N values and using previously calculated K values and known blank values, calculates C, H and N percentages.

75505 - MOLECULAR WEIGHT BY VPO
Calculates molecular weight for an unknown based on a series of vapor pressure osmometer (VPO) readings at various dilutions by extrapolating least squares curve fit to infinite dilution.

75506 - MEMBRANE OSMOMETER
Determines the number-average molecular weight by extrapolating a least squares curve fit to infinite dilution.

HEWLETT·PACKARD
HEWLETT·PACKARD
HEWLETT·PACKARD
HEWLETT·PACKARD



PART NO.
09100-75502

ELEMENTAL PERCENTAGE AND MOLECULAR WEIGHT
6 ELEMENT

This program calculates relative elemental percentages of up to and including 6 elements. The atomic weight of the elements are programmed into the calculator. The total molecular weight of the compound is also calculated.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

N_1
- PRESS: GO TO (0) (4)
- SET: | PROGRAM | |
- Enter program steps for atomic weight of N_1; maximum of 6 steps is allowed.
- SET: | | RUN |

N_2
- PRESS: GO TO (1) (4)
- SET: | PROGRAM | |
- Enter program steps for atomic weight of N_2 as before.
- SET: | | RUN |

N_3
- PRESS: GO TO (2) (4)
- SET: | PROGRAM | |
- Enter atomic weight of N_3 as before.
- SET: | | RUN |

N_4
- PRESS: GO TO (3) (4)
- SET: | PROGRAM | |
- Enter atomic weight of N_4 as before.
- SET: | | RUN |

N_5
- PRESS: GO TO (4) (4)
- SET: | PROGRAM | |
- Enter atomic weight of N_5 as before.
- SET: | | RUN |

N_6
- PRESS: GO TO (5) (4)
- SET: | PROGRAM | |
- Enter atomic weight of N_6 as before.
- SET: | | RUN |

PRESS: GO TO (0) (0) [or End]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
i	X

(i indicates number of element to be entered)

ENTER DATA: $N_i \rightarrow X$ (N_i = the number of atoms of each element.)

USER INSTRUCTIONS con't

i = 6? NO → (return to PRESS: CONTINUE above) YES ↓

PRESS: CONTINUE

DISPLAY

M.W.	Z
0	Y
0	X

PRESS: CONTINUE

DISPLAY

0	Z
$\%N_i$	Y
i	X

PRESS: CONTINUE

i = 6? NO → (return to DISPLAY above) YES → (return to PRESS: CONTINUE at start)

Note: To reset problem with different elements, repeat user instructions; otherwise, enter new number of atoms of each element when initial display appears after completion of last problem.

EXAMPLE

Elements:	N_1(C)	N_2(H)	N_3(S)	N_4(N)	N_5(O)	N_6(Si)
At. Wt.:	12.01	1.008	32.07	14.01	16.00	28.09
# of Atoms	6	2	8	1	2	3

Solution:

% C = 15.63%

% H = .44%

% S = 55.66%

% N = 3.04%

% O = 6.94%

%Si = 18.28%

Molecular Weight = 460.92

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a	Notes
0 0	CLEAR	20										
1	1	01										
2	STOP	41	N_1	0	0							ENTER N_1
3	↑	27										
4	CONTINUE	47										AREA FOR INSERTING ATOMIC WEIGHT OF N_1 (steps 4–9)
5	↓	↓										
6	↓	↓										
7	↓	↓										
8	↓	↓										
9	CONTINUE	47										
a	×	36										CALCULATE PARTIAL MOLECULAR WEIGHT (steps a–b)
b	ACC +	60										
c	y→( )	40										STORE X_1 (steps c–d)
d	a	13										
1 0	↓	25										
1	2	02										
2	STOP	41	N_2	0	0							ENTER N_2
3	↑	27										
4	CONTINUE	47										AREA FOR INSERTING ATOMIC WEIGHT OF N_2 (steps 4–9)
5	↓	↓										
6	↓	↓										
7	↓	↓										
8	↓	↓										
9	CONTINUE	47										
a	×	36										CALCULATE PARTIAL MOLECULAR WEIGHT (steps a–b)
b	ACC +	60										
c	y→( )	40										STORE X_2 (steps c–d)
d	b	14										
2 0	↓	25										
1	3	03										
2	STOP	41	N_3	0	0							ENTER N_3
3	↑	27										
4	CONTINUE	47										AREA FOR INSERTING ATOMIC WEIGHT OF N_3 (steps 4–9)
5	↓	↓										
6	↓	↓										
7	↓	↓										
8	↓	↓										
9	CONTINUE	47										
a	×	36										CALCULATE PARTIAL MOLECULAR WEIGHT (steps a–b)
b	ACC +	60										
c	y→( )	40										STORE X_3 (steps c–d)
d	c	16										

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	↓	25									
	1	4	04									
	2	STOP	41	N_4	0	0	ENTER N_4					
	3	↑	27									
	4	CONTINUE	47									
	5											
	6			AREA FOR INSERTING								
	7			ATOMIC WEIGHT OF N_4								
	8											
	9	CONTINUE	47									
	a	×	36	CALCULATE PARTIAL MOLECULAR WEIGHT								
	b	ACC +	60									
	c	y→( )	40	STORE X_4								
	d	d	17									
4	0	↓	25									
	1	5	05									
	2	STOP	41	N_5	0	0	ENTER N_5					
	3	↑	27									
	4	CONTINUE	47									
	5											
	6			AREA FOR INSERTING								
	7			ATOMIC WEIGHT OF N_5								
	8											
	9	CONTINUE	47									
	a	×	36	CALCULATE PARTIAL MOLECULAR WEIGHT								
	b	ACC +	60									
	c	y→( )	40	STORE X_5								
	d	f	15									
5	0	↓	25									
	1	6	06									
	2	STOP	41	N_6	0	0	ENTER N_6					
	3	↑	27									
	4	CONTINUE	47									
	5											
	6			AREA FOR INSERTING								
	7			ATOMIC WEIGHT OF N_6								
	8											
	9	CONTINUE	47									
	a	×	36									
	b	e	12									
	c	x⇄y	30	CALCULATE AND STORE								
	d	+	33	TOTAL MOLECULAR WEIGHT								

Step		Key	Code	Display x	y	z	Storage f	e	d	c	b	a
6	0	$x \rightleftarrows y$	30									
	1	$x \rightarrow ()$	23									
	2	e	12									
	3	↑	27									
	4	ENTER EXP	26									
	5	2	02	CALCULATE MOLECULAR WEIGHT								
	6	÷	35									
	7	↓	25	CALCULATE $\%N_6$								
	8	÷	35									
	9	$y \rightleftarrows ()$	24									
	a	a	13	CALCULATE $\%N_1$								
	b	÷	35									
	c	$y \rightleftarrows ()$	24									
	d	b	14	CALCULATE $\%N_2$								
7	0	÷	35									
	1	$y \rightleftarrows ()$	24									
	2	c	16	CALCULATE $\%N_3$								
	3	÷	35									
	4	$y \rightleftarrows ()$	24									
	5	d	17	CALCULATE $\%N_4$								
	6	÷	35									
	7	$y \rightleftarrows ()$	24									
	8	f	15	CALCULATE $\%N_5$								
	9	÷	35									
	a	CLEAR x	37									
	b	$y \rightleftarrows ()$	24									
	c	e	12	RECALL AND POSITION MOLECULAR WEIGHT								
	d	↑	27									
8	0	STOP	41	0	0	M.W.	DISPLAY MOLECULAR WEIGHT					
	1	b	14									
	2	↑	27									
	3	1	01									
	4	STOP	41	1	$\%N_1$	0	DISPLAY $\%N_1$					
	5	$y \rightleftarrows ()$	24									
	6	c	16									
	7	2	02									
	8	STOP	41	2	$\%N_2$	0	DISPLAY $\%N_2$					
	9	$y \rightleftarrows ()$	24									
	a	d	17									
	b	3	03									
	c	STOP	41	3	$\%N_3$	0	DISPLAY $\%N_3$					
	d	$y \rightleftarrows ()$	24									

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
9	0	f	15									
	1	4	04									
	2	STOP	41	4	%N_4	0	DISPLAY		% N_4			
	3	y⇄( )	24									
	4	e	12									
	5	5	05									
	6	STOP	41	5	%N_5	0	DISPLAY		%N_5			
	7	y⇄( )	24									
	8	a	13									
	9	6	06									
	a	END	46	6	%N_6	0	DISPLAY		%N_6			
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

CHN ANALYSIS - K VALUES

This program calculates the K values (K_C, K_H, and K_N), for compounds containing carbon (C), hydrogen (H), and nitrogen (N), from blank values (B_C, B_H, and B_N), percentages of C, H, and N from a known standard ($\%C_s$, $\%H_s$, and $\%N_s$), and measured quantities of C, H, and N of the known standard (C_{s_i}, H_{s_i}, and N_{s_i}). Two sets of measurements of C-H-N quantities of the standard are normally taken, although one set of measurements is sufficient for operation of the program.

The following equations are used:

$$K_C = \frac{\%C_s}{\overline{C}_s - B_C}$$

$$K_H = \frac{\%H_s}{\overline{H}_s - B_H}$$

$$K_N = \frac{\%N_s}{\overline{N}_s - B_N}$$

where $\overline{M}_s \left[M = C, H, \text{or } N \right]$ is the average of M_{s_1} and M_{s_2}.

If only one set of C-H-N quantities of the standard are used, then:

$$\overline{M}_s \left[M = C, H, \text{or } N \right] = M_{s_1}$$

$$K_C = \frac{\%C_s}{C_1 - B_C} \quad \text{, etc.}$$

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (3) (1)

SET: [PROGRAM |]

ENTER: Program steps for the constant - $\%N_s$ (five spaces available).

SET: [| RUN]

PRESS: GO TO (4) (3)

SET: [PROGRAM |]

ENTER: Program steps for the constant - $\%C_s$ (five spaces available).

SET: [| RUN]

PRESS: GO TO (5) (3)

SET: [PROGRAM |]

ENTER: Program steps for the constant - $\%H_s$ (five spaces available).

SET: [| RUN]

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
0 ——	X

ENTER DATA: $B_N \rightarrow Z$, $B_H \rightarrow Y$, $B_C \rightarrow X$

PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
1 ——	X

ENTER DATA: $N_{s_1} \rightarrow Z$, $H_{s_1} \rightarrow Y$, $C_{s_1} \rightarrow X$

PRESS: CONTINUE

DISPLAY

0 ——	Z
0 ——	Y
2 ——	X

ENTER DATA: $N_{s_2} \rightarrow Z$, $H_{s_2} \rightarrow Y$, $C_{s_2} \rightarrow X$

USER INSTRUCTIONS (con't)

PRESS: CONTINUE

DISPLAY

K_N ——	Z
K_H ——	Y
K_C ——	X

Note: If only one set of standard values is to be used, N_{s_2}, H_{s_2}, and C_{s_2} must be set to zero. Do not press "clear" at any time during program operation.

To reset program for a different standard, repeat user instructions.

EXAMPLES

(A) Only 1 set of standard values is available

Given: $\%C_s = 51.79$, $\%H_s = 5.07$, $\%N_s = 20.14$

$B_C = 1.6$, $B_H = 30$, $B_N = 1.0$

$C_{s_1} = 125.0$, $H_{s_1} = 119.5$, $N_{s_1} = 88.0$

Note: Since only 1 set of standards is available, the 2nd set of standards must be set to zero at the appropriate data input of the program. Do not press "CLEAR" to zero the 2nd set of standards.

Solution: $K_C = .420$, $K_H = .057$, $K_N = .231$

(B) Given: $\%C_s = 51.79$, $\%H_s = 5.07$, $\%N_s = 20.14$

$B_C = 1.6$, $B_H = 30$, $B_N = 1.0$

$C_{s_1} = 121.0$, $H_{s_1} = 121.5$, $N_{s_1} = 79.5$

$C_{s_2} = 120.5$, $H_{s_2} = 115.5$, $N_{s_2} = 79.5$

Solution: $K_C = .435$, $K_H = .057$, $K_N = .257$

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20									
	1	$x \to (\ )$	23	CLEAR DISPLAY AND STORAGE REGISTERS								
	2	d	17									
	3	STOP	41	B_C	B_H	B_N	ENTER B_C, B_H AND B_N					
	4	$x \to (\ )$	23									
	5	a	13									
	6	$y \to (\ )$	40									
	7	b	14	STORE B_C, B_H AND B_N								
	8	ROLL ↓	31									
	9	$y \to (\ )$	40									
	a	c	16									
	b	CLEAR	20									
	c	1	01									
	d	STOP	41	C_{S_i}	H_{S_i}	N_{S_i}	ENTER C_{S_i}'s H_{S_i}'s AND N_{S_i}'s					
1	0	ACC +	60									
	1	CLEAR x	37									
	2	ROLL ↓	31									
	3	d	17	FORM ΣC_i, ΣH_i AND ΣN_i AND STORE								
	4	+	33									
	5	$y \to (\ )$	40									
	6	d	17									
	7	IF FLAG	43									
	8	2	02	BRANCH IF 2nd SET OF C, H AND N HAS BEEN ENTERED								
	9	2	02									
	a	SET FLAG	54	SET FLAG AFTER 1st SET OF C, H AND N HAS BEEN ENTERED								
	b	↓	25									
	c	2	02									
	d	GO TO ( ) ( )	44									
2	0	0	00	BRANCH TO ENTER 2nd SET OF C, H AND N								
	1	d	17									
	2	IF $x = y$	50									
	3	2	02	SET COUNTER TO SEE IF 2nd SET OF C, H AND N ARE ZERO								
	4	9	11									
	5	2	02									
	6	GO TO ( ) (	44	SET COUNTER TO TWO IF 2nd SET OF C, H AND N ARE NOW ZERO.								
	7	2	02									
	8	a	13									
	9	1	01									
	a	÷	35									
	b	$x \rightleftarrows y$	30									
	c	↑	27									
	d	c	16									

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
3 0	−	34									
1	CONTINUE	47									
2	CONTINUE	47									
3	·	21									
4	CONTINUE	47	CALCULATE AND STORE K_N								
5	CONTINUE	47	*STEPS 3-1 THROUGH 3-5 ARE*								
6	$x \rightleftarrows y$	30	*THE BLANK SPACES FOR*								
7	÷	35	*ENTERING % N_s*								
8	$y \to ( )$	40									
9	d	17									
a	f	15									
b	ROLL ↑	22									
c	÷	35									
d	$x \rightleftarrows y$	30									
4 0	↑	27									
1	a	13									
2	−	34									
3	CONTINUE	47									
4	CONTINUE	47	CALCULATE AND STORE K_C								
5	·	21	*STEPS 4-3 THROUGH 4-7 ARE*								
6	CONTINUE	47	*THE BLANK SPACES FOR*								
7	CONTINUE	47	*ENTERING % C_s*								
8	$x \rightleftarrows y$	30									
9	÷	35									
a	$y \to ( )$	40									
b	f	15									
c	e	12									
d	ROLL ↑	22									
5 0	÷	35									
1	b	14									
2	−	34									
3	CONTINUE	47									
4	CONTINUE	47									
5	·	21	CALCULATE AND STORE K_H								
6	CONTINUE	47	*STEPS 5-3 THROUGH 5-7 ARE*								
7	CONTINUE	47	*THE BLANK SPACES FOR*								
8	$x \rightleftarrows y$	30	*ENTERING % H_s*								
9	÷	35									
a	$y \to ( )$	40									
b	e	12									
c	d	17	RECALL AND POSITION K_C, K_H								
d	$x \rightleftarrows y$	30	AND K_N FOR DISPLAY								

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
6	0	ROLL ↑	22									
	1	END	46	K_C	K_H	K_N	DISPLAY					
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											

PART NO.
09100-75504

CHN PERCENTAGES

Percentages of carbon (C), hydrogen (H), and nitrogen (N) in an unknown substance are calculated from measured quantities of C, H, and N, blank values (B_C, B_H, and B_N), and K values (K_C, K_H, K_N) which may be calculated with the K value program (09100-75503). K and B values must be stored prior to program operation either manually by the operator, or automatically by the use of the K value program.

The following equations are used:

$$\% C_1 = K_C (C_1 - B_C)$$

$$\% C_2 = K_C (C_2 - B_C)$$

or, in general:

$$\% M_1 \left[M = C, H, \text{ or } N \right] = K_M (M_1 - B_M)$$

$$\% M_2 \left[M = C, H, \text{ or } N \right] = K_M (M_2 - B_M)$$

The percentage evaluation is considered successful if M_1 and M_2 differ by .3% or less.

If only one set of CHN measurements are available, then C_2, H_2, and N_2 may be made any value at the corresponding data input. In this case only % C_1, % H_1, and % N_1 are useful information; disregard answers for % C_2, % H_2, and % N_2.

USER INSTRUCTIONS

ENTER PROGRAM [(Starting Address is 0 - 0)]

Have K and B values been stored by use of the K-Values program?

YES → go to PRESS: GO TO (0) (0)

NO ↓

ENTER DATA: $K_N \rightarrow Z$, $K_H \rightarrow Y$, $K_C \rightarrow X$

PRESS:
x →
f
y →
e
R↓
y →
d

ENTER DATA: $B_N \rightarrow Z$, $B_H \rightarrow Y$, $B_C \rightarrow X$

PRESS:
x →
a
y →
b
R↓
y →
c

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
1	———	X

ENTER DATA: $N_1 \rightarrow Z$, $H_1 \rightarrow Y$, $C_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	———	Z
0	———	Y
2	———	X

ENTER DATA: $N_2 \rightarrow Z$, $H_2 \rightarrow Y$, $C_2 \rightarrow X$

USER INSTRUCTIONS (con't)

Note: If only one C-H-N set is available, N_2, H_2, and C_2 may be set to zero or any other value--Simply disregard the solutions for % C_2, % H_2, and % N_2.

PRESS: CONTINUE

DISPLAY

% C_2	——	Z
% C_1	——	Y
1	——	X

PRESS: CONTINUE

DISPLAY

% H_2	——	Z
% H_1	——	Y
2	——	X

PRESS: CONTINUE

DISPLAY

% N_2	——	Z
% N_1	——	Y
3	——	X

To reset problem for different C, H, and N samples. (return to PRESS: GO TO (0) (0))

To reset program for different sets of K and B, repeat user instructions.

EXAMPLES

(A)

Example (A) of the K-value program (09100-75503) has previously been run, and the K and B values have already been stored.

Data: $C_1 = 118.3$, $H_1 = 75.0$, $N_1 = 49.1$
$C_2 = 128.6$, $H_2 = 69.7$, $N_2 = 53.4$

Solution: $\%C_1 = 48.98$, $\%C_2 = 53.30$
$\%H_1 = 2.55$, $\%H_2 = 2.25$
$\%N_1 = 11.13$, $\%N_2 = 12.13$

(B)

The K-value program (09100-75503) has not been run; K and B values must therefore be stored manually. (see "user instructions").

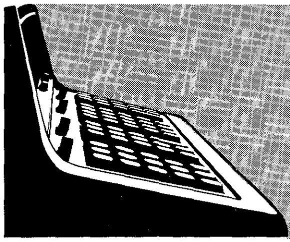
Data: $B_C = 2.1$, $B_H = 28$, $B_N = 3$
$K_C = .520$, $K_H = .102$, $K_N = .235$
$C_1 = 132.1$, $H_1 = 92.0$, $N_1 = 57.9$

Solution: $\%C_1 = 67.6$
$\%H_1 = 6.53$
$\%N_1 = 12.90$

Note: Disregard answers for $\% C_2$, $\%H_2$, and $\%N_2$ since a second set of C-H-N was not available.

Step	Key	Code	Display x	Display y	Display z	Storage (f e d c b a) / Remarks
0 0	CLEAR x	37				
1	↑	27				
2	↑	27				
3	1	01				
4	STOP	41	C_1	H_1	N_1	ENTER 1st SET OF C, H, & N
5	x→( )	23				
6	9	11				
7	y→( )	40				
8	8	10	STORE C_1, H_1 AND N_1			
9	ROLL ↓	31				
a	y→( )	40				
b	7	07				
c	CLEAR x	37				
d	↑	27				
1 0	↑	27				
1	2	02				
2	STOP	41	C_2	H_2	N_2	ENTER 2nd SET OF C, H, & N
3	x→( )	23	STORE C_2			
4	6	06				
5	b	14				
6	—	34				
7	e	12				
8	×	36				
9	y⇄( )	24				
a	8	10	CALCULATE AND STORE % H_2 AND % H_1			
b	b	14				
c	—	34				
d	e	12				
2 0	×	36				
1	y⇄( )	24				
2	9	11				
3	a	13				
4	—	34				
5	f	15				
6	×	36				
7	y⇄( )	24				
8	6	06	CALCULATE AND STORE % C_1 AND % C_2			
9	a	13				
a	—	34				
b	f	15				
c	×	36				
d	y⇄( )	24				

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3 0	7	07									
1	c	16									
2	−	34									
3	d	17									
4	×	36									
5	c	16									
6	x⇄y	30	CALCULATE AND STORE % N_1 AND % N_2								
7	ROLL ↓	31									
8	−	34									
9	d	17									
a	×	36									
b	y⇄( )	24									
c	6	06									
d	↓	25									
4 0	y⇄( )	24	RECALL AND POSITION % C_1 AND % C_2 FOR DISPLAY.								
1	7	07									
2	↑	27									
3	1	01									
4	STOP	41	1	% C_1	% C_2						
5	y⇄( )	24									
6	8	10									
7	↑	27	RECALL AND POSITION % H_1 AND % H_2 FOR DISPLAY.								
8	y⇄( )	24									
9	9	11									
a	2	02									
b	STOP	41	2	% H_1	% H_2						
c	y⇄( )	24									
d	6	06									
5 0	↑	27	RECALL AND POSITION % N_1 AND % N_2 FOR DISPLAY.								
1	y⇄( )	24									
2	7	07									
3	3	03									
4	END	46	3	% N_1	% N_2						
5											
6											
7											
8											
9											
a											
b											
c											
d											



PART NO.
09100-75505

MOLECULAR WEIGHT BY VPO

Data obtained from the vapor pressure osmometer (VPO) is used to calculate either the depression constant (K_d) for a substance with standard molecular weight, or the molecular weight (MW) of an unknown substance. In normal application, VPO readings (V) for various concentrations (C) of a substance with known molecular weight are taken, and K_d is determined. The molecular weight of an unknown substance may then be determined from VPO readings obtained from various concentrations of the unknown, and from the previously determined K_d.

The same procedure is used to calculate either K_d or MW. In both cases, a linear regression is performed with V/C and C as the dependent and independent variables respectively. The regression is similar to that of the regression program (09100-70803) for equations of the form Y = mx + b; in this case the defining equation is V/C = mc + b, where m is the slope of the V/C versus C curve. The V/C versus C curve is then extrapolated to infinite dilution (C = 0) to obtain the Y = V/C intercept (b); i.e. $\lim_{c \to 0} V/C = b$.

K_d is then calculated using the equation:

$$K_d = b(MW)$$ where MW, in this case, is the molecular weight of a known substance.

Likewise, MW may be calculated by the equation:

$$MW = \frac{K_d}{b}$$ where now MW is the molecular weight (of an unknown substance) which is to be determined.

A correlation coefficient (r) is also calculated. r is a measure of reproducibility and also how ideal the system is.

USER INSTRUCTIONS

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
i	X

ENTER DATA: $V_i \rightarrow Y$, $C_i \rightarrow X$

PRESS: CONTINUE

Has all data been entered? NO (return to DISPLAY) / YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	Z
1	Y
0	X

Wish to Calculate MW? NO / YES

NO: [Wish to Calculate K_d]

ENTER DATA: $MW \rightarrow X$

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

m	Z
r	Y
K_d	X

YES:

ENTER DATA: $K_d \rightarrow X$

PRESS: CONTINUE

DISPLAY

m	Z
r	Y
MW	X

To reset problem (return to PRESS: CONTINUE)

EXAMPLES

(A) Calculate K_d for the following standard molecular weight, and set of VPO and C readings.

Data: MW = 1500

i	V_i	C_i
1	500	34
2	330	24
3	205	16
4	125	8

Solution: m = -.017
r = -.156
K_d = 21858.377

(B) Calculate MW for the following data:

Data: K_d = 337.576

i	V_i	C_i
1	56.10	121.95
2	23.85	57.32
3	11.45	27.85
4	5.68	13.72

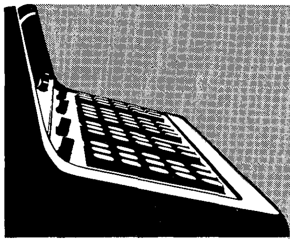
Solution: m = .0005
r = .9419
MW = 843.5813

Step		Key	Code	Display x	Display y	Display z	Storage / Remarks
0	0	CLEAR	20				
	1	x→( )	23				
	2	d	17				
	3	x→( )	23				CLEAR DISPLAY AND STORAGE REGISTERS.
	4	c	16				
	5	x→( )	23				
	6	b	14				
	7	1	01				
	8	x→( )	23				INITIALIZE COUNTER (FROM 3-4)
	9	a	13				
	a	STOP	41	C_i	V_i	0	ENTER DATA
	b	IF FLAG	43				
	c	3	03				BRANCH IF ALL DATA HAS BEEN ENTERED
	d	5	05				
1	0	÷	35				CALCULATE $\Sigma X = \Sigma C$ AND $\Sigma Y = \Sigma V$
	1	ACC +	60				
	2	↑	27				
	3	×	36				
	4	x⇄y	30				
	5	y⇄( )	24				CALCULATE ΣX^2
	6	d	17				
	7	+	33				
	8	y⇄( )	24				
	9	d	17				
	a	↓	25				
	b	×	36				
	c	b	14				CALCULATE ΣXY
	d	+	33				
2	0	y→( )	40				
	1	b	14				
	2	ROLL ↑	22				
	3	↑	27				
	4	×	36				
	5	c	16				CALCULATE ΣY^2
	6	+	33				
	7	y→( )	40				
	8	c	16				
	9	a	13				
	a	↑	27				INCREMENT COUNTER
	b	1	01				
	c	+	33				
	d	CLEAR x	37				

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	↑	27									
	1	ROLL ↑	22									
	2	GO TO ( )( )	44									
	3	0	00	BRANCH TO ENTER NEXT SET OF DATA								
	4	8	10									
	5	a	13	FROM-0-d								
	6	↑	27	DECREMENT COUNTER								
	7	1	01									
	8	−	34									
	9	e	12									
	a	$x \rightleftarrows y$	30	CALCULATE $\bar{Y}$								
	b	÷	35									
	c	$y \rightleftarrows ()$	24									
	d	f	15	CALCULATE $\bar{X}$								
4	0	÷	35									
	1	$x \rightarrow ()$	23									
	2	a	13	STORE $\bar{X}$ AND n								
	3	$y \rightarrow ()$	40									
	4	e	12									
	5	×	36									
	6	e	12									
	7	×	36	CALCULATE $\Sigma(X-\bar{X})^2$								
	8	d	17									
	9	$x \rightleftarrows y$	30									
	a	−	34									
	b	$y \rightleftarrows ()$	24									
	c	c	16									
	d	f	15									
5	0	↑	27									
	1	×	36	CALCULATE $\Sigma(Y-\bar{Y})^2$								
	2	a	13									
	3	×	36									
	4	↓	25									
	5	−	34									
	6	$y \rightleftarrows ()$	24									
	7	b	14									
	8	↑	27									
	9	RCL	61									
	a	×	36									
	b	a	13									
	c	×	36									
	d	↓	25									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	−	34	CALCULATE AND STORE r								
	1	c	16									
	2	↑	27									
	3	↓	25									
	4	√x	76									
	5	÷	35									
	6	b	14									
	7	√x	76									
	8	÷	35									
	9	y→( )	40									
	a	a	13									
	b	c	16	CALCULATE AND STORE m								
	c	ROLL ↑	22									
	d	x⇄y	30									
7	0	÷	35									
	1	y→( )	40									
	2	b	14									
	3	e	12	CALCULATE AND STORE b								
	4	×	36									
	5	f	15									
	6	x⇄y	30									
	7	−	34									
	8	y→( )	40									
	9	c	16									
	a	CLEAR	20									
	b	1	01									
	c	x⇄y	30									
	d	STOP	41	MW or K_d	0	0	ENTER DATA −MW or K_d					
8	0	↑	27									
	1	c	16	RECALL b								
	2	IF FLAG	43	CALCULATE K_d IF FLAG IS SET, OTHERWISE CALCULATE MW								
	3	×	36									
	4	×	36									
	5	÷	35									
	6	b	14	RECALL m AND r AND POSITION FOR DISPLAY.								
	7	↑	27									
	8	a	13									
	9	ROLL ↑	22									
	a	END	46	MW or K_d	r	m	DISPLAY					
	b											
	c											
	d											

Step	Key	Code	Display			Storage					
			x	y	z	f	e	d	c	b	a
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
a											
b											
c											
d											



MEMBRANE OSMOMETER

The membrane osmometer is used to obtain data for calculation of either the constant RT (R = gas constant, T = absolute temperature), or the number average molecular weight ($\overline{M}_n$). In normal application, osmotic pressure (P) measurements are taken for various concentrations (C) of a substance with known $\overline{M}_n$, and RT is subsequently calculated. The number average molecular weight of an unknown substance may then be determined from membrane osmometer measurements of P obtained from various concentrations of the unknown, and from the previously determined RT.

The same procedure is used to calculate either RT or $\overline{M}_n$. In both cases, a linear regression is performed with $(P/C)^{1/2}$ and C as the dependent and independent variables respectively. The regression is similar to that of the regression program (09100-70803) for equations of the form y = mx + b; in this case the defining equation is $P/C = mc + \frac{RT}{\overline{M}_n}$, where m is the slope of the (P/C) versus C curve. The $(P/C)^{1/2}$ versus C curve is then extrapolated to infinite dilution (c=0) to obtain the Y = P/C intercept (b), i.e. $\lim_{c \to 0} P/C = b$. $(P/C)^{1/2}$ rather than P/C is used in the regression to avoid extrapolation error when the P/C and C data results in a P/C versus C plot with pronounced curvature. In many cases where the first power plot has pronounced curvature, the square-root plot will produce a straight line suitable for extrapolation. The desired intercept (b) of the P/C versus C curve is then the square of the $(P/C)^{1/2}$ versus C intercept.

RT is calculated using the equations:

$RT = \overline{M}_n\ (b)$ where $\overline{M}_n$ is a known number average molecular weight.

Likewise, $\overline{M}_n$ may be calculated by the equation:

$\overline{M}_n = \frac{RT}{b}$ where now $\overline{M}_n$ is the number average molecular weight (of an unknown substance) which is to be determine.

A correlation coefficient (r) is also calculated. r is a measure of reproducibility and also how ideal the system is.

USER INSTRUCTION

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: END

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
i	X

ENTER DATA: P_i → Y, C_i → X

PRESS: CONTINUE

Has all data been entered? NO (return to DISPLAY) / YES

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	Z
1	Y
0	X

Wish to calculate $\overline{M}_n$?

YES:

ENTER DATA: RT → X

PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

m	Z
r	Y
$\overline{M}_n$	X

[Wish to calculate RT]

ENTER DATA: $\overline{M}_n$ → X

PRESS: CONTINUE

DISPLAY

m	Z
r	Y
RT	X

To reset problem: (return to PRESS: CONTINUE)

EXAMPLES

(A) Determine RT for a substance with known number average molecular weight ($\overline{M}_n$) using the following data:

Data: $\overline{M}_n = 1.2 \times 10^5$

i	P_i	C_i
1	.58	2
2	1.75	5
3	4.45	10
4	12.64	20

Solution: m = .014
r = .998
RT = 32,185.017

(B) Calculate $\overline{M}_n$ for an unknown substance using the following data:

Data: $RT = 3.65 \times 10^4$

i	P_i	C_i
1	.67	2.2
2	1.50	4.3
3	3.72	8.4
4	10.41	18.6

Solution: m = .012
r = .977
$\overline{M}_n$ = 124,403.599

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20	CLEAR DISPLAY AND STORAGE REGISTERS.								
1	x→( )	23									
2	d	17									
3	x→( )	23									
4	c	16									
5	x→( )	23									
6	b	14									
7	1	01	INITIALIZE COUNTER								
8 (FROM 3-7)	x→( )	23									
9	a	13									
a	STOP	41	C	P	O	ENTER DATA					
b	IF FLAG	43	BRANCH IF ALL DATA HAS BEEN ENTERED								
c	3	03									
d	8	10									
1 0	÷	35	CALCULATE $(P/C)^{1/2}$								
1	ROLL ↓	31									
2	$\sqrt{x}$	76									
3	ROLL ↑	22									
4	ACC +	60	CALCULATE $\Sigma C=\Sigma X$ AND $\Sigma(P/C)^{1/2}=\Sigma Y$								
5	↑	27	CALCULATE ΣX^2								
6	×	36									
7	x⇄y	30									
8	y⇄( )	24									
9	d	17									
a	+	33									
b	y⇄( )	24									
c	d	17									
d	↓	25	CALCULATE ΣXY								
2 0	×	36									
1	b	14									
2	+	33									
3	y→( )	40									
4	b	14									
5	ROLL ↑	22	CALCULATE ΣY^2								
6	↑	27									
7	×	36									
8	c	16									
9	+	33									
a	y→( )	40									
b	c	16									
c	a	13	INCREMENT COUNTER								
d	↑	27									

Step		Key	Code	Comment
3	0	1	01	
	1	+	33	
	2	CLEAR x	37	
	3	↑	27	
	4	ROLL ↑	22	
	5	GO TO ( )( )	44	BRANCH TO ENTER NEXT SET OF DATA.
	6	0	00	
	7	8	10	
	8	a	13	FROM-0-d; DECREMENT AND STORE COUNTER.
	9	↑	27	
	a	1	01	
	b	−	34	
	c	y→( )	40	
	d	a	13	
4	0	e	12	CALCULATE $\bar{Y}$
	1	x⇄y	30	
	2	÷	35	
	3	y⇄( )	24	CALCULATE AND STORE $\bar{X}$
	4	f	15	
	5	÷	35	
	6	y→( )	40	
	7	e	12	
	8	×	36	CALCULATE AND STORE $\Sigma(X-\bar{X})^2$.
	9	e	12	
	a	×	36	
	b	d	17	
	c	x⇄y	30	
	d	−	34	
5	0	y⇄( )	24	
	1	c	16	
	2	f	15	CALCULATE AND STORE $\Sigma(Y-\bar{Y})^2$.
	3	↑	27	
	4	×	36	
	5	a	13	
	6	×	36	
	7	↓	25	
	8	−	34	
	9	y⇄( )	24	
	a	b	14	
	b	↑	27	
	c	RCL	61	
	d	×	36	

Display: x y z — Storage: f e d c b a

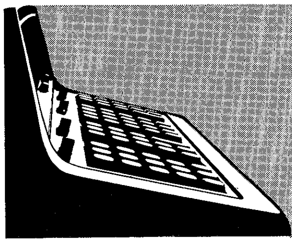
Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
6	0	a	13	CALCULATE AND STORE r.								
	1	×	36									
	2	↓	25									
	3	−	34									
	4	c	16									
	5	↑	27									
	6	↓	25									
	7	$\sqrt{x}$	76									
	8	÷	35									
	9	b	14									
	a	$\sqrt{x}$	76									
	b	÷	35									
	c	y→( )	40									
	d	a	13									
7	0	c	16	CALCULATE AND STORE m.								
	1	ROLL ↑	22									
	2	x⇄y	30									
	3	÷	35									
	4	y→( )	40									
	5	b	14									
	6	e	12	CALCULATE AND STORE C.								
	7	×	36									
	8	f	15									
	9	x⇄y	30									
	a	−	34									
	b	y→( )	40									
	c	c	16									
	d	CLEAR	20									
8	0	1	01									
	1	x⇄y	30									
	2	STOP	41	$\bar{M}_n$ or RT	0	0	ENTER DATA: $\bar{M}_n$ or RT					
	3	↑	27	RECALL b								
	4	c	16									
	5	IF FLAG	43	BRANCH IF $\bar{M}_n$ IS TO BE CALCULATED.								
	6	8	10									
	7	d	17									
	8	×	36	CALCULATE RT								
	9	×	36									
	a	GO TO ( )( )	44	BRANCH TO RECALL r AND m.								
	b	9	11									
	c	1	01									
	d	÷	35	CALCULATE $\bar{M}_n$								

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
9	0	÷	35									
	1	b	14	RECALL r AND m AND POSITION FOR DISPLAY								
	2	↑	27									
	3	a	13									
	4	ROLL ↑	22									
	5	END	46	RT or $\bar{M}_n$	r	m	DISPLAY					
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

75802 - PRIME NUMBERS
Calculates all prime numbers between any two numbers.

9100B ONLY

75901 - CONIC SECTION DETERMINATION WITH PLOT
Given the generating angle of a cone, β, and the intersection angle with a plane, α, this program determines and plots the conic section.



PART NO.
09100-75802

PRIME NUMBER PROGRAM

This program displays all prime numbers between specified upper and lower limits and the total number of prime numbers found between the limits.

A prime number n is an integer, not a unit (± 1), that is divisible by only ± n and ± 1.

Reference:

Elementary Theory of Numbers
Harriet Griffin
1954

USER INSTRUCTIONS	EXAMPLE

ENTER PROGRAM: (Starting Address is 0 - 0)

→ PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
0	X

ENTER DATA: Upper limit → X

PRESS: CONTINUE

DISPLAY

0	Z
UL	Y
0	X

ENTER DATA: Lower Limit → X

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
Prime Number	X

Has Upper Limit been reached?

NO → (return to PRESS: CONTINUE)

YES ↓

PRESS: CONTINUE

DISPLAY

Upper Limit	Z
Lower Limit	Y
Number of Primes	X

To enter new limits: (return to PRESS: GO TO (0) (0))

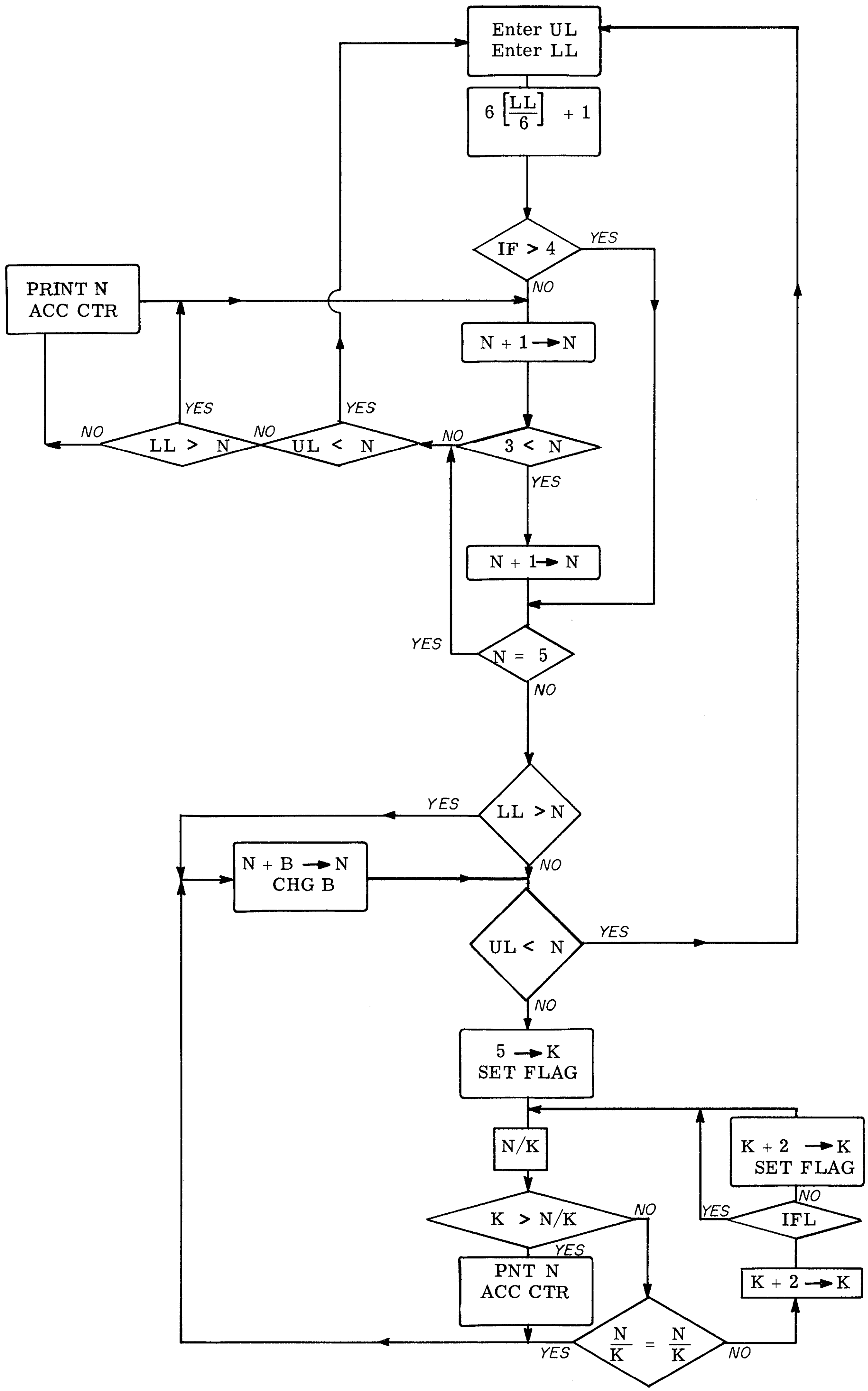
EXAMPLE

Upper Limit = 100

Lower Limit = 0

Primes = 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Number of Primes = 26


Enter UL
Enter LL
6 [LL/6] + 1
IF > 4
YES
NO
PRINT N
ACC CTR
N + 1 → N
YES
YES
NO
LL > N
NO
UL < N
NO
3 < N
YES
N + 1 → N
YES
N = 5
NO
YES
LL > N
NO
N + B → N
CHG B
UL < N
YES
NO
5 → K
SET FLAG
N/K
K + 2 → K
SET FLAG
K > N/K
NO
NO
YES
IFL
YES
PNT N
ACC CTR
K + 2 → K
YES
N/K = N/K
NO

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	STOP	41	U.L.	0	0	ENTER UPPER LIMIT					
2	y→( )	40									
3	a	13									
4	x⇄y	30									
5	STOP	41	L.L.	0	0	ENTER LOWER LIMIT					
6	x→( )	23									
7	d	17	STORE LIMITS								
8	y→( )	40									
9	c	16									
a	x⇄y	30									
b	6	06									
c	÷	35									
d	x⇄y	30									
1 0	int x	64									
1	×	36									
2	1	01									
3	+	33									
4	4	04									
5	x→( )	23									
6	b	14									
7	IF x<y	52									
8	4	04									
9	2	02									
a	CLEAR	20									
b	1	01									
c	+	33									
d	3	03									
2 0	IF x<y	52									
1	3	03									
2	a	13									
3	c	16									
4	IF x<y	52									
5	8	10									
6	4	04									
7	d	17									
8	IF x>y	53									
9	1	01									
a	b	14									
b	↓	25									
c	STOP	41	PRIME NO.	0	0	DISPLAY					
d	ROLL ↓	31									

FROM 3-9 (→ step 1c)

FROM 4-1 (→ step 2-3)

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	y⇄( )	24									
	1	a	13									
	2	1	01									
	3	+	33									
	4	y⇄( )	24									
	5	a	13									
	6	ROLL ↑	22									
	7	GO TO ( )( )	44									
	8	1	01									
	9	c	16									
	a	1	01									
	b	+	33									
	c	5	05									
	d	IF x = y	50									
4	0	2	02									
	1	3	03									
	2	ACC +	60									
	3	d	17									
	4	IF x > y	53									
	5	7	07									
	6	4	04									
	7	c	16									
	8	IF x < y	52									
	9	8	10									
	a	4	04									
	b	5	05									
	c	x→( )	23									
	d	f	15									
5	0	SET FLAG	54									
	1	RCL	61									
	2	÷	35									
	3	IF x > y	53									
	4	6	06									
	5	9	11									
	6	↓	25									
	7	↑	27									
	8	int x	64									
	9	IF x = y	50									
	a	7	07									
	b	4	04									
	c	↓	25									
	d	2	02									

FROM 2-2 (beside steps 3-8, 3-9)

FROM 1-9 (beside steps 3-a to 3-d)

FROM 8-3 (beside steps 4-7 to 4-b)

FROM 6-3 & 6-8 (beside steps 5-1 to 5-9)

JULY, 1969
PART NO 09100-75802

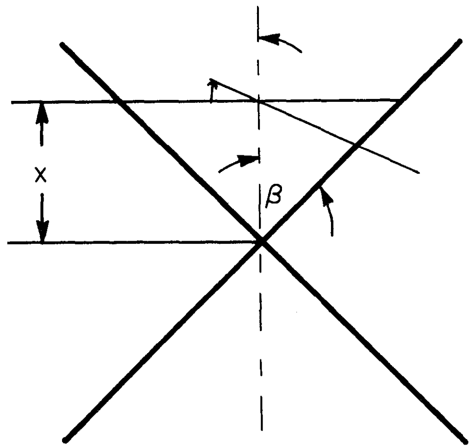
	Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
	6 0	ACC +	60									
	1	IF FLAG	43									
	2	5	05									
	3	1	01									
	4	ACC +	60									
	5	SET FLAG	54									
	6	GO TO ( )( )	44									
	7	5	05									
	8	1	01									
FROM-5-5	9	↓	25									
	a	e	12									
	b	STOP	41	PRIME NUMBER	0	0	DISPLAY					
	c	y⇄( )	24									
	d	a	13									
	7 0	1	01									
	1	+	33									
	2	y⇄( )	24									
	3	a	13									
FROM 4-6 & 5-b	4	y⇄( )	24									
	5	b	14									
	6	ACC +	60									
	7	2	02									
	8	IF x = y	50									
	9	8	10 ▲									
	a	a	13 ▲									
	b	−	34									
FROM 8-d	c	y→( )	40									
	d	b	14									
	8 0	RCL	61									
	1	GO TO ( )( )	44									
	2	4	04									
	3	7	07									
FROM 4-a & 2-6	4	c	16									
	5	↑	27									
	6	d	17									
	7	↑	27									
	8	a	13									
	9	STOP	41 ▲	NUMBER OF PRIMES	LOWER LIMIT	UPPER LIMIT	DISPLAY					
FROM 7-c	a	+	33 ▲									
	b	GO TO ( )( )	44 ▲									
	c	7	07									
	d	c	16 ▲									

▲ Denotes Revision

Step		Key	Code	Display			Storage					
				x	y	z	f	e	d	c	b	a
9	0	END	46									
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											

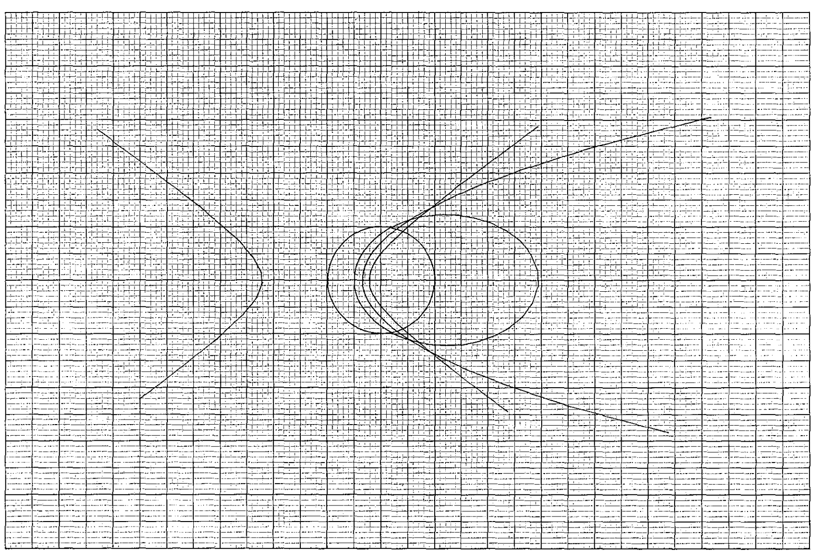
CONIC SECTION WITH PLOT

This program determines a conic section (circle, ellipse, parabola, hyperbola), calculates the equations of the curve, and plots the curve. Program inputs are the generating angle of a cone, β, and the intersecting angle, α, with a plane. Figure 1 illustrates the geometry.



α = Plane intersecting angle with axis of cone
β = Cone generating angle
X = Distance from plane to vertex of cone on axis

FIGURE 1



USER INSTRUCTIONS

SET: Decimal Wheel at 6 or less.

PRESS: END

Using orgin controls, place pen in the center of the paper.

ENTER PROGRAM: Side A followed by Side B

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $\alpha \rightarrow Y$, $\beta \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: $X \rightarrow X$

PRESS: CONTINUE

DISPLAY: Conic Code

PRESS: CONTINUE

RESULTS: Plotter will plot conic

To run another case.

CONIC CODE APPEARS IN Z - REGISTER

Z	1 = Circle	2 = Parabola	3 = Ellipse	4 = Hyperbola
Y	0	0	B	B
X	R	P	A	A

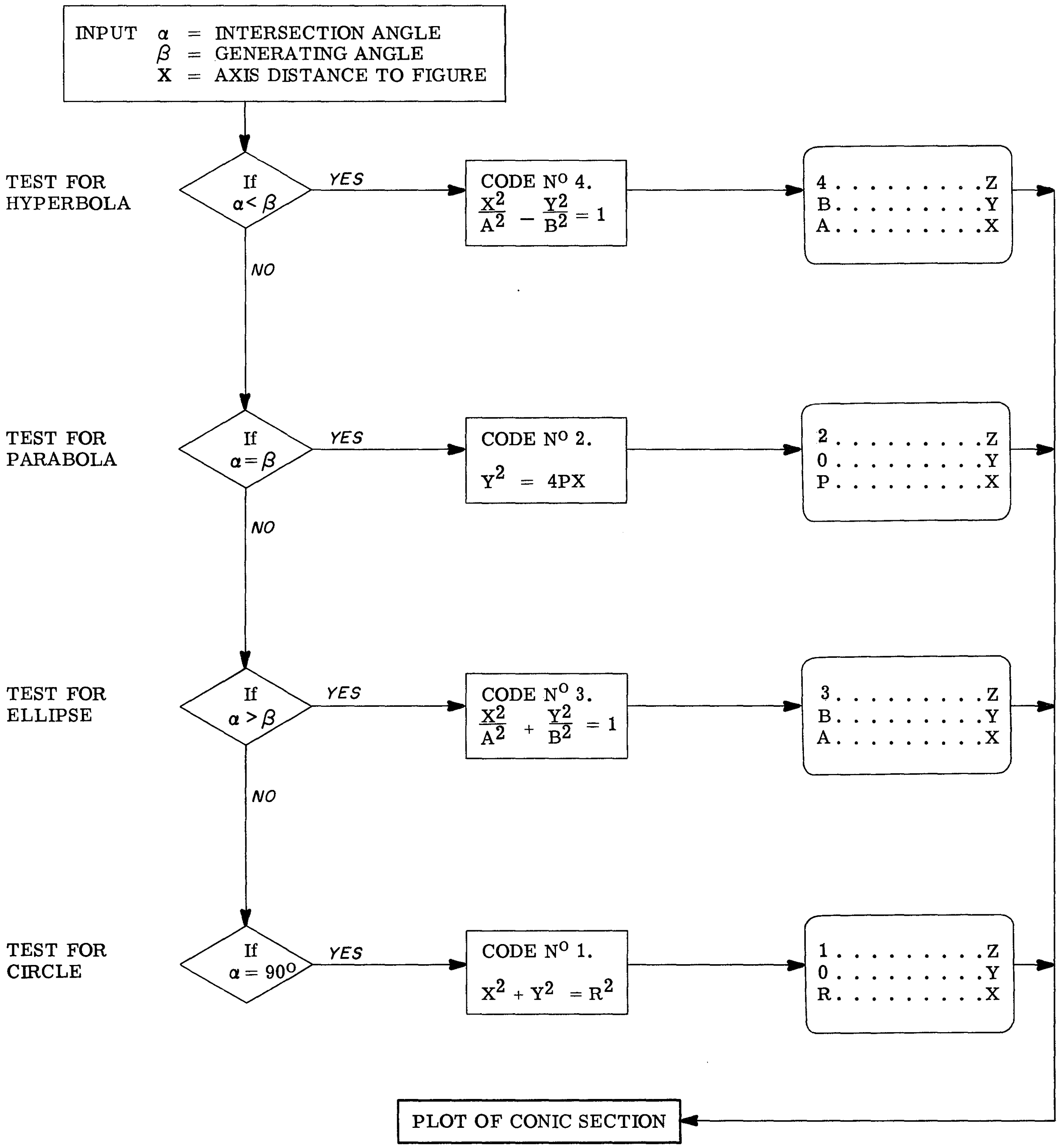
EXAMPLES

SET: DEGREES

Circle	Parabola	Ellipse	Hyperbola
$\alpha = 90^o$	$\alpha = 45^o$	$\alpha = 60^o$	$\alpha = 30^o$
$\beta = 45^o$	$\beta = 45^o$	$\beta = 45^o$	$\beta = 45^o$
X = 2	X = 2	X = 2	X = 2
R = 2	P = 0.70711	B = 2.44949	B = 1.41421
		A = 3.46410	A = 2.0000

Note: Scale 1/2" = 1 unit (1.25 cm. = 1 unit)
Resolution can be increased or decreased
By changing value of ΔQ in (−)(6)(4)

CONIC SECTION PROGRAM



HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	1	01	ENTER		
2	STOP	41	β	α	0
3	y→( )	40			
4	a	13			
5	x→( )	23			
6	b	14			
7	sin x	70			
8	ROLL ↓	31			
9	sin x	70			
a	ROLL ↓	31			
b	1	01			
c	x⇄y	30			
d	÷	35			
1 0	1	01			
(+) 1	ROLL ↑	22			
2	÷	35			
3	↓	25			
4	+	33			
5	y→( )	40			
6	c	16			
7	CLEAR	20			
8	2	02	ENTER		
9	STOP	41	X	0	0
a	x→( )	23			
b	e	12			
c	↑	27			
d	c	16			
2 0	÷	35			
(+) 1	y→( )	40			
2	c	16			
3	a	13			
4	↑	27			
5	b	14			
6	+	33			
7	2	02			
8	÷	35			
9	9	11			
a	0	00			
b	x⇄y	30			
c	−	34			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
3 0	tan x	71			
(+) 1	↑	27			
2	c	16			
3	x⇄y	30			
4	÷	35			
5	y→( )	40			
6	d	17			
7	a	13			
8	cos x	73			
9	↑	27			
a	b	14			
b	cos x	73			
c	÷	35			
d	CONT	47			
4 0	y→( )	40			
(+) 1	c	16			
2	CONT	47			
3	CONT	47			
4	a	13			
5	↑	27			
6	9	11			
7	0	00			
8	IF x=y	50			
9	5	05			
a	7	07			
b	b	14			
c	IF x>y	53			
d	6	06			
5 0	7	07			
(+) 1	IF x=y	50			
2	8	10			
3	3	03			
4	IF x<y	52			
5	8	10			
6	c	16			
7	CLEAR x	37			
8	↑	27			
9	b	14			
a	tan x	71			
b	↑	27			
c	e	12			
d	×	36			

Step	Key	Code	Display x	Display y	Display z
6 0	1	01			
(+) 1	ROLL ↓	31	DISPLAY		
2	STOP	41	R	0	1
3	GO TO ( )	44			
4	−	34			
5	0	00			
6	9	11			
7	d	17			
8	↑	27			
9	c	16			
a	↑	27			
b	1	01			
c	−	34			
d	ROLL ↓	31			
7 0	÷	35			
(+) 1	c	16			
2	↑	27			
3	×	36			
4	1	01			
5	−	34			
6	↓	25			
7	√x	76			
8	×	36			
9	4	04			
a	ROLL ↓	31			
b	x⇄y	30	DISPLAY		
c	STOP	41	A	B	4
d	GO TO ( )	44			

Storage

Register	Contents
f	A/ R
e	X/ B/ θ
d	
c	e
b	β
a	α
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step		Key	Code	Display x	Display y	Display z
8	0	—	34			
(+)	1	0	00			
	2	9	11			
	3	CLEAR	20			
	4	2	02			
	5	ROLL ↓	31			
	6	d	17	DISPLAY		
	7	STOP	41	P	0	2
	8	GO TO ( )	44			
	9	—	34			
	a	0	00			
	b	9	11			
	c	d	17			
	d	↑	27			
9	0	1	01			
(+)	1	↑	27			
	2	c	16			
	3	—	34			
	4	ROLL ↓	31			
	5	÷	35			
	6	c	16			
	7	↑	27			
	8	×	36			
	9	1	01			
	a	GO TO ( )	44			
	b	—	34			
	c	0	00			
	d	0	00			

Storage

HEWLETT · PACKARD

HEWLETT · PACKARD

Step	Key	Code	x	y	z
0 0	x⇄y	30			
(−) 1	−	34			
2	↓	25			
3	√x	76			
4	×	36			
5	3	03			
6	ROLL ↓	31			
7	x⇄y	30		DISPLAY	
8	STOP	41	A	B	3
9	ACC +	60			
a	ROLL ↓	31			
b	2	02			
c	IF x<y	52			
d	2	02			
1 0	0	00			
(−) 1	f	15			
2	x→( )	23			
3	d	17			
4	1	01			
5	IF x=y	50			
6	CLEAR x	37			
7	CONT	47			
8	x→( )	23			
9	c	16			
a	GO TO ( )	44			
b	2	02			
c	0	00			
d	CONT	47			
2 0	CLEAR x	37			
(−) 1	x→( )	23			
2	e	12			
3	c	16			
4	↑	27			
5	1	01			
6	+	33			
7	d	17			
8	×	36			
9	2	02			
a	5	05			
b	0	00			
c	×	36			
d	e	12			

Step	Key	Code	x	y	z
3 0	cos x	73			
(−) 1	↑	27			
2	c	16			
3	×	36			
4	1	01			
5	x⇄y	30			
6	−	34			
7	↓	25			
8	÷	35			
9	y→( )	40			
a	f	15			
b	RCL	61			
c	TO RECT	66			
d	↑	27			
4 0	3	03			
(−) 1	7	07			
2	5	05			
3	0	00			
4	|y|	55			
5	IF x<y	52			
6	7	07			
7	3	03			
8	2	02			
9	5	05			
a	0	00			
b	0	00			
c	ROLL ↑	22			
d	x⇄y	30			
5 0	|y|	55			
(−) 1	IF x<y	52			
2	7	07			
3	3	03			
4	RCL	61			
5	TO RECT	66			
6	FMT	42			
7	↓	25			
8	RCL	61			
9	3	03			
a	6	06			
b	0	00			
c	IF x<y	52			
d	7	07			

Step	Key	Code	x	y	z
6 0	a	13			
(−) 1	↑	27			
2	f	15			
3	÷	35			
4	4	04			
5	×	36			
6	|y|	55			
7	IF x>y	53			
8	x⇄y	30			
9	CONT	47			
a	ROLL ↓	31			
b	+	33			
c	y→( )	40			
d	e	12			
7 0	GO TO ( )	44			
(−) 1	2	02			
2	3	03			
3	CLEAR x	37			
4	↑	27			
5	FMT	42			
6	↑	27			
7	GO TO ( )	44			
8	5	05			
9	8	10			
a	CLEAR	20			
b	FMT	42			
c	↑	27			
d	END	46			

Storage

Register	Contents
f	A
e	B
d	
c	e
b	β
a	α
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage

Register	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

HEWLETT · PACKARD
HEWLETT · PACKARD
HEWLETT · PACKARD

9100B MISCELLANEOUS PROGRAM LISTING

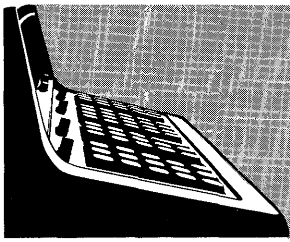
76003 - NAVIGATIONAL COURSE CALCULATION
Calculates course settings for one or more adjoining legs of a proposed journey, the length of each leg, and the total distance covered on completion of the journey.

76004 - CIRCLE DETERMINED BY THREE POINTS
Calculates the radius and center point (in rectangular coordinates) of the circle defined by three given points.

76005 - AREA OF A RECTILINEAR SURFACE POLYGON
Calculates the area of any rectilinear polygon given the rectangular coordinates of the vertices.

9100B ONLY

76501 - DIAGNOSTIC (EXERCISER)
The program exercises each calculator operation and memory location with the exception of the FMT, PRINT, and error conditions.



PART NO.
09100-76003

NAVIGATIONAL COURSE CALCULATION

This program is designed for the navigator concerned with courses consisting of several adjoining legs, each leg being relatively short (500 miles or less). Although there is no theoretical limit to the leg length, only one heading from source to destination is given. Long legs should therefore be subdivided to improve calculated results.

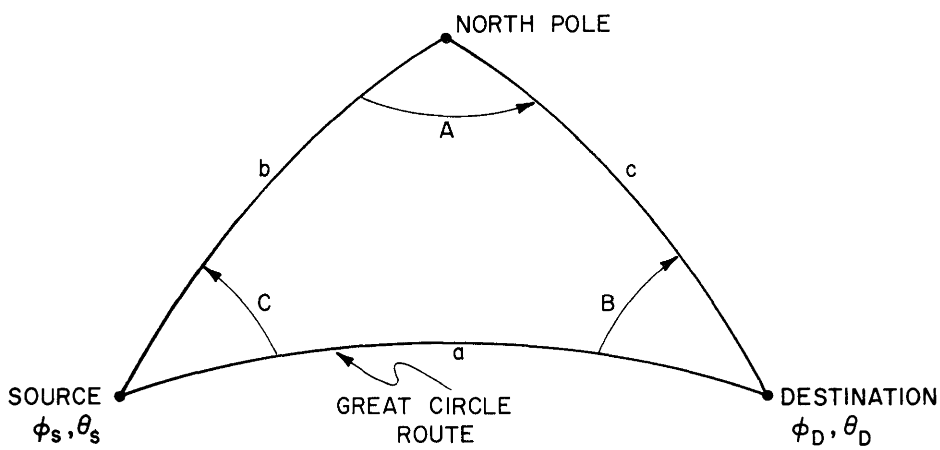
All headings lie between 0° and 360° with either 0° or 360° being a true north course; true west is 90°. Leg lengths are given in nautical miles (one nautical mile equals 6,080 feet). All legs are portions of great circles and all calculations are performed in spherical coordinates.

Referring to the diagram below, the angles A, B, and C are found from the coordinates of the source (ϕ_S, θ_S) and destination (ϕ_D, θ_D). The great circle route a is found from the equation:

$$a = \cos^{-1}\left[\cos b \cos c + \sin b \sin c \cos A\right]$$

The heading (c) is then:

$$c = \cos^{-1}\left[\frac{\cos c - \cos a \cos b}{\sin a \ \sin b}\right]$$



Notes:

(1) Coordinates are entered in degrees, minutes and decimal fractions of minutes. The decimal point is used to separate degrees and minutes. Example: 30.173 means 30 degrees, 17.3 minutes. 45 degrees, 5 minutes, 30 seconds would be written 45.0550

degrees (45) minutes (0550) (i.e. 5.5 min.)

(2) Headings and magnetic variations are entered in decimal degrees (i.e. 17.5° = 17 degrees, 30 minutes)

(3) Northern hemisphere latitudes and western hemisphere longitude are indicated as positive number, southern and eastern coordinates are indicated as negative numbers.

(4) The program is operable over all coordinates except for legs starting or stopping at either the north or south pole, routes going directly over a pole, or a single leg which includes points which are diametrically opposite each other.

USER INSTRUCTIONS

The following notation is used:

- T. H. = True heading
- M. H. = Magnetic heading
- L = Distance of each leg
- $\sum D$ = Total distance traversed over more than one leg
- ϕ = Latitude coordinate
- θ = Longitude coordinate
- ϕ_s, θ_s = Coordinates of the source of each leg
- ϕ_D, θ_D = Coordinates of the endpoint of each leg
- M.V. = Magnetic Variation
- N.M. = Nautical miles

ENTER PROGRAM (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: ϕ_s (1) → Y, θ_s (1) → X

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

NO

ENTER DATA: ϕ_D (n) → Y, θ_D (n) → X

(n indicates the number of the leg under consideration)

PRESS: CONTINUE

DISPLAY

T. H.	Z
L	Y
2	X

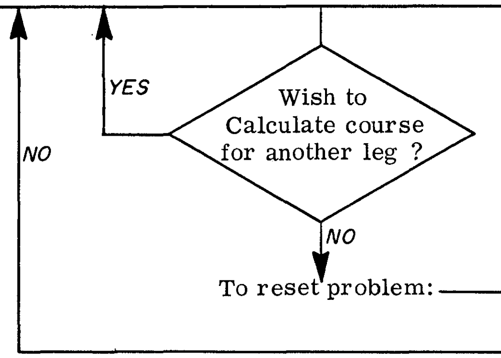
ENTER DATA: M.V. → X (do not alter the Y and Z register contents)

PRESS: CONTINUE

DISPLAY

M. H.	Z
$\sum$ D	Y
1	X

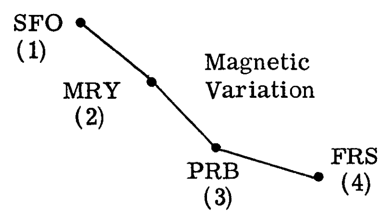
USER INSTRUCTIONS (con't)



Note: The beginning (source) coordinates and end coordinates are both entered for the first leg only; for additional legs the end coordinates of the previous leg are retained as the source coordinates of the next leg. Therefore, after the first leg only endpoint coordinates are entered as data.

EXAMPLE

A flight from San Francisco, California, to Monterey, to Paso Robles, to Fresno is to be charted.



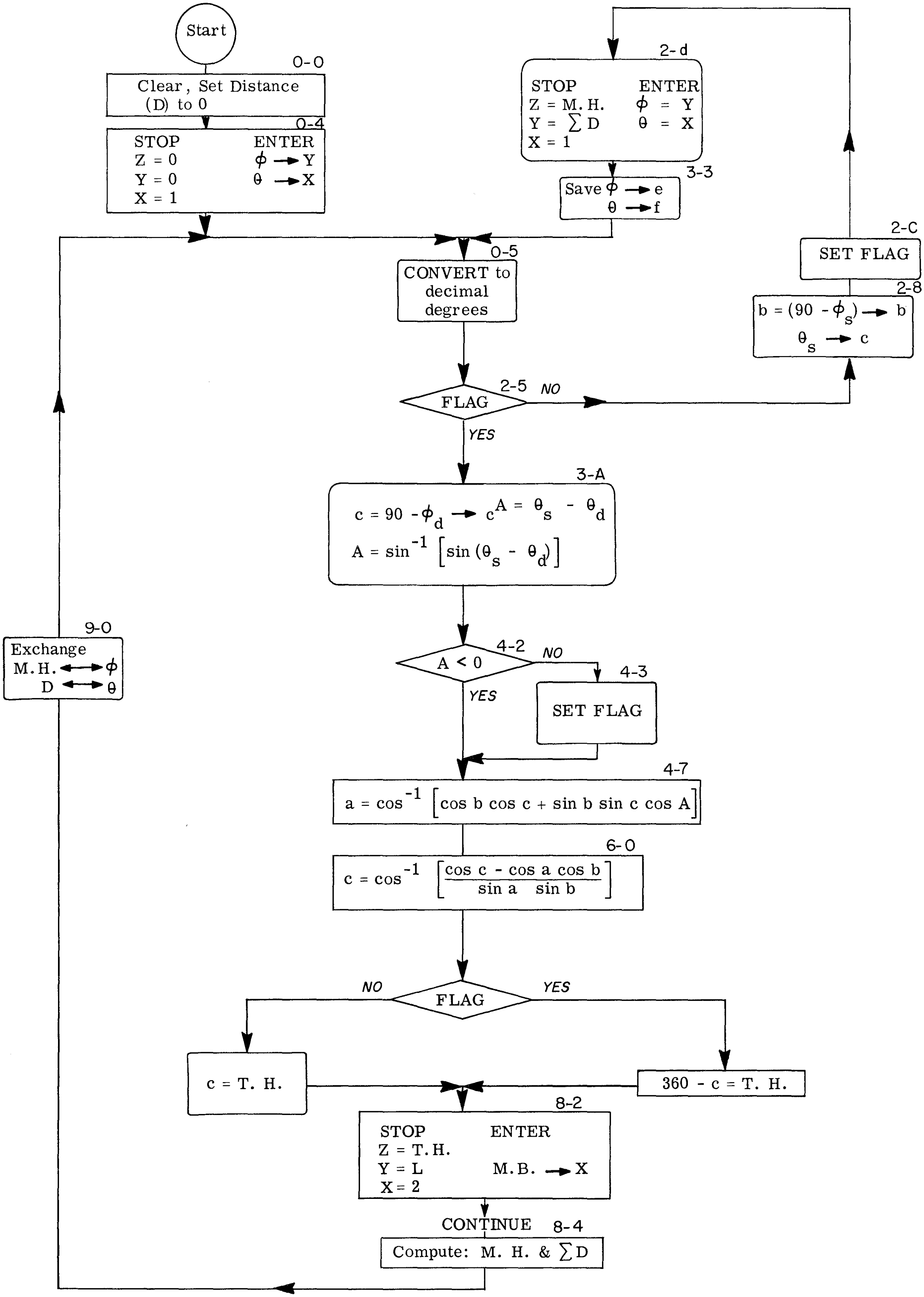
Data:

Points	ϕ	θ	M.V.
(1)	N37.37°	W122.23°	
(2)	N36.35°	W121.51°	17°E
(3)	N35.40°	W120.38°	16.5°E
(4)	N36.44°	W119.49°	16.3°E

Magnetic variation will be input as a negative number due to the easterly coordinate.

Solution:

LEG	T. H.	M. H.	L(N.M.)	$\sum$D(N.M.)
First leg	157.46°	140.46°	67.05	67.05
Second leg	132.65°	116.15°	80.63	147.68
Third leg	31.47°	15.17°	75.23	222.91



Start
0-0
Clear, Set Distance (D) to 0
0-4
STOP Z = 0 Y = 0 X = 1
ENTER φ → Y θ → X
2-d
STOP Z = M.H. Y = ∑D X = 1
ENTER φ = Y θ = X
3-3
Save φ → e θ → f
0-5
CONVERT to decimal degrees
2-5
FLAG
NO
YES
2-C
SET FLAG
2-8
b = (90 - φs) → b
θs → c
3-A
c = 90 - φd → c
A = θs - θd
A = sin-1 [sin (θs - θd)]
4-2
A < 0
NO
YES
4-3
SET FLAG
4-7
a = cos-1 [cos b cos c + sin b sin c cos A]
6-0
c = cos-1 [cos c - cos a cos b / sin a sin b]
FLAG
NO
YES
c = T. H.
360 - c = T. H.
8-2
STOP Z = T.H. Y = L X = 2
ENTER M.B. → X
CONTINUE
8-4
Compute: M. H. & ∑D
9-0
Exchange M.H. ↔ φ D ↔ θ

Step	Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	$x \rightarrow ()$	23									
2	a	13									
3	1	01				ENTER LATITUDE AND LONGITUDE OF THE ORIGIN OF EACH LEG					
4	STOP	41	θ	ϕ	0						
5 (FROM 3-7 & 9-c)	$y \rightarrow ()$	40	STORE ϕ								
6	d	17									
7	↑	27									
8	int x	64									
9	−	34									
a	$x \rightleftarrows y$	30									
b	↑	27	CONVERT θ TO DECIMAL DEGREES AND STORE								
c	·	21									
d	6	06									
1 0	÷	35									
1	ROLL ↓	31									
2	+	33									
3	$y \rightleftarrows ()$	24									
4	d	17									
5	↓	25									
6	↑	27									
7	int x	64									
8	−	34									
9	$x \rightleftarrows y$	30									
a	ROLL ↑	22	CONVERT ϕ TO DECIMAL DEGREES AND CALCULATE $90° - \phi$								
b	÷	35									
c	9	11									
d	0	00									
2 0	ROLL ↓	31									
1	+	33									
2	↓	25									
3	−	34									
4	d	17									
5	IF FLAG	43	BRANCH IF FLAG IS SET TO CALCULATE DISTANCE AND TRUE HEADING OF EACH LEG. FLAG IS SET AUTOMATICALLY AFTER BOTH END POINTS OF EACH LEG HAVE BEEN ENTERED								
6	3	03									
7	8	10									
8	$x \rightarrow ()$	23									
9	c	16									
a	$y \rightarrow ()$	40	STORE θ_s AND $(90° - \phi) = b$								
b	b	14									
c	SET FLAG	54									
d	RCL	61	RECALL ACCUMULATED DISTANCE AND MAGNETIC HEADING FOR DISPLAY. CLEAR e AND f REGISTERS								

Step	Key	Code	Display x	Display y	Display z	Remarks
3 0	ACC −	63				
1	↑	27				
2	1	01				ENTER LATITUDE AND LONGITUDE OF ENDPOINT OF EACH LEG
3	STOP	41	θ	ϕ	0	
4	ACC +	60				STORE COORDINATES OF ENDPOINTS OF EACH LEG
5	GO TO ( )( )	44				BRANCH TO CONVERT ENDPOINT COORDINATES TO DECIMAL DEGREES (steps 5–7)
6	0	00				
7	5	05				
8	y⇄( )	24				FROM 2-7 (CALCULATE DISTANCE (a) OF EACH LEG: steps 3-8 to 5-a)
9	c	16				
a	−	34				
b	0	00				
c	x⇄y	30				
d	↑	27				
4 0	sin x	70				
1	ROLL ↑	22				
2	IF x < y	52				SET THE FLAG IF $\sin(\theta_S - \theta_D) > 0$ (steps 2–4)
3	SET FLAG	54				
4	SET FLAG	54				
5	ROLL ↑	22				
6	cos x	73				
7	↑	27				
8	b	14				
9	sin x	70				CALCULATE DISTANCE (a) OF EACH LEG
a	×	36				
b	c	16				
c	sin x	70				
d	×	36				
5 0	c	16				
1	cos x	73				
2	↑	27				
3	b	14				
4	cos x	73				
5	×	36				
6	↓	25				
7	+	33				
8	↓	25				
9	arc ▾	72				
a	cos x	73				
b	x→( )	23				
c	d	17				
d	cos x	73				

JULY, 1969
PART NO. 09100-76003

Step	Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a	Remarks
6 0	↑	27										
1	b	14										
2	cos x	73										
3	×	36										
4	c	16										
5	cos x	73										
6	−	34										
7	d	17										
8	sin x	70										
9	÷	35										CALCULATE TRUE HEADING (c) OF EACH LEG (steps 60–75)
a	b	14										
b	sin x	70										
c	÷	35										
d	3	03										
7 0	6	06										
1	0	00										
2	x⇄y	30										
3	CHG SIGN	32										
4	arc ▾	72										
5	cos x	73										
6	IF FLAG	43										
7	↑	27										CALCULATE TRUE HEADING = (360−c) IF $\sin(\theta_S-\theta_D)<0$; OTHERWISE (c) IS THE TRUE HEADING. (steps 76–79)
8	0	00										
9	−	34										
a	d	17										
b	↑	27										
c	6	06										
d	0	00										
8 0	×	36										
1	2	02										
2	STOP	41	2	D	T.H.							DISPLAY DISTANCE AND TRUE HEADING OF EACH LEG; ENTER MAGNETIC VARIATION.
3	ROLL ↑	22										CALCULATE MAGNETIC VARIATION (M.V.) PLUS TRUE HEADING (T.H.) (steps 83–84)
4	+	33										
5	3	03										
6	6	06										
7	0	00										
8	IF x>y	53										ADD 360° IF (M.V.+T.H.)<360°
9	arc ▾	72 ▲										RESULT: MAGNETIC HEADING (M.H.) = (M.V.+T.H.)+360°
a	+	33 ▲										
b	IF x<y	52										SUBTRACT 360° IF (M.V.+T.H.)>360°
c	arc ▾	72 ▲										RESULT: M.H. = (M.V.+T.H.) − 360°
d	−	34 ▲										

▲ Denotes Revision

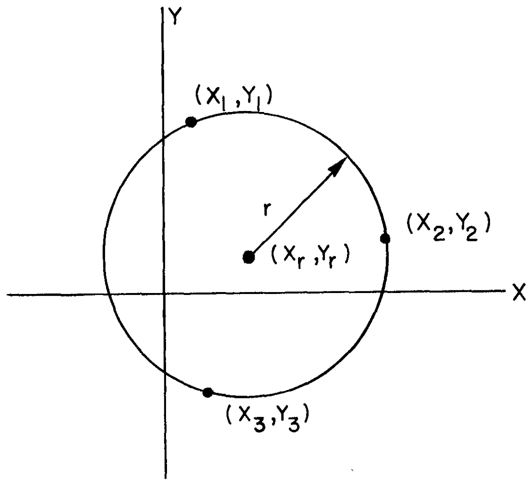
Step		Key	Code	Display / Comments
9	0	y⇄( )	24	STORE M.H. FOR RECALL AND FUTURE DISPLAY; RECALL ϕ_D.
	1	e	12	
	2	a	13	SUM THE DISTANCES TRAVERSED BY EACH LEG (ΣD)
	3	ROLL ↑	22	
	4	+	33	
	5	y→( )	40	
	6	a	13	
	7	y⇄( )	24	STORE ΣD FOR RECALL AND FUTURE DISPLAY; RECALL θ_D.
	8	f	15	
	9	↓	25	
	a	GO TO ( )( )	44	BRANCH TO DISPLAY ΣD AND M.H. AND ENTER NEW ENDPOINT COORDINATES FOR NEXT LEG.
	b	0	00	
	c	5	05	
	d	END	46	
	0			
	1			
	2			
	3			
	4			
	5			
	6			
	7			
	8			
	9			
	a			
	b			
	c			
	d			
	0			
	1			
	2			
	3			
	4			
	5			
	6			
	7			
	8			
	9			
	a			
	b			
	c			
	d			

PART NO.
09100-76004

CIRCLE DETERMINED BY THREE POINTS

This program solves for the radius and center point of a circle which is determined by three given points in rectangular coordinates; i.e. (X_i, Y_i), $(i = 1, 2, \text{or } 3)$

Consider the following representative diagram:



The following equations are used to solve for X_r, Y_r and r.

1) $(X_1 - X_r)^2 + (Y_1 - Y_r)^2 = r^2$

2) $(X_2 - X_r)^2 + (Y_2 - Y_r)^2 = r^2$

3) $(X_3 - X_r)^2 + (Y_3 - Y_r)^2 = r^2$

Combining Equations 1 and 2:

4) $X_2^2 - 2X_2X_r + X_r^2 + Y_2^2 - 2Y_2Y_r + Y_r^2 = X_1^2 - 2X_1X_r + X_r^2 + Y_1^2 - 2Y_1Y_r + Y_r^2$

Rearranging:

5) $\underbrace{(X_2 - X_1)}_{A}\underbrace{(X_2 + X_1)}_{B} + \underbrace{(Y_2 - Y_1)}_{C}\underbrace{(Y_2 + Y_1)}_{D} = 2X_r(X_2 - X_1) + 2Y_r(Y_2 - Y_1)$

Substituting the variables indicated in equation (5) we have:

6) $AB + CD = 2AX_r + 2CY_r$

Rearranging:

7) $X_r = \underbrace{\frac{AB + CD}{2A}}_{K_1} - \underbrace{\frac{C}{A}}_{N_1} Y_r$

From equation (7) we then have:

8) $X_r = K_1 - N_1 Y_r$

Combining equations 1 & 3 we obtain:

$$\underbrace{(X^3 - X_1)}_{a} \underbrace{(X_3 + X_1)}_{b} + \underbrace{(Y_3 - Y_1)}_{c} \underbrace{(Y_3 + Y_1)}_{d} = 2 X_r (X_3 - X_1) + 2Y_r (Y_3 - Y_1)$$

OR

9) $$X_r = \underbrace{\frac{ab + cd}{2a}}_{K_2} - \underbrace{\frac{c}{a}}_{N_2} Y_r$$

Therefore:

10) $$X_r = K_2 - N_2 Y_r$$

Combining equations (8) and (10):

11) $$Y_r = \frac{K_2 - K_1}{N_2 - N_1}$$

Equations (10) and (11) are used to solve for X_r and Y_r.

The radius (r) is found from equation (1):

$$r = \sqrt{(X_1 - X_r)^2 + (Y_1 - Y_r)^2}$$

USER INSTRUCTIONS	EXAMPLE

ENTER PROGRAM: (Starting Address is 0 - 0)

PRESS: GO TO (0) (0) [or END]

→ PRESS: CONTINUE

DISPLAY

0	Z
0	Y
1	X

ENTER DATA: $Y_1 \rightarrow Y$, $X_1 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
2	X

ENTER DATA: $Y_2 \rightarrow Y$, $X_2 \rightarrow X$

PRESS: CONTINUE

DISPLAY

0	Z
0	Y
3	X

ENTER DATA: $Y_3 \rightarrow Y$, $X_3 \rightarrow X$

PRESS: CONTINUE

DISPLAY

R	Z
Y_R	Y
X_R	X

└─ To enter new data points:

EXAMPLE

$X_1 = 2$
$Y_1 = 1$

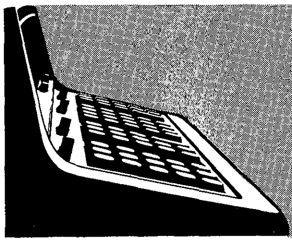
$X_2 = 2.5$
$Y_2 = .9$

$X_3 = 1$
$Y_3 = -3$

R = 2.092
$Y_R = -1.086$
$X_R = 1.843$

Step	Key	Code	Display x	y	z	Storage f	e	d	c	b	a
0 0	CLEAR	20									
1	1	01									
2	STOP	41	X_1	Y_1	0	ENTER DATA					
3	x→( )	23									
4	a	13	STORE X_1 AND Y_1								
5	y→( )	40									
6	b	14									
7	CLEAR	20									
8	2	02									
9	STOP	41	X_2	Y_2	0	ENTER DATA					
a (FROM 4-1)	↑	27									
b	a	13	CALCULATE A or a								
c	−	34									
d	y→( )	40									
1 0	c	16									
1	+	33	CALCULATE B or b								
2	+	33									
3	c	16									
4	×	36	CALCULATE AB or ab								
5	y→( )	40									
6	d	17									
7	b	14									
8	ROLL ↑	22									
9	+	33									
a	↑	27									
b	b	14	CALCULATE C D or c d								
c	−	34									
d	↓	25									
2 0	×	36									
1	ROLL ↓	31									
2	y⇄( )	24	CALCULATE AB + CD or ab + cd								
3	d	17									
4	+	33									
5	y⇄( )	24									
6	d	17									
7	↓	25	CALCULATE $\frac{C}{A}$ or $\frac{c}{a}$								
8	c	16									
9	÷	35									
a	↑	27									
b	+	33	CALCULATE $\frac{AB+CD}{2A}$ or $\frac{ab+cd}{2a}$								
c	d	17									
d	x⇄y	30									

Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
3	0	÷	35									
	1	↓	25									
	2	x⇄y	30									
	3	IF FLAG	43									
	4	4	04	BRANCH AFTER ENTRY OF X_3 AND Y_3								
	5	2	02									
	6	ACC −	63	CALCULATE $-N_1$ AND $-K_1$								
	7	CLEAR x	37									
	8	↑	27									
	9	↑	27									
	a	3	03									
	b	STOP	41	X_3	Y_3	0	ENTER DATA					
	c	SET FLAG	54									
	d	GO TO ( )( )	44									
4	0	0	00	BRANCH FOR EQUATIONS INVOLVING X_1, Y_1 AND OR X_3, Y_3								
	1	a	13									
	2	ACC +	60	CALCULATE $N_2 - N_1$ AND $K_2 - K_1$								
	3	y→( )	40	STORE K_2								
	4	c	16									
	5	RCL	61	CALCULATE Y_r								
	6	÷	35									
	7	↓	25									
	8	×	36									
	9	ROLL ↓	31	CALCULATE X_r								
	a	y⇄( )	24									
	b	c	16									
	c	−	34									
	d	b	14									
5	0	ROLL ↑	22									
	1	x→( )	23									
	2	d	17									
	3	−	34	CALCULATE $r = \sqrt{(X_1 - X_r)^2 + (Y_1 - Y_r)^2}$								
	4	a	13									
	5	ROLL ↑	22									
	6	−	34									
	7	ROLL ↓	31									
	8	TO POLAR	62									
	9	ROLL ↓	31									
	a	d	17	RECALL AND POSITION DISPLAY								
	b	x⇄y	30									
	c	END	46	X_r	Y_r	r	DISPLAY					
	d											



PART NO.
09100-76005

AREA OF A POLYGON

This program calculates the area of a polygon of n sides, where $n \geq 3$. The endpoint coordinates of each side of the polygon must be in rectangular form.

The equation used to calculate the area sums the area of trapezoids and is given below:

$$A = \frac{(X_1 + X_2)(Y_1 - Y_2) + (X_2 + X_3)(Y_2 - Y_3) + \ldots + (X_n + X_1)(Y_n - Y_1)}{2}$$

Reference: Analytic Geometry
C. E. Love and E. D. Rainville
MacMillan Co., 5th Edition

USER INSTRUCTIONS

ENTER PROGRAM: (Starting Address is 0 - 0)

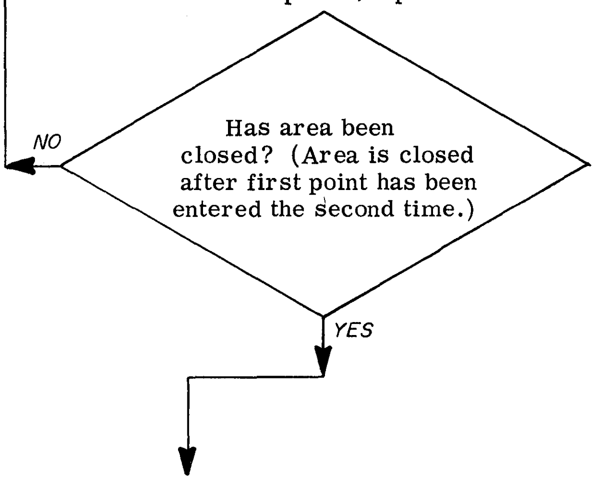
PRESS: GO TO (0) (0) [or END]

PRESS: CONTINUE

DISPLAY

0	——	Z
0	——	Y
i	——	X

ENTER DATA: $Y_i \rightarrow Y$, $X_i \rightarrow X$



PRESS: SET FLAG

PRESS: CONTINUE

DISPLAY

0	——	Z
area	——	Y
n+ 1	——	X

Note: If there are n distinct data sets, there will be n+1 entries (the first point entered twice).

EXAMPLES

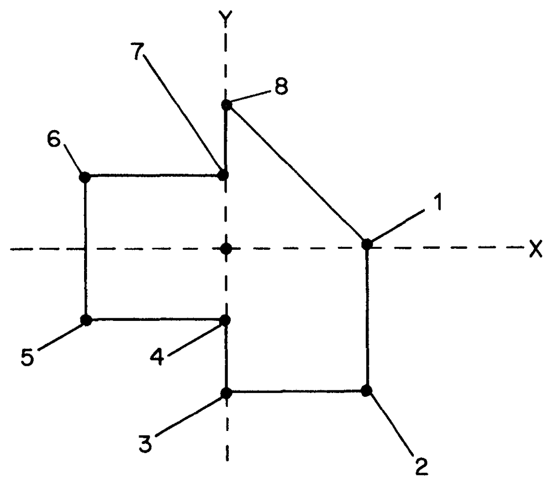
Note: When traversing counterclockwise, the area will be displayed as a negative number.

DATA:

Point (i)	Coordinates (x, y)
1	1, 0
2	1, -1
3	0, -1
4	0, -.5
5	-1, -.5
6	-1, .5
7	0, .5
8	0, 1
1	1, 0

SOLUTION:

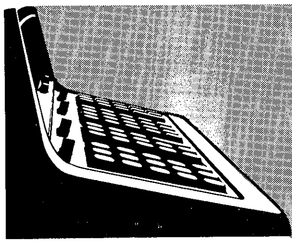
Area = 2.5
n + 1 = 9



Step		Key	Code	Display x	Display y	Display z	Storage f	e	d	c	b	a
0	0	CLEAR	20	CLEAR AND INITIALIZE REGISTERS AND DISPLAY 1 TO INDICATE FIRST ENTRY								
	1	$x\to()$	23									
	2	c	16									
	3	1	01									
	4	$x\to()$	23									
	5	d	17									
	6	STOP	41	X_i	Y_i	0	ENTER Y_i AND X_i					
	7	ACC +	60	INCREMENT COUNTER								
	9 (← FROM 3-2)	$x\rightleftarrows y$	30									
	8	d	17									
	a	1	01									
	b	+	33									
	c	$y\to()$	40									
	d	d	17									
1	0	↓	25	POSITION i TO INDICATE ENTRY								
	1	STOP	41	X_i	Y_i	0	ENTER Y_i AND X_i					
	2	↑	27	CALCULATE $(Y_{i-1} - Y_i)$								
	3	e	12									
	4	ROLL ↑	22									
	5	−	34									
	6	ROLL ↑	22	LET Y_i REPLACE Y_{i-1} IN STORAGE								
	7	$y\rightleftarrows()$	24									
	8	e	12									
	9	$x\rightleftarrows y$	30	LET X_i REPLACE X_{i-1} IN STORAGE								
	a	↓	25									
	b	↑	27									
	c	$y\rightleftarrows()$	24									
	d	f	15									
2	0	+	33	CALCULATE $(X_i + X_{i-1})$								
	1	2	02	CALCULATE $(Y_i - Y_{i-1})\left(\frac{X_i + X_{i-1}}{2}\right)$								
	2	÷	35									
	3	CLEAR x	37									
	4	ROLL ↓	31									
	5	×	36									
	6	c	16	RECALL Σ AREA AND ADD $AREA_i$								
	7	+	33									
	8	$y\to()$	40									
	9	c	16									
	a	d	17	RECALL COUNTER								
	b	IF FLAG	43	BRANCH WHEN FLAG IS SET AFTER ALL DATA HAS BEEN ENTERED								
	c	3	03									
	d	3	03									

FROM 2-d

Step		Key	Code	Display x	Display y	Display z	Storage f	Storage e	Storage d	Storage c	Storage b	Storage a
3	0	GO TO ( )( )	44									
	1	0	00	BRANCH TO DATA ENTRY								
	2	9	11									
	3	END	46	n + 1	AREA	0	DISPLAY					
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											
	0											
	1											
	2											
	3											
	4											
	5											
	6											
	7											
	8											
	9											
	a											
	b											
	c											
	d											



9100B ONLY
PART NO.
09100-76501

9100B DIAGNOSTIC

This program exercises every calculator operation to verify that the calculator is functioning correctly.

USER INSTRUCTIONS

SET: [ ] RADIANS   FLOATING [ ]

PRESS: END

ENTER PROGRAM: Side A followed by Side B

PRESS: CONTINUE

CORRECT DISPLAY   FLASHING

```
-- --- --- --- ---     Z
n nnn nnn nnn nn       Y
-- --- --- --- ---     X
```

n = 0, 1, 2 . . . 9, 0; CYCLIC

Step	Key	Code	Display x	Display y	Display z
0 0	CLEAR	20			
(+) 1	IF $x=y$	50			
2	0	00			
3	5	05			
4	STOP	41			
5	IF FLAG	43			
6	STOP	41			
7	STOP	41			
8	SET FLAG	54			
9	IF FLAG	43			
a	0	00			
b	d	17			
c	STOP	41			
d	IF FLAG	43			

Step	Key	Code	Display x	Display y	Display z
1 0	STOP	41			
(+) 1	STOP	41			
2	3	03			
3	•	21			
4	1	01			
5	4	04			
6	1	01			
7	5	05			
8	9	11			
9	2	02			
a	6	06			
b	5	05			
c	3	03			
d	6	06			

Step	Key	Code	Display x	Display y	Display z
2 0	0	00			
(+) 1	↑	27			
2	π	56			
3	IF $x=y$	50			
4	2	02			
5	7	07			
6	STOP	41			
7	$\sqrt{x}$	76			
8	$y\rightarrow()$	40			
9	e	12			
a	ROLL ↓	31			
b	$\sqrt{x}$	76			
c	ROLL ↑	22			
d	×	36			

Step	Key	Code	Display x	Display y	Display z
3 0	e	12			
(+) 1	CHG SIGN	32			
2	+	33			
3	a	13			
4	$\lvert y\rvert$	55			
5	IF $x>y$	53			
6	3	03			
7	9	11			
8	STOP	41			
9	$x\rightleftarrows y$	30			
a	IF $x<y$	52			
b	4	04			
c	0	00			
d	STOP	41			

Step	Key	Code	Display x	Display y	Display z
4 0	•	21			
(+) 1	7	07			
2	8	10			
3	↑	27			
4	7	07			
5	8	10			
6	ENTER EXP	26			
7	CHG SIGN	32			
8	1	01			
9	−	34			
a	IF $x>y$	53			
b	CLEAR x	37			
c	ROLL ↑	22			
d	IF $x=y$	50			

Step	Key	Code	Display x	Display y	Display z
5 0	STOP	41			
(+) 1	STOP	41			
2	IF $x<y$	52			
3	STOP	41			
4	STOP	41			
5	↓	25			
6	IF $x>y$	53			
7	STOP	41			
8	STOP	41			
9	π	56			
a	int x	64			
b	↑	27			
c	$y\rightleftarrows()$	24			
d	e	12			

Step	Key	Code	Display x	Display y	Display z
6 0	↓	25			
(+) 1	int x	64			
2	↑	27			
3	CLEAR x	37			
4	$x\leftarrow()$	67			
5	e	12			
6	IF $x=y$	50			
7	CLEAR x	37			
8	3	03			
9	IF $x=y$	50			
a	6	06			
b	d	17			
c	STOP	41			
d	π	56			

Step	Key	Code	Display x	Display y	Display z
7 0	$x\rightleftarrows y$	30			
(+) 1	÷	35			
2	×	36			
3	π	56			
4	IF $x=y$	50			
5	7	07			
6	8	10			
7	STOP	41			
8	CLEAR	20			
9	π	56			
a	↑	27			
b	+	33			
c	ACC +	60			
d	ACC +	60			

Storage

f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

8 (+)

Step	Key	Code	Display x	Display y	Display z
0	ACC −	63			
1	ACC +	60			
2	RCL	61			
3	↑	27			
4	π	56			
5	−	34			
6	↓	25			
7	−	34			
8	−	34			
9	−	34			
a	IF $x=y$	50			
b	b	14			
c	8	10			
d	STOP	41			

9 (+)

Step	Key	Code	Display x	Display y	Display z
0	sin x	70			
1	arc ▼	72			
2	sin x	70			
3	cos x	73			
4	arc ▼	72			
5	cos x	73			
6	tan x	71			
7	arc ▼	72			
8	tan x	71			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	b	14			
c	0	00			
d	RETURN	77			

a (+)

Step	Key	Code	Display x	Display y	Display z
0	0	00			
1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	1	01			
a	9	11			
b	CLEAR	20			
c	0	00			
d	0	00			

b (+)

Step	Key	Code	Display x	Display y	Display z
0	−	34			
1	|y|	55			
2	a	13			
3	IF $x<y$	52			
4	SET FLAG	54			
5	x→( )	23			
6	RETURN	77			
7	STOP	41			
8	3	03			
9	÷	35			
a	↓	25			
b	↑	27			
c	x→( )	23			
d	−	34			

c (+)

Step	Key	Code	Display x	Display y	Display z
0	e	12			
1	GO TO ( )	44			
2	▲SUB▼	77			
3	9	11			
4	0	00			
5	IF FLAG	43			
6	STOP	41			
7	STOP	41			
8	x←( )	67			
9	−	34			
a	e	12			
b	↑	27			
c	GO TO ( )	44			
d	▲SUB▼	77			

d (+)

Step	Key	Code	Display x	Display y	Display z
0	−	34			
1	b	14			
2	a	13			
3	IF FLAG	43			
4	STOP	41			
5	STOP	41			
6	y⇄( )	24			
7	−	34			
8	e	12			
9	↓	25			
a	↑	27			
b	SET FLAG	54			
c	GO TO ( )	44			
d	▲SUB▼	77			

0 (−)

Step	Key	Code	Display x	Display y	Display z
0	−	34			
1	8	10			
2	6	06			
3	−	34			
4	|y|	55			
5	a	13			
6	IF $x<y$	52			
7	STOP	41			
8	STOP	41			
9	GO TO ( )	44			
a	▲SUB▼	77			
b	3	03			
c	3	03			
d	CLEAR	20			

1 (−)

Step	Key	Code	Display x	Display y	Display z
0	SET FLAG	54			
1	e	12			
2	f	15			
3	y→( )	40			
4	−	34			
5	f	15			
6	y⇄( )	24			
7	−	34			
8	f	15			
9	ACC +	60			
a	ACC +	60			
b	ACC −	63			
c	RCL	61			
d	0	00			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
2 0	0	00			
1	0	00			
2	0	00			
3	0	00			
4	0	00			
5	0	00			
6	0	00			
7	0	00			
8	0	00			
9	IF x = y	50			
a	2	02			
b	d	17			
c	STOP	41			
d	GO TO ( )	44			
3 0	+	33			
1	0	00			
2	0	00			
3	CLEAR	20			
4	ENTER EXP	26			
5	1	01			
6	2	02			
7	x→( )	23			
8	f	15			
9	1	01			
a	↑	27			
b	GO TO ( )	44			
c	▲SUB▼	77			
d	8	10			
4 0	6	06			
1	RCL	61			
2	CLEAR x	37			
3	IF x = y	50			
4	7	07			
5	a	13			
6	9	11			
7	IF x = y	50			
8	7	07			
9	3	03			
a	RCL	61			
b	x⇄y	30			
c	↑	27			
d	↓	25			

Step	Key	Code	Display x	Display y	Display z
5 0	÷	35			
1	↓	25			
2	+	33			
3	ENTER EXP	26			
4	1	01			
5	1	01			
6	×	36			
7	1	01			
8	x⇄y	30			
9	ACC +	60			
a	x→( )	23			
b	f	15			
c	↓	25			
d	9	11			
6 0	GO TO ( )	44			
1	▲SUB▼	77			
2	b	14			
3	0	00			
4	RCL	61			
5	•	21			
6	1	01			
7	×	36			
8	↓	25			
9	↑	27			
a	GO TO ( )	44			
b	3	03			
c	b	14			
d	STOP	41			
7 0	STOP	41			
1	STOP	41			
2	STOP	41			
3	RCL	61			
4	↑	27			
5	GO TO ( )	44			
6	▲SUB▼	77			
7	b	14			
8	1	01			
9	RETURN	77			
a	1	01			
b	x→( )	23			
c	e	12			
d	GO TO ( )	44			

Step	Key	Code	Display x	Display y	Display z
8 0	▲SUB▼	77			
1	b	14			
2	0	00			
3	GO TO ( )	44			
4	3	03			
5	b	14			
6	TO POLAR	62			
7	TO RECT	66			
8	e^x	74			
9	log x	75			
a	↑	27			
b	ENTER EXP	26			
c	1	01			
d	ln x	65			
9 0	×	36			
1	↓	25			
2	IF FLAG	43			
3	x←( )	67			
4	RETURN	77			
5	SET FLAG	54			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	b	14			
9	a	13			
a	IF FLAG	43			
b	STOP	41			
c	STOP	41			
d	RETURN	77			

Storage

Register	Contents
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	

Step	Key	Code	Display x	Display y	Display z
a 0	CHG SIGN	32			
(−) 1	CHG SIGN	32			
2	CHG SIGN	32			
3	CHG SIGN	32			
4	CHG SIGN	32			
5	CHG SIGN	32			
6	CHG SIGN	32			
7	CHG SIGN	32			
8	CHG SIGN	32			
9	CHG SIGN	32			
a	CHG SIGN	32			
b	CHG SIGN	32			
c	CHG SIGN	32			
d	CHG SIGN	32			
b 0	÷	35			
(−) 1	x←( )	67			
2	−	34			
3	a	13			
4	↑	27			
5	ROLL ↓	31			
6	PAUSE	57			
7	PAUSE	57			
8	PAUSE	57			
9	RETURN	77			
a	hyper ▾	67			
b	sin x	70			
c	arc ▾	72			
d	hyper ▾	67			
c 0	sin x	70			
(−) 1	hyper ▾	67			
2	cos x	73			
3	arc ▾	72			
4	hyper ▾	67			
5	cos x	73			
6	hyper ▾	67			
7	tan x	71			
8	arc ▾	72			
9	hyper ▾	67			
a	tan x	71			
b	IF FLAG	43			
c	d	17			
d	6	06			

Step	Key	Code	Display x	Display y	Display z
d 0	GO TO ( )	44			
(−) 1	▲SUB▼	77			
2	+	33			
3	b	14			
4	0	00			
5	RETURN	77			
6	GO TO ( )	44			
7	▲SUB▼	77			
8	+	33			
9	9	11			
a	0	00			
b	RETURN	77			
c	END	46			
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Step	Key	Code	Display x	Display y	Display z
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
a					
b					
c					
d					

Storage	
f	
e	
d	
c	
b	
a	
9	
8	
7	
6	
5	
4	
3	
2	
1	
0	